*Kragujevac J. Sci.* 32 (2010) 47-56. UDC 541.27:541.61

# THE TRUNCATED RANDIĆ-TYPE INDICES

**Modjtaba Ghorbani**,[a] **Mohammad Ali Hosseinzadeh**,[b] **Ivan Gutman**[c]

[a]*Department of Mathematics, Faculty of Science, Shahid Rajaei Teacher Training University, Tehran, 16785-136, I. R. Iran*

[b]*Department of Mathematical Science, Sharif University of Technology, Tehran, 11365-9415, I. R. Iran*

[c]*Faculty of Science, University of Kragujevac, P. O. Box 60, 34000 Kragujevac, Serbia*

(*Received January 30, 2009*)

ABSTRACT. For a graph *G* = (*V*,*E*), the general Randić index is defined as $R_{-\alpha}(G)=\sum_{uv\in E}\left(\deg_G(u)\deg_G(v)\right)^{-\alpha}$, where α is an arbitrary real number. In this paper we introduce the truncated version of this index, $R_{-\alpha}^{(U)}(G)$, in which the summands pertaining to a selected subset U of vertices of G are abandoned. The truncated higher-order Randić index ${}^{m}R_{-\alpha}^{(U)}(G)$, $m\geq 1$, is also put forward. The indices $R_{-\alpha}^{(U)}(G)$ and ${}^{m}R_{-\alpha}^{(U)}(G)$ of chain graphs are computed.

## 1. Introduction

In 1975 Milan RANDIĆ [1] proposed a topological index $R_{-1/2}$ in order to measure the extent of branching of the carbon-atom skeleton of saturated hydrocarbons. Already Randić noticed that there is a good correlation between his index and several physico-chemical properties of alkanes, such as boiling point, chromatographic retention time, enthalpy of formation, parameters in the Antoine equation for vapor pressure, etc. Later, in 1998 BOLLOBÁS and ERDŐS [2] generalized this index by replacing the exponent $1/2$ by any real number α. For a graph $G=(V(G),E(G))$, the general Randić index $R_{-\alpha}(G)$ of *G* is thus defined as the sum of the terms $\left(\deg_G(u)\deg_G(v)\right)^{-\alpha}$ over all edges *uv* of $G$, where $\deg_G(u)$ denotes the degree of the vertex $u$ of $G$, that is

$$R_{-\alpha}(G) = \sum_{uv \in E(G)} \left(\deg_G(u)\deg_G(v)\right)^{-\alpha}.$$

Let $U = \{u_1, u_2, ..., u_k\}$ be a subset of $V(G)$. We now define a new version of the general Randić index and name it the *truncated Randić index* $R_\alpha^{(U)}$ as

$$R_{-\alpha}^{(u_1,u_2,...,u_k)}(G) = \sum_{\substack{uv \in E(G) \\ u,v \notin U}} \left(\deg_G(u)\deg_G(v)\right)^{-\alpha}$$

i. e.,

$$R_{-\alpha}^{(U)}(G) = \sum_{\substack{uv \in E(G) \\ u,v \notin U}} \left(\deg_G(u)\deg_G(v)\right)^{-\alpha}.$$

The *m*-th order connectivity index of an organic molecule whose molecule graph is $G$ is defined by [3]

$${}^{m}R(G) = \sum_{i_1 i_2 ... i_{m+1} \subset G} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

where the summation runs over all paths $i_1 i_2 ... i_{m+1}$ of length $m$ in $G$.. Similarly we define the *truncated m-th order connectivity index* as:

$${}^{m}R^{(u_1,u_2,...,u_k)}(G) = \sum_{\substack{i_1 i_2 ... i_{m+1} \subset G \\ i_1, i_2, ..., i_{m+1} \notin U}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

i. e.,

$${}^{m}R^{(U)}(G) = \sum_{\substack{i_1 i_2 ... i_{m+1} \subset G \\ i_1, i_2, ..., i_{m+1} \notin U}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}.$$

If $G - U$ denotes the subgraph obtained by deleting the vertices $u_1, u_2, ..., u_k$ from the graph $G$, then it is immediately seen that

$$R_{-\alpha}^{(U)}(G) = \sum_{uv \in E(G-U)} \left(\deg_G(u)\deg_G(v)\right)^{-\alpha}$$

and

$${}^{m}R^{(U)}(G) = \sum_{i_1 i_2 ... i_{m+1} \subset (G-U)} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}.$$

## 2. Main Results and Discussion

In this section we compute the truncated Randić index of the chain graphs. Then we use this method to compute the general Randić index for an infinite class of nanostar dendrimers.

