

To Academic Council of the Doctoral School of Mathematics

EVALUATION REPORT OF THE DOCTORAL DISSERTATION

At the session of Academic Council of the Doctoral School of Mathematics, held on 18/04/2022, at the session of Academic Council of the Faculty of Science in Kragujevac, held on 27/04/2022 (No. 230/X-1), as well as at the session of the Council for Natural and Mathematical Sciences, University of Kragujevac, held on 18/05/2022 (No. IV-01-352/9), I was appointed as a member (chairman) of the commission for evaluation and defense of the doctoral dissertation thesis written by the candidate **Bogdan Pirković** and titled:

"Contribution to the theory of random environment integer-valued autoregressive processes".

The candidate submitted the manuscript of his doctoral dissertation to the Academic Council of the Doctoral School of Mathematics and the Academic Council of the Faculty of Science in Kragujevac for evaluation. After a detailed review of the manuscript, I assessed the quality of the dissertation and pointed out to the candidate the corrections need to be made. The candidate adopted all the proposals and incorporated them into the final version of the dissertation, which allows me to submit the following

REPORT

1. Description of the doctoral dissertation

Doctoral dissertation "Contribution to the theory of random environment integer-valued autoregressive processes" belongs to the field of Mathematical statistics. Results of the previous research in the field of integer-valued autoregressive (INAR) time series created the starting point for conducted research. INAR processes were introduced, by McKenzie (1985) and Al-Osh and Alzaid (1987). The first one who introduced the INAR model that can take both positive and negative values was Freeland (2010) was, while Nastić et al. (2016) introduced the notion of random environment, and alongside with this, the random environment INAR model. Proposed dissertation discusses the development of INAR models that can take values over the entire set of integers, development of random environment INAR models and their possible combining.

Then text of the dissertation has been written in English. It contains 117 pages and consists of four chapters, a conclusion, a list of 50 references and three appendices. An integral part of the dissertation are abstracts in English and Serbian language, a word from the author and his biography. The manuscript itself contains 28 images and 21 tables.

Chapter 1 - INAR models-from the very beginnings to the present day. In the first chapter, the historical development of the observed models is presented. Terms necessary for following the course of the dissertation are defined, as well as the most important results from previous research in this area. This chapter is divided into four sections. In the first section, the definitions of the terms used and the motivation for the INAR model are given. In the second section, the chronological

development of the INAR model from its inception until today is described. The third section contains the definitions and most important properties of the distributions used in the dissertation. In the last, fourth section, the research results of other authors, which will be used in the dissertation, are given.

Original results of the doctoral dissertation are presented in Chapter 2, Chapter 3 and Chapter 4, as well as in Appendix A, Appendix B and Appendix C.

Chapter 2 - Extracting and predicting latent components of the skewed TINAR(1) time series. In the second section of the dissertation time series that can result in values from the entire set of integers. These results are based on Freeland's paper from 2010. In this chapter, INAR time series with Skellam marginal distributions, both symmetric and asymmetric, are described. Their basic characteristics are presented, as well as new methods for identifying and predicting the latent components of these INAR time series, which is part of the original results of the candidate. This chapter concludes by presenting the application of the obtained results to simulated data sets as well as to real-life data.

Chapter 3 - Random environment integer-valued autoregressive model with discrete Laplace marginal distributions. The focus of the third chapter is the new INAR time series model. This is a model of non-stationary time series in a random environment, and it is a model of time series that can take values from the entire set of integers. First, the construction of the model and the most important features are given. After that, the candidate discussed the estimation of the unknown parameters of the model and the behavior of the obtained statistics. At the end, the application of the new model is given both on simulated data and on data from real life.

Chapter 4 - Random environment estimation (RENES) method for generalized random environment INAR models of higher order. In the fourth chapter, a modification of the K-means clustering technique, which is used for the estimation of the environmental state, is described. This adaptation is called the RENNES method. This modified technique proved to be very suitable for evaluating the environment state of realizations corresponding to the general $RrINAR$ time series of higher order. At the beginning, the construction of the RENNES method is presented. After that, the candidate discusses the application of the newly introduced clustering technique on simulated data sets. At the end of the chapter, the application of this method on real data is shown, which confirms the exceptional effectiveness of the RENNES method. Additionally, many results concerning the application of the RENNES method on simulated and real data sets are presented in appendices A, B and C. In this way, the candidate has successfully increased the visibility of the entire dissertation manuscript.

