Alternative approach to the problem of spin and statistics

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Summary: *From infinitesimal 4-rotations via Noether's theorem one obtains the law of conservation of 4-angular momentum. Also, which can easily be seen, from infinitesimal 3-rotations the law of conservation of orbital angular momentum follows analogously to the case of 3-dimensinal space translations, which give the conservation of momentum. Accordingly, as the time translation results in the conservation of energy, if pure time rotation were introduced, the conservation of spin [1] should follow. The conservation here is to be understood as the conservation of fermion or boson properties of particles. Also we gave the explanation why fermions obey Pauli exclusion principle and bosons do not, without ever leaving Quantum Mechanical approach. So this means that we do not need to go over to Quantum Field Theory arguments to prove the so called spin and statistics theorem.*

1. **INTRODUCTION**

Famous Noether's theorem[1] introduces laws of conservation directly in terms of symmetry requirements in Lagrangian (***L***). One of the advantages of this theorem is that it is easily applied to quantized field theories. This is the reason it has been often used in developing new theories as well as in textbooks [3-5].

Yet, as it seems, the potentials of this outstanding theorem are far from being exhausted [see, also, 2, 10]. For instance, we are proposing here the new application of Noether's theorem to the problem of the origin and conservation of spin, drawing, also, the consequences which explain the need for fermions to be described by antisymmetric functions, thus, unlike bosons, being subject to Pauli exclusion principle. Though those results are already obtained [3,6] in the frame of Quantum Field Theory, it could be of certain interest to show their alternative obtaining in the frame of Standard Quantum Theory, because mathematics is straight forward, and the physical interpretation may be fruitful in its innovations.

The line of thought is the following: From infinitesimal, belonging to the continues Lie group, 4-rotations

$$\overline{x}^{\nu} = x^{\nu} + x_{\mu}\delta\Omega^{\nu\mu}, \tag{1}$$

where with $\overline{x}^{\nu}$ we denote new, transformed, 4 - coordinates, the law of conservation of 4 - angular momentum follows (that is, of course, orbital angular momentum and spin are conserved). Also, from infinitesimal 3-rotations the law of conservation of orbital angular

[1] To any S-parametric continuous transformation of field functions and coordinates, which keeps variation of action zero, there correspond S-dynamic invariants (i.e. constant in time combinations of field functions and their derivatives), [3].

momentum should follow [and this can easily be shown,7], as is the case, for instance, for 3-dimensional space translations, which give the conservation of momentum. Accordingly, as the time translation results in the conservation of energy, if the pure time rotation were introduced, some conservation connected with spin should follow. But of course, for rotation one needs more than one dimension. So, our time should have at least two dimensions (and maybe more)[2]. It has been said that such theories have difficulties with causality, but new development of the two-time physics has shown that in the case of two dimensional time the gauge can be found, which resolves the problem of causality and ghosts (negative norm states) [9].Yet it seems "we can assume that the number of times is greater than 2, but than one does not have enough constraints to eliminate all the possible ghosts" [10]. Still, as from our reasoning it follows that rotation of time coordinate will result in explanation of the origin of Pauli principle (see end of paragraph 2), only if two time was parallely used with three time, and so give us a deeper insight into the nature of spin, then we may be forced to reach for the hypothetical solution of the problem of ghosts. Hopping that "search for missing constrains" will give results soon, as the difficulty mentioned in [10], is only of technical nature, and not the principal one.

This difficulty could be overcome, for the moment, by introducing different time which is connected with each set of elementary particles (that is to say with fermions and with bosons), whichis justified by Einsteins implication that each coordinate system carryies its own time with it [11]. So, as we shall soon see, Eq.s (10) and (11) , fermions

[2] At this point we tried to speculate and rotate the time coordinate among space coordinates. But mathematics was strict; no reasonable result could be obtained.

are connected with two-time physics and are originated from that kind of manifold legally [see comment in the previous passage]. Yet, bosons, which are responsible for interactions and, as such, also fundamental to our picture of the world, are carrying with themselves the three-dimensional time which is full of ghosts so we have to treat these dimensions as the ones that are not actual in the everyday physical world, but are under all circumstances bound. As this approach could give us a deeper insight into the nature of spin, we are forced to use this seemingly fictive manifold. Yet the physical values could be divided, in respect of their treatment in Noether's theorem, into spacelike (momentum, angular momentum), and timelike (energy, spin).

The former discussion relates fermions, matter building particles, to the two-time physics, and bosons, the carriers of interaction, to the three-time physics. The two-time physics is being developed by various physicists [see 9,10, and references thereof], and could be considered already established, yet three-time physics still has problems, but obviously, if we want to describe interactions in our world we have to introduce three-dimensional time, one way or another. *Also, this indicates that hypothesis of gravitons having spin 2 cannot be taken seriously into account, because it needs more time dimensions (6!) which is obviously impossible to satisfy.*

There, also, have been attempts to obtain the spin conservation, but based on an extension of phase-space via Grassmannian variables (such an attempt is illustrated in [1]) and not on the extension of notion of time. In [5] the authors touched the problem of using "time approach", though without going through all the consequences. Besides, in nature there is no, strictly speaking, the conservation of spin as there is the conservation of angular momentum, because, for example the spin selection rules are, as one knows, only approximative.