Let $G_i$ $(1 \le i \le n)$ be some graphs and $v_i \in V(G_i)$. A chain graph denoted by $G = G(G_1,...,G_n,v_1,...,v_n)$ is obtained from the union of the graphs $G_i$, $i = 1,2,...,n$, by adding these edges $v_iv_{i+1}$ $(1 \le i \le n-1)$, see Fig. 1. Then $|V(G)| = \sum_{i=1}^{n} |V(G_i)|$ and $|E(G)| = (n-1) + \sum_{i=1}^{n} |E(G_i)|$.

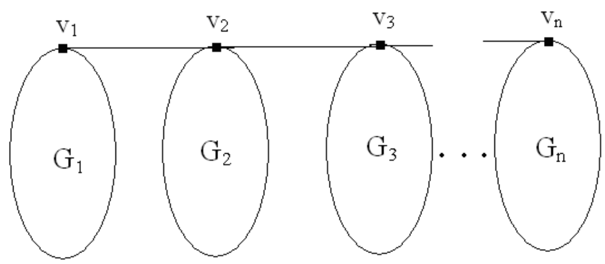


**Fig. 1.** The chain graph $G = G(G_1,...,G_n,v_1,...,v_n)$.

It is worth noting that the above specified class of chain graphs embraces, as special cases, all trees (among which are the molecular graphs of alkanes) and all unicyclic graphs (among which are the molecular graphs of monocycloalkanes). Also the molecular graphs of many polymers and dendrimers are chain graphs.

**Lemma 1.** Suppose that $G = G(G_1, G_2,..., G_n, v_1, v_2,..., v_n)$ is a chain graph. Then:

(i) $G(G_1, G_2,..., G_n, v_1, v_2,..., v_n)$ is connected if and only if $G_i$ $(1 \le i \le n)$ are connected.

(ii) $$\deg_G(a) = \begin{cases} \deg_{G_i}(a) & a \in V(G_i) \text{ and } a \ne v_i \\ \deg_{G_i}(a) + 1 & a = v_i,\ i = 1, n \\ \deg_{G_i}(a) + 2 & a = v_i,\ 2 \le i \le n-1 \end{cases}.$$

**Proof** is straightforward.

**Theorem 2.** The truncated Randić index of the chain graph $G = G(G_1, G_2, v_1, v_2)$ $(v_1, v_2 \ne u_1,...,u_k)$ is:

$$R_{-\alpha}^{(u_1,\ldots,u_k)}(G) = R_{-\alpha}^{(u_1,\ldots,u_k,v_1)}(G_1) + R_{-\alpha}^{(u_1,\ldots,u_k,v_2)}(G_2) + \sum_{i=1}^{2} \sum_{\substack{uv_i \in E(G_i) \\ u \neq u_1,\ldots,u_k}} \left( \left( \deg_{G_i}(v_i) + 1 \right) \deg_{G_i}(u) \right)^{-\alpha} + \left( \left( \deg_{G_1}(v_1) + 1 \right) \left( \deg_{G_2}(v_2) + 1 \right) \right)^{-\alpha}.$$

**Proof.** By using the definition of the truncated Randić index one can see that

$$R_{-\alpha}^{(u_1,\ldots,u_k)}(G) = \sum_{\substack{uv \in E(G) \\ u,v \neq u_1,\ldots,u_k}} \left( \deg_G(u) \deg_G(v) \right)^{-\alpha}$$

$$= \sum_{\substack{uv \in E(G_1) \\ u,v \neq u_1,\ldots,u_k,v_1}} \left( \deg_G(u) \deg_G(v) \right)^{-\alpha} + \sum_{\substack{uv \in E(G_2) \\ u,v \neq u_1,\ldots,u_k,v_2}} \left( \deg_G(u) \deg_G(v) \right)^{-\alpha}$$

