New gnosiological aspects of Noether’s theorem

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Absract: *Here, a rather new gnosiological concept is proposed, or pretty renewed one. Because, Noether’s theorem is not applied to obtain laws of conservation of natural, physical objects and concepts, which is to the general opinion, the main and most important purpose of this theorem, but to a somewhat different goal. I.e. to check the validity of some theories from the point of view of their fitting to describe natural processes which are obeying (as all natural processes are, in fact) the laws of conservation of physical quantities such as energy, momentum, angular momentum etc. As it was mentioned elswhere, interestingly enough, Noether’s theorem was even discovered to check one theory (General Theory of Relativity) in that sense, but this has been ever since forgotten, and this theorem was recgnized as the most useful tool for connecting symetries and conservation in modern physics.*

It is a well known fact that theories describing behavior of atoms in strong laser fields, treat an atom as quantized object and electromagnetic field classically. Yet these theories which combine classical and quantum approach have shown their vitality not only in the case of strong laser fields but also for super strong fields [see, for instance, N.B. Delone, and V.P. Krainov: *Multiphoton Processes in Atoms*, Springer, Berlin, 2000, for so called ADK-theoey], and even when relativistic effects are included. Let such theories be called ″mixed″.

In order to check the reliability of *mixed* theories, concerning conservation laws, here shall be used Noether's theorem. That means that a rather new gnoseologicall concept is proposed, or pretty renewed one. Because, Noether's theorem is not applied to obtain laws of conservation of natural, physical objects and concepts, which is to the general opinion, the main and most important purpose of this theorem, but to a somewhat different goal. I.e. to check the validity of some theories from the point of view of their fitting to describe natural processes which are obeying (as all natural processes are, in fact) the laws of conservation of physical quantities such as energy, momentum, angular momentum etc. As it was mentioned elswhere, interestingly enough, Noether's theorem was even discovered to check one theory (General Theory of Relativity) in that sense, but this has been ever since forgotten, and this theorem was recgnized as the most useful tool for connecting symetries and conservation in modern physics.

Maybe it was too erly then, in the beginning of XX century, to use the cocept of the metatheory in scientific research. This concept was developed by Hilbert at that time and was not so widespread. Emy Noether, who was working with Hilbert, among others, was the most suitable person to move forward with such ideas and to even discover something that can be considered a metatheorem.

Following the definition given elswhere in ***wikipedia***, we could repeat the next formulation of Noether's theorem: To any *s*-parametric continuous transformation of field functions and coordinates, which keeps variation of action zero, there correspond s-dynamic invariants (i.e. constant in time combinations of field functions and their derivatives).

In what follows the results and notation of book by N.N. Bogoliubov and D.V. Shirkov, shall be used, except that Greek indices are denoting 4 space-time coordinates (0,1,2,3) and Latin are denoting spatial 3-coordinates (1,2,3). So, one should start with expression (2.5 in the aforementioned book)

$$\theta_\nu^\kappa = \frac{\partial L}{\partial u_{\alpha;\kappa}}(u_{\alpha;\lambda} X_\nu^\lambda - \Psi_{\alpha\nu}) - L(x) X_\nu^\kappa , \quad (1)$$

which appears in the action integral obtained for s-parametric transformations (here L(x) is the Lagrangian of the system). Standard procedure is to introduce infinitesimal transformation of 4-coordinates, for which infinitesimal space-time rotations could be chosen:

$$x^{\kappa} \to x'^{\kappa} = x^{\kappa} + \delta x^{\kappa} \tag{2}$$

and, for transformation of field functions, one has

$$u_{\alpha}(x) \to u'_{\alpha}(x') = u_{\alpha}(x) + \delta u_{\alpha}(x)\,. \tag{3}$$

Variations δx^{κ} and δu_{α} could be expressed using infinitesimal linearly independent parameters of transformation $\delta\omega^{\nu}$:

$$\delta x^{\kappa} = \sum_{1\le\nu\le s} X^{\kappa}_{\nu}\,\delta\omega^{\nu}, \quad \delta u_{\alpha}(x) = \sum_{1\le\nu\le s} \Psi_{\alpha\nu}\,\delta\omega^{\nu}\,, \tag{4}$$

where s is the number of parameters of transformation and is not restricted by our confining Latin indices to three dimensions, i.e. it could be a number greater than three; $\delta\omega^{\nu}$ are parameters themselves.

So, if one goes back to expression (2), choosing for parameters of transformation values δx^{κ}, one obtains, from (4)

$$X^{\kappa}_{\lambda} = \delta^{\kappa}_{\lambda}, \quad \Psi_{\alpha\lambda} = 0\,. \tag{5}$$

Because of this, θ^{κ}_{ν} from equation (1) becomes a mixed second rank tensor

$$T^{\kappa}_{\nu} = \frac{\partial L}{\partial u_{\alpha;\kappa}(x)}\frac{\partial u_{\alpha}}{\partial x^{\nu}} - L\,\delta^{\kappa}_{\nu}\,, \tag{6}$$

which could be transformed into fully contravariant form

$$T^{\lambda\kappa} = \frac{\partial L}{\partial u_{\alpha;\kappa}(x)}\frac{\partial u_{\alpha}}{\partial x^{\lambda}} - L\,g^{\kappa\lambda}\,. \tag{7}$$

It is shown in Bogolibov's book that integrals over threedimensional configuration space, integrals of the type

$$C_{\nu}(x^{0}) = \int \theta_{\nu}^{0} d\vec{x} \,, \tag{8}$$

are constant in time.

For tensor $T^{\kappa\nu}$ from (7), such an integral would give constant in time 4-vector

$$P^{\lambda} = \int T^{\lambda 0}\, d\vec{x} \,. \tag{9}$$

Zero component of this vector is, in fact, Hamilton function of Classical mechanics, i.e. energy. As time and space, which are involved in transformation (2) that is responsible for the resulting conservation law, are not quantized in all physical theories, except, maybe, when the ultra relativictic effects and energies are involved, we can smoothly connect the part of the theory which uses classical approach with that using quantum. Thus, one has the law of conservation of momentum-energy, and especially of energy in the case of the theories which combine classical and quantum approach (we call these *mixed* theories).

If one wishes to separate the energetical part from that of momentum, one could take only double zero component of tensor (7), i.e. T^{00}

$$E = P^{0} = \int T^{00}\, d\vec{x} = \text{const} \,, \tag{10}$$

obtaining thus the pure energy conservation law, which is the result of time translation, and because the time is not quantized, it is applicable to the *mixed* theories.

In order to illustrate previous arguments further we shall add following. If we chose 4-rotations of space-time instead of translations it would result in conservation of 4-angular momentum (i.e. 3-angular momentum and spin), but this would not be applicable to mixed theories, because the rotations which are continuus in the classical theory, have to be quantized in the quantum theory, so there is no smooth connection between the classical and quantum part of the theory, hence for such theories the conservation of angular momentum and spin is not working.

Thus here has been proven the following

Corollary to Noether's theorem

Physical theories wich combine (″mix″) the quantum and classical approach are subject to the laws of energy and momentum conservation, but are **breaking** *the law of angular momentum - spin conservation.*

This is the shortcoming of such theories, but one that does not affect the results of ADK-theory, for instance, because that theory never operates with spin, and also because the most probable ejected electrons described by that theory have orbital quantum number $l = 0$.

This proof, being all-inclusive, holds for other mixed theories and is not restricted to theories that describe laser-atom interaction. Yet it is applyed here to such theories solely, not only because they are most familiar, but also because one feels that there is no need for further illustration of this rather general principle.

References

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