Conclusion. In conclusion, results of the entire dissertation are summarized. In addition, possible directions for further research are presented.

2. Significance and contribution of the doctoral dissertation regarding the current situation in a certain scientific field

Doctoral dissertation "Contribution to the theory of random environment integer-valued autoregressive processes" belongs to the field of Mathematics, and the corresponding subfield is Mathematical statistics. The first random environment INAR model has been introduced in 2016,

and numerous generalizations of this model have emerged later. The given results from the literature created a good starting point for making new research and obtaining original scientific results.

In this dissertation, candidate Bogdan Pirković keeps forcing two research directions. The first one refers to the construction of the random environment integer-valued autoregressive time series that can take both, positive and negative values. The realization of this goal would create new possibilities in integer-valued data modeling. In addition, since the environment state estimation of each individual realization is one of the crucial steps in real-life modeling by usage of models in random environment, the second direction refers to adapting existing clustering procedures in order to make environment state estimates as accurate as possible.

The dissertation presents original results obtained as a product of the candidate's scientific research. First of all, new statistics for extracting and predicting latent components of the skewed TINAR(1) time series have been introduced. Further, a new non-stationary random environment INAR model with discrete Laplace marginal distributions has been constructed. This model enabled more efficient modeling of the data that take both positive and negative values. Furthermore, the dissertation offers the new method for estimating environment states of the data corresponding to the generalized random environment INAR models of higher order. This method, called RENES, has proven to be more efficient than the standard K-means technique mainly used for environment state estimation so far.

All hypotheses rely on well known results from the field of INAR time series. First of all, candidate assumed that is possible to construct new random environment INAR models that can take values over entire set of integers by combining stationary INAR models with positive and negative values and nonnegative random environment INAR models. The second hypothesis assumed that is possible to improve performances of the K-means clustering method, used to estimate environment states of random environment INAR models of higher order, by including estimates of all parameters of the model (distribution parameter, thinning parameter, and model order) in the clustering procedure.

The dissertation contains methods widely used for introduction of new models. First, new models are constructed in the way to satisfy the set criteria regarding the stationarity and allowable model values. Further, new techniques for unknown parameter estimation are derived. New techniques represent an adaptation of familiar techniques to the specific problem of parameter estimation in newly introduced random environment INAR models. The efficiency of estimates such obtained is tested on simulated data. Finally, the model quality is examined on adequately selected real-life data, whereby the level of the model quality is determined using the root mean square (RMS) error value.

3. Evaluation of the originality of results presented in the doctoral dissertation

Based on the insight into the existing research and scientific contributions in the field of Mathematical statistics, I confirm that the doctoral dissertation of the candidate Bogdan Pirković is an original scientific work. The results presented in doctoral dissertation have not been the subject of previous research.

4. Review of achieved scientific results from the candidate's doctoral dissertation

Candidate Bogdan Pirković has two scientific papers in international journals from the SCI list (category M22), as well as one statement from the international conference (category M34). **All scientific attainments that the candidate has achieved so far are based on results presented in his doctoral dissertation.** In this way, the candidate successfully fulfilled all the conditions for defending the dissertation provided by the University of Kragujevac Rulebook on application, preparation and defense of the doctoral dissertation and Doctoral School of Mathematics Rulebook on realization of joint doctoral academic studies. In addition, the candidate has one more scientific paper that relies on results from the dissertation. This paper is in the process of being published.