$$+ \sum_{\substack{uv_1 \in E(G_1) \\ u \neq u_1,\ldots,u_k}} \left( \left( \deg_{G_1}(v_1) + 1 \right) \deg_{G_1}(u) \right)^{-\alpha} + \sum_{\substack{uv_2 \in E(G_2) \\ u \neq u_1,\ldots,u_k}} \left( \left( \deg_{G_2}(v_2) + 1 \right) \deg_{G_2}(u) \right)^{-\alpha}$$

$$+ \left( \left( \deg_{G_1}(v_1) + 1 \right) \left( \deg_{G_2}(v_2) + 1 \right) \right)^{-\alpha}$$

$$= R_{-\alpha}^{(u_1,\ldots,u_k,v_1)}(G_1) + R_{-\alpha}^{(u_1,\ldots,u_k,v_2)}(G_2) + \sum_{i=1}^{2} \sum_{\substack{uv_i \in E(G_i) \\ u \neq u_1,\ldots,u_k}} \left( \left( \deg_{G_i}(v_i) + 1 \right) \deg_{G_i}(u) \right)^{-\alpha}$$

$$+ \left( \left( \deg_{G_1}(v_1) + 1 \right) \left( \deg_{G_2}(v_2) + 1 \right) \right)^{-\alpha}.$$

**Theorem 3.** If $n \geq 3$ and $v_1,\ldots,v_n \neq u_1,\ldots,u_k$, then for $G = G(G_1, G_2,\ldots,G_n, v_1, v_2,\ldots,v_n)$ it holds:

$$R_{-\alpha}^{(u_1,\ldots,u_k)}(G) = \sum_{i=1}^{n} R_{-\alpha}^{(u_1,\ldots,u_k,v_i)}(G_i) + \sum_{i=1}^{n} \sum_{\substack{uv_i \in E(G_i) \\ u \neq u_1,\ldots,u_k}} \left( \deg_G(v_i) \deg_G(u) \right)^{-\alpha}$$

$$+ \sum_{i=2}^{n-1} \left( \left( \deg_{G_i}(v_i) + 2 \right) \left( \deg_{G_{i+1}}(v_{i+1}) + 2 \right) \right)^{-\alpha}$$

$$+ \left( \left( \deg_{G_1}(v_1) + 1 \right) \left( \deg_{G_2}(v_2) + 2 \right) \right)^{-\alpha}$$

$$+ \cdots + \left( \left( \deg_{G_{n-1}}(v_{n-1}) + 2 \right) \left( \deg_{G_n}(v_n) + 1 \right) \right)^{-\alpha}.$$

**Proof** is analogous as that of Theorem 2.

## 3. Applications

**Example 1.** Consider the graph $G_1$ shown in Fig. 2. It is easy to see that

$$R_{-\alpha}(G_1) = 3\times 2^{2(1-\alpha)} + 3^{1-2\alpha} + 6^{1-\alpha},$$

$$R_{-\alpha}^{(v_1)}(G_1) = R_{-\alpha}^{(v_2)}(G_1) = R_{-\alpha}^{(v_3)}(G_1) = R_{-\alpha}^{(v)}(G_1) = 5\times 2^{1-2\alpha} + 3^{1-2\alpha} + 6^{1-\alpha}$$

and so,

$$R_{-\alpha}^{(v,v)}(G_1) = R_{-\alpha}^{(v_i,v_j)}(G_1) = 2^{3-2\alpha} + 3^{1-2\alpha} + 6^{1-\alpha}$$

for $1 \le i, j \le 3, i \ne j$.

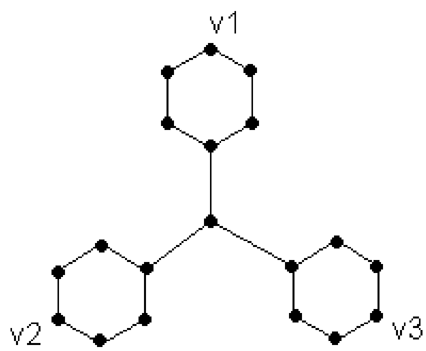


**Fig. 2** The graph of nanostar $G_n$ for n=1.

Consider now the chain graph $G_n = G(G_{n-1}, H_1, v_1, u_1)$, shown in Fig. 2 (for $n=1$) and Fig. 3, respectively. It is easy to see that $H_i \cong G_1$ ($1 \le \mathrm{i} \le$ n-1) and