Even so, there is a candidate for conservation of spin. Indeed, elementary particles never change from fermions to bosons, and vice versa. So, this is the conservation to be sought for via Noether's theorem (see, the end of the following paragraph). This is the fact that is being confirmed many times experimentally, and also proven using completely different thinking by Pauli [3,6]. Also, accepting the idea of multidimensional time which every set of elementary particles carries with itself, one can easily explain the problem why only fermions are subject to Pauli exclusion principle {see discussion at the end of the following paragraph) without ever using the results and methods of Quantum Field Theory like in spin-statistics theorem of Pauli.

2. OBTAINING SPIN CONSERVATION VIA NOETHER'S THEOREM

In what follows we shall use the results and notation of [3], except that we shall use Greek indices to denote 4 space-time coordinates (0,1,2,3) and Latin for spatial 3-coordinates (1,2,3). So, we shall start with expression (2.5 in [3])

$$\theta^{\kappa}_{(\nu)}(x) = -\frac{\partial L}{\partial u_{\alpha;x}}(\psi_{\alpha(\nu)} - u_{\alpha;\lambda}X^{\lambda}_{(\nu)}) - L(x)X^{\kappa}_{(\nu)}, \qquad (2)$$

which appears in the action integral obtained for S-parametric transformations (here L(x) is the Lagrangian of the system). Standard procedure is to introduce the infinitesimal 4-

rotations (1), where as the parameters of transformation could be chosen six linearly independent parameters: $\delta\omega^{\nu\mu} = \delta\Omega^{\nu\mu}$, $\nu < \mu$.

After some calculations, and obtaining expressions

$$X^{\kappa}_{\nu\mu} = x_{\mu}\delta^{\kappa}_{\nu} - x_{\nu}\delta^{\kappa}_{\mu}, (\nu \le \mu), \tag{3}$$

and

$$\psi_{\alpha\nu\mu} = A^{\beta}_{\alpha\nu\mu} u_{\beta}(x), \tag{4}$$

where, for vector fields,

$$A^{\beta}_{\kappa\alpha\lambda} = g_{\alpha\kappa}\delta^{\beta}_{\lambda} - g_{\alpha\lambda}\delta^{\beta}_{\kappa}, \kappa \le \lambda, \tag{5}$$

one can get the 4-angular momentum tensor

$$\begin{gathered} M^{\kappa}_{\lambda\mu} = \frac{\partial L}{\partial(\partial u_{\alpha}/\partial x^{k})}\left\{\frac{\partial u_{\alpha}}{\partial x^{\lambda}} x_{\mu} - \frac{\partial u_{\alpha}}{\partial x^{\mu}} x_{\lambda}\right\} + L(x_{\lambda}\delta^{\kappa}_{\mu} - x_{\mu}\delta^{\kappa}_{\lambda}) - \\ \frac{\partial L}{\partial(\partial u_{\alpha}/\partial x^{\kappa})} A^{\beta}_{\alpha\lambda\mu} u_{\beta}(x) = (x_{\mu}T^{\kappa}_{\lambda} - x_{\lambda}T^{\kappa}_{\mu}) - \frac{\partial L}{\partial(\partial u_{\alpha}/\partial x^{\kappa})} A^{\beta}_{\alpha\lambda\mu} u_{\beta}(x). \end{gathered} \tag{6}$$

It is easily seen that the first term in the last part of expression (6) corresponds to an orbital angular momentum of the wave field, and the second part, which shall be denoted in the following manner

$$S^{\kappa}_{\lambda\mu} = -\frac{\partial L}{\partial(\partial u_{\alpha}/\partial x^{\kappa})} A^{\beta}_{\alpha\lambda\mu} u_{\beta}(x), \tag{7}$$

characterizes the polarization properties of the field, and in the quantized case corresponds to the spin of the particle described by the quantized field. Which is already standard, well established result, beyond any doubt.

But to perform decoupling of the orbital momentum and spin in the theory based on assumptions (3-5) is not possible. So we are suggesting going the other way round, i.e. to use methods of obtaining isotopic spin (see [3]), for standard spin.

Let us deal now with rotations in two-dimensional time continuum related to the one set of particles (fermions, as shall be revealed later on, and especially at the end of this section). Since wave functions do not depend explicitly on coordinates of this continuum, and standard coordinates x_κ do not transform under the rotations of two-dimensional time continuum, we shall start with expressions for infinitesimal transformations only for wave functions

$$\overline{u}_\alpha = u_\alpha + \delta u_\alpha, \quad \delta u_\alpha = K^{ij}_{\alpha\beta} u_\beta \delta\omega_{ij}. \tag{8}$$

Here $\delta\omega_{ij}$ are, antisymmetric in indices i, j (=1, 2), infinitesimal angles of rotation of two-dimensional time continuum.