Scientific papers published in scientific journals of international importance (M20)

- [1] M. S. Đorđević, M. M. Ristić, **B. A. Pirković** (2021) Identifying latent components of the TINAR model, *Filomat*, **35**(13), 4469-4482. (ISSN:0354-5180, **M22**, IF2020=0.844, 197/330, **Category**: Mathematics)
<https://doi.org/10.2298/FIL2113469D>
- [2] **B. A. Pirković**, P. N. Laketa, A. S. Nastić (2021) On generalized random environment INAR models of higher order: estimation of random environment states, *Filomat*, **35**(13), 4545-4576. (ISSN:0354-5180, **M22**, IF2020=0.844, 197/330, **Category**: Mathematics)
<https://doi.org/10.2298/FIL2113545P>

Statements from conferences of international importance (M34)

- [1] **B. A. Pirković**, P. N. Laketa, A. S. Nastić (2021) Random Environment Estimation Method for Generalized Random Environment INAR Models of Higher Order, *Third International Workshop on Nonlinear Analysis and its Applications*, Niš, Serbia, October 13-16, 2021 Book of Abstracts-page 40.

5. Assessment of the fulfillment of the scope and quality of the dissertation in relation to the applied topic

The scope of the research work, goals, hypotheses, and methodological approach in the research realization, planned within the application of dissertation topic, have been fully realized.

6. Applicability of results in theory and practice

The most important theoretical (scientific) results, that represent the scientific contribution of this dissertation, are:

- formulation of statistics for extracting and predicting latent components of the skewed TINAR(1) time series, provided that one realization sequence of the mentioned time series is known;
- estimation of unknown parameters of the skewed TINAR(1) time series;
- construction of a new non-stationary INAR model in random environment that can take both positive and negative values;

- collection of theorems describing the properties of the newly introduced model;
- adaptation of the Yule-Walker method and Conditional least squares method in order to successfully estimate unknown model parameters;
- construction of a new clustering technique used to estimate environment states of realizations corresponding to the generalized random environment INAR models of higher order.

Applicability of results in theory. A generalization of results from the second and third chapter of the doctoral dissertation leads to solving the problem of modeling non-stationary integer-valued data sequences with more complex correlation structure. In addition, the process of adapting the K-means technique, outlined in the fourth chapter, can be applied to other clustering techniques.

Applicability of results in practice. All results presented in the doctoral dissertation are successfully applied to the appropriate real-life data sequences. In this way, the applicability of results in practice is unequivocally confirmed. Beside the applications described in the dissertation text, given results can find practical application in medicine, telecommunications, gambling and many other aspects of life.

7. Ways of presenting the results to the scientific public

Scientific results of the doctoral dissertation were presented to the public through two scientific papers in prominent international journals (category M22). Some of the results are presented in the third paper, which is in the process of being published. Beside this, a portion of results from the doctoral dissertation was presented to the scientific public at international conference *Third International Workshop on Nonlinear Analysis and its Applications*.

CONCLUSION

Dissertation manuscript of the candidate Bogdan Pirković entitled "**Contribution to the theory of random environment integer-valued autoregressive processes**" is an original science achievement in the field of Mathematical statistics under the mentorship of **prof. dr Aleksandar Nastić**.

Original scientific results presented in the dissertation, such as: new statistics for extracting and predicting latent components of the skewed TINAR(1) time series, new non-stationary INAR model in random environment with discrete Laplace marginal distributions and improved method for estimating environment states of the data that correspond to the generalized random environment INAR models of higher order, brought new possibilities in modeling data sequences with integer values and data sequences of non-stationary nature.

The quality of scientific results of the doctoral dissertation has been confirmed by publishing two papers in prominent international journals from the SCI list (category M22) and by the statement from the conference of international importance (category M34).

Accordingly, I think that all scientific, professional and administrative conditions for accepting the proposed doctoral dissertation as an original scientific work have been fulfilled. **In that sense, my position on the proposed doctoral dissertation is: THE DISSERTATION IS ACCEPTED.** In this regard, I suggest to Academic Council of the Doctoral School of Mathematics to approve the public defense of the doctoral dissertation to Bogdan Pirković under the stated title.

In Niš,

July 07, 2022



dr Miodrag Đorđević (**member**)

associate professor, Faculty of Sciences and Mathematics, University of Niš
field of research: Mathematics