$$G_n = G(G_{n-1}, H_1, v_1, u_1)$$

$$G_{n-1} = G(G_{n-2}, H_2, v_2, u_2)$$

$$\vdots$$

$$G_{n-i} = G(G_{n-i-1}, H_{i+1}, v_i, u_i)$$

$$\vdots$$

$$G_2 = G(G_1, H_{n-1}, v_{n-2}, u_{n-2})$$

Then by using Theorem 2, we have the following relations:

$$R_{-\alpha}(G_n) = R_{-\alpha}^{(v_1)}(G_{n-1}) + R_{-\alpha}^{(u_1)}(H_1) + 3^{-2\alpha} + 4\times 6^{-\alpha}$$

$$R_{-\alpha}^{(v_1)}(G_{n-1}) = R_{-\alpha}^{(v_2)}(G_{n-2}) + R_{-\alpha}^{(v_1,u_2)}(H_2) + 3^{-2\alpha} + 4\times 6^{-\alpha}$$

$$\vdots$$

$$R_{-\alpha}^{(v_i)}(G_{n-i}) = R_{-\alpha}^{(v_{i+1})}(G_{n-i-1}) + R_{-\alpha}^{(v_i,u_{i+1})}(H_{i+1}) + 3^{-2\alpha} + 4\times 6^{-\alpha}$$

$$\vdots$$

$$R_{-\alpha}^{(v_{n-2})}(G_2) = R_{-\alpha}^{(v_{n-1})}(G_1) + R_{-\alpha}^{(v_{n-2},u_{n-1})}(H_{n-1}) + 3^{-2\alpha} + 4\times 6^{-\alpha}$$

Summation of these relations yields

$$R_{-\alpha}(G_n) = R_{-\alpha}^{(v_{n-1})}(G_1) + R_{-\alpha}^{(u_1)}(H_1) + \sum_{i=2}^{n-1} R_{-\alpha}^{(v_{i-1},u_i)}(H_i) + (n-1)(3^{-2\alpha} + 4\times 6^{-\alpha})$$