It follows that tensor (6) in this case does not have orbital part, so

$$S^\kappa_i = -\frac{\partial L}{\partial u_{\alpha;\kappa}} K^i_{\alpha\beta} u_\beta, \tag{9}$$

it gives only spin tensor from which it follows that the rotation of two-dimensional time continuum gives conservation of the half integral spin (but conservation here must be taken strictly as conservation of the half integral spin, i.e. ***fermions always stay fermions***):

$$S_i = \int S_i^0 d\boldsymbol{x} = -\int d\boldsymbol{x} \frac{\partial L}{\partial u_{\alpha;0}} K^i_{\alpha\beta} u_\beta , \tag{10}$$

d***x*** being differential of two-dimensional time continuum.

Equation (10) represents the quantity which has only two components (i=1, 2) and is behaving like a spinor, which indicates its connection with half integral spin.

Yet if our time-continuum were three-dimensional our spin tensor should give us when rotated the conservation of the integral spin (i.e. ***bosons always stay bosons***):

$$\mathrm{S}_{\mathrm{ij}} = \int \mathrm{S}^0_{\mathrm{ij}} \mathrm{d}\boldsymbol{x} = -\int \mathrm{d}\boldsymbol{x} \frac{\partial \mathrm{L}}{\partial \mathrm{u}_{\alpha;0}} \mathrm{K}^{\mathrm{ij}}_{\alpha\beta} \mathrm{u}_\beta , \tag{11}$$

d***x*** being differential of three-dimensional time continuum.

Contracting three-dimensional components with antisymmetric tensor of the third order $\varepsilon_{\mathrm{ijp}}$, we obtain components of three-dimensional (pseudo) vector of spin (that is to say, the vector describing bosons):

$$\mathrm{S}_\mathrm{i} = \varepsilon_{\mathrm{ijp}} \mathrm{S}_{\mathrm{jp}} . \tag{12}$$

And according to equations (10) and (11) in both cases (of fermions and of bosons), there is no change of the type of particles from one to another. I. e. an electron is a fermion and it can not be changed. As mentioned already this is no new result [see 3,6], but it is the first time it has been obtained in the frame of Quantum Mechanics via Noether's theorem, and confirming the idea of multi-dimensional time.

But there is more to it. For if one consideres going over from the left hand coordinate system to the right one in the case of two-dimensional manifold, one sees that for this operation are needed one rotation through π and one inversion of the coordinate system, i.e. the inversion of one of its axes, say x. The determinant of the rotation is definitely +1 and of the inversion is –1. These two multiplied give the determinant of the system equal to -1.

Thus in the case of the two-dimensional time manifold one has the antisymmetry of functions involved. So, the spin function is antisymmetric in the case of the half-integral spin. This is in part Pauli principle [6] given in a broader definition, because it is usually the total wave function, including also spatial variables, which is to be either antisymmetric or symmetric.Yet it could be espected that only the spin part of the total wave function influences its symmetry. But, until now, the separate wave functions for spin, i.e. the wave functions which depend only on spin variables. Thus, it is obvious that by eq. (10) the quantity is defined, which has to be described with anticommutative operators.

Also, in exactly the same situation, but now in three-dimensional manifold, one needs a rotation through π and two inversions of the coordinate system. The determinants of three, are, mutatis mutandis, exactly the same as above, so multiplied the three of them give the determinant of the system equal to 1. Analogously to the previous case, in the case of three-dimensional time manifold one has the symmetry of functions. Accordingly, the spin function is symmetric in the case of the integral spin. And that, together with the result for the half-integral spin, gives Pauli principle, without ever using the results and methods of Quantum Field Theory like in spin-statistics theorem of Pauli. Because, without this theorem, until now it was not clear why particles with the half-integral spin are subject to the law of antisymmetricity of functions describing particles (fermions) that have that feature, thus obeying Pauli exclusion principle [8].

3. FINAL REMARKS

We have shown rather striking result, that rotation of time results in conservation of spin. Here should be stressed, once again, that conservation is to be understood as keeping the status: *fermions stay fermions*, and *bosons stay bosons*. Also, this result leads to a deeper insight into the origin of Pauli principle, explaining that two-dimensional time manifold leads to antisymmetric states, logically described by antisymmetrical spin functions, or anticommuting spin operators, and three-dimensional time manifold produces symmetrical states, described by symmetrical spin functions, as this principle states. We are thus producing spin and statistics theorem, without ever leaving the results and methods of pure Quantum Theory.

Of course there are lots of problems open yet, and the most obvious one is the interpretation of fictive multi - dimensional time continuum. But, the extra dimensions in time continuum could be interpreted as bound by the spatial selection rules or energy requirements, and not effective in everyday physical world. It is not an entirely new situation in physics: quark confinement is the example that there are physical entities which could not be measured and yet are underlying very reliable physical theory. Also strings are not observable at the moment, but are revealing many of the, until now, poorly understood features of elementary particles, black holes etc.

This interpretation is strongly supported by mathematics of the problem which is definitely giving the aforementioned results. The problem of spin has been extensively treated in many papers ([12-14] to mention a few), but, as to our knowledge, never on the basis of time extension. For there is a prejudice that spin is exclusively spatial phenomenon, based on the model of spin as intrinsic rotation of a particle, which, of course, does not have a classical analogue, so the "intrinsic rotation" is a mere image.

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