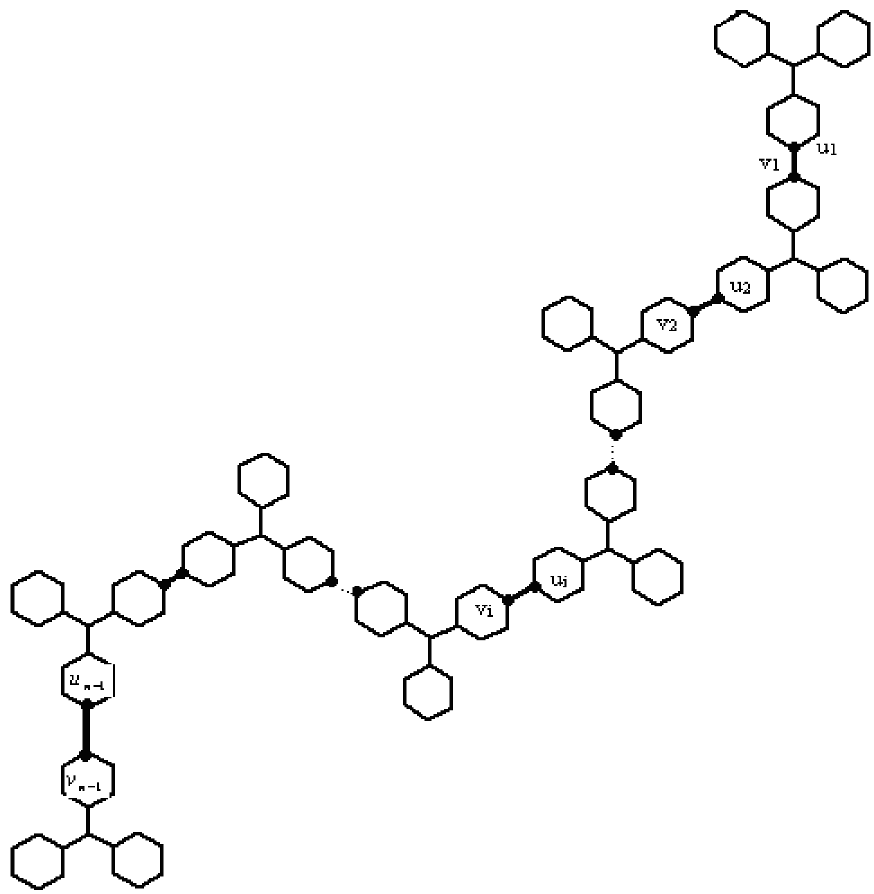


**Fig. 3.** The chain graph $G_n$ and the labeling of its vertices.

and so by using Example 1, it is easy to obtain

$$\begin{aligned} R_{-\alpha}(G_n) &= 2R_{-\alpha}^{(v_1)}(G_1) + (n-2)R_{-\alpha}^{(v_1,v_2)}(G_1) + (n-1)(3^{-2\alpha} + 4\times 6^{-\alpha}) \\ &= (2n+1)2^{2(1-\alpha)} + (4n-1)3^{-2\alpha} + (10n-4)6^{-\alpha}. \end{aligned}$$

In other words we arrived at the following:

**Theorem 4.** Consider the chain graph $G_n = G(G_{n-1}, H_1, v_1, u_1)$, shown in Fig. 3. Then,

$$R_{-\alpha}(G_n) = (2n+1)2^{2(1-\alpha)} + (4n-1)3^{-2\alpha} + (10n-4)6^{-\alpha}.$$

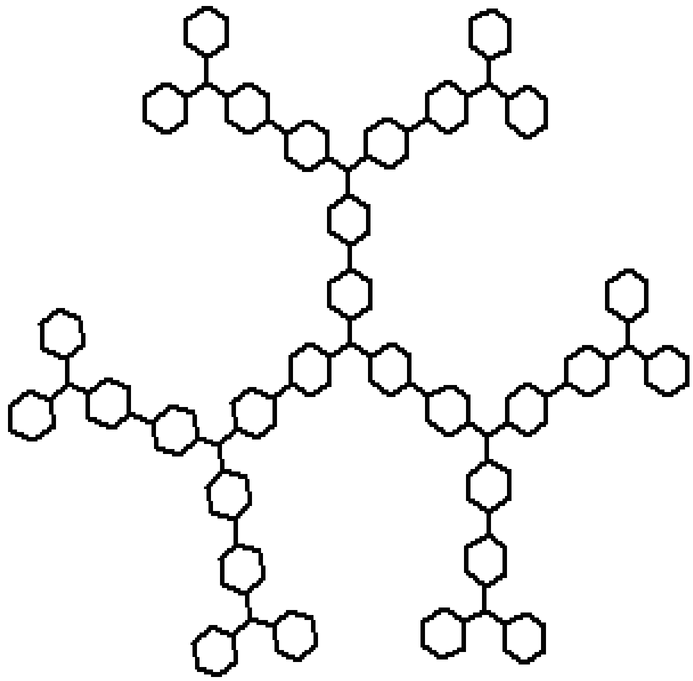

**Fig. 4:** The graph of the nanostar dendrimer D.

**Corollary 4.1.** Consider the nanostar dendrimer D, shown in Fig. 4. Then,

$$R_{-\alpha}(D) = (2n+1)2^{2(1-\alpha)} + (4n-1)3^{-2\alpha} + (10n-4)6^{-\alpha}$$

where $n$ is the number of repetition of the fragment $G_1$.

**Theorem 5.** For the chain graph $G = G(G_1, G_2, ..., G_n, v_1, v_2, ..., v_n)$, $u_1, ..., u_r \neq v_1, ..., v_n$, $1 \leq j, p, q \leq m+1$,

$${}^{m}R^{(u_1,...,u_r)}(G) = \sum_{k=1}^{n} {}^{m}R^{(u_1,...,u_r,v_k)}(G_k) + \sum_{h=0}^{m}\sum_{k=1}^{n-h} \sum_{i_1 i_2 ... i_{m+1}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

in which in the lasr summation we have the conditions $i_1, i_2, ..., i_{m+1} \neq u_1, ..., u_r$. There is a variable j such that $i_j = v_k, ..., i_{j+h} = v_{k+h}$ and for all $p \leq j$ and $q \geq j+h$ we have $i_p \in V(G_k)$ and $i_q \in V(G_{k+h})$, respectively.

**Proof .** By the conditions of the theorem we have:

$${}^{m}R^{(u_1,...,u_r)}(G) = \sum_{\substack{i_1 i_2 ... i_{m+1} \\ i_1, i_2, ..., i_{m+1} \neq u_1, ..., u_r}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)...\deg_G(i_{m+1})}}$$

$$= \sum_{k=1}^{n} \sum_{\substack{i_1 i_2 ... i_{m+1} \\ i_1, i_2, ..., i_{m+1} \neq u_1, ..., u_r, v_k \\ i_j \in V(G_k)}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

$$+\sum_{k=1}^{n}\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_1,i_2,\dots,i_{m+1}\neq u_1,\dots,u_r\\ i_j\in V(G_k)\\ \exists j:i_j=v_k}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

$$+\sum_{k=1}^{n-1}\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_1,i_2,\dots,i_{m+1}\neq u_1,\dots,u_r\\ \exists j:i_j=v_k,i_{j+1}=v_{k+1}\\ \forall p\le j:i_p\in V(G_k)\\ \forall q\ge j+1:i_q\in V(G_{k+1})}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

$$+\sum_{k=1}^{n-2}\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_1,i_2,\dots,i_{m+1}\neq u_1,\dots,u_r\\ \exists j:i_j=v_k,i_{j+1}=v_{k+1},i_{j+2}=v_{k+2}\\ \forall p\le j:i_p\in V(G_k)\\ \forall q\ge j+2:i_q\in V(G_{k+2})}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

$$+\sum_{k=1}^{n-m}\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_1,i_2,\dots,i_{m+1}\neq u_1,\dots,u_r\\ \exists j:i_j=v_k,i_{j+1}=v_{k+1},\dots,i_{j+m}=v_{k+m}\\ \forall p\le j:i_p\in V(G_k)\\ \forall q\ge j+m:i_q\in V(G_{k+m})}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}$$

$$=\sum_{k=1}^{n}{}^{m}R^{(u_1,\dots,u_r,v_k)}(G_k)+\sum_{h=0}^{m}\sum_{k=1}^{n-h}\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_1,i_2,\dots,i_{m+1}\neq u_1,\dots,u_r\\ \exists j:i_j=v_k,\dots,i_{j+h}=v_{k+h}\\ \forall p\le j:i_p\in V(G_k)\\ \forall q\ge j+h:i_q\in V(G_{k+h})}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}.$$

**Example 2.** Consider the graph $G_1$ depicted in Fig. 2. We have the results displayed in Table 1, in which $G=G(G_t,G_1,v_1,v_2)$ and

$$c_m=\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_j\in V(G_t),\,1\le j\le m+1\\ \exists j:i_j=v_1}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}+$$

$$\sum_{\substack{i_1 i_2\dots i_{m+1}\\ i_j\in V(G_1),\,1\le j\le m+1\\ \exists j:i_j=v_2}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}+\sum_{\substack{i_1 i_2\dots i_{m+1}\\ \exists j:i_j=v_1,i_{j+1}=v_2}}\frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}.$$

| $m$ | ${}^{m}R^{(v)}(G_1)$ | ${}^{m}R^{(v,v)}(G_1)$ | $c_m$ |
|---|---|---|---|
| 2 | $\frac{5}{\sqrt{2}}+\frac{11}{2\sqrt{3}}$ | $\frac{7}{2\sqrt{2}}+\frac{11}{2\sqrt{3}}$ | $\frac{4}{3\sqrt{2}}+\frac{3}{\sqrt{3}}$ |
| 3 | $2+\frac{9}{\sqrt{6}}$ | $\frac{3}{2}+\frac{8}{\sqrt{6}}$ | $2+\frac{2}{\sqrt{6}}$ |

| | | | |
|---|---|---|---|
| 4 | $\frac{7}{6\sqrt{2}}+\frac{73}{12\sqrt{3}}$ | $\frac{7}{12\sqrt{2}}+\frac{61}{12\sqrt{3}}$ | $\frac{8}{3\sqrt{2}}+\frac{11}{6\sqrt{3}}$ |
| 5 | $\frac{1}{3}+\frac{25}{3\sqrt{6}}$ | $\frac{1}{6}+\frac{37}{6\sqrt{6}}$ | $2+\frac{8}{3\sqrt{6}}$ |
| 6 | $\frac{1}{3\sqrt{2}}+\frac{8}{3\sqrt{3}}$ | $\frac{1}{6\sqrt{2}}+\frac{5}{3\sqrt{3}}$ | $\frac{16}{9\sqrt{2}}+\frac{22}{9\sqrt{3}}$ |
| 7 | $\frac{10}{3\sqrt{6}}$ | $\frac{5}{3\sqrt{6}}$ | $\frac{13}{9}+\frac{28}{9\sqrt{6}}$ |

**Table 1.**

Now consider the chain graph $G_n = G(G_{n-1}, H_1, v_1, u_1)$ shown in Fig. 3. One can see that for $2 \le m \le 7$

$$\begin{aligned}
{}^{m}R(G_n) &= {}^{m}R^{(v_1)}(G_{n-1}) + {}^{m}R^{(u_1)}(H_1) + c_m \\
{}^{m}R^{(v_1)}(G_{n-1}) &= {}^{m}R^{(v_2)}(G_{n-2}) + {}^{m}R^{(v_1,u_2)}(H_2) + c'_m \\
&\vdots \\
{}^{m}R^{(v_i)}(G_{n-i}) &= {}^{m}R^{(v_{i+1})}(G_{n-i-1}) + {}^{m}R^{(v_i,u_{i+1})}(H_{i+1}) + c'_m \\
&\vdots \\
{}^{m}R^{(v_{n-2})}(G_2) &= {}^{m}R^{(v_{n-1})}(G_1) + {}^{m}R^{(v_{n-2},u_{n-1})}(H_{n-1}) + c'_m
\end{aligned}$$

where for 1 ≤ i ≤ n-1 and $G_{n-i} = G(G_{n-i-1}, H_{i+1}, v_i, u_i)$,

$$\begin{aligned}
c'_m = &\sum_{\substack{i_1 i_2 \ldots i_{m+1} \\ i_j \in V(G_{n-i-1}),\, 1\le j\le m+1 \\ \exists j: i_j = v_i}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}} \\
&+ \sum_{\substack{i_1 i_2 \ldots i_{m+1} \\ i_j \in V(H_{i+1}),\, 1\le j\le m+1 \\ i_j \ne v_{i+1} \\ \exists j: i_j = u_i}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}} \\
&+ \sum_{\substack{i_1 i_2 \ldots i_{m+1} \\ i_j \ne v_{i+1},\, 1\le j\le m+1 \\ \exists j: i_j = u_i,\, i_{j+1} = v_i}} \frac{1}{\sqrt{\deg_G(i_1)\deg_G(i_2)\cdots\deg_G(i_{m+1})}}.
\end{aligned}$$

Because $2 \le m \le 7$, it is easy to see that $c_m = c'_m$ for $u_1 \equiv u_2 \equiv v$. Therefore, by summation of these relations we get:

$$ {}^{m}R(G_n) = 2\times {}^{m}R^{(v)}(G_1) + (n-2)\times {}^{m}R^{(v,v)}(G_1) + c_m + (n-2)c'_m . $$

By this we have proven the following:

**Theorem 6.** For the nanostar dendrimer $G_n$ shown in Fig. 4,

$$ {}^{m}R(G_n) = 2\times {}^{m}R^{(v)}(G_1) + (n-2)\times {}^{m}R^{(v,v)}(G_1) + (n-1)c_m $$

in which for $G = G(G_t, G_1, u_1, u_2)$, $c_m$ is same as above, and $u_1 \equiv u_2 \equiv v$.

The *m*-connectivity indices of $G_n$ for 2≤ m ≤7 are given in Table 2.

| $m$ | ${}^{m}R(G_n)$ | $m$ | ${}^{m}R(G_n)$ |
|---|---|---|---|
| 2 | $\frac{29n+10}{6\sqrt{2}}+\frac{17n-6}{2\sqrt{3}}$ | 5 | $\frac{13n-10}{6}+\frac{53n+10}{6\sqrt{6}}$ |
| 3 | $\frac{7n-2}{2}+\frac{10n}{\sqrt{6}}$ | 6 | $\frac{35n-26}{18\sqrt{2}}+\frac{37n-4}{9\sqrt{3}}$ |
| 4 | $\frac{13n-6}{4\sqrt{2}}+\frac{83n+2}{12\sqrt{3}}$ | 7 | $\frac{13n-13}{9}+\frac{43n+2}{9\sqrt{6}}$ |

**Table 2.**

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