

## ELASTIC-PLASTIC TRANSITIONAL STRESSES DISTRIBUTION AND DISPLACEMENT FOR TRANSVERSELY ISOTROPIC CIRCULAR DISC WITH INCLUSION SUBJECT TO MECHANICAL LOAD

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(Received September 6, 2014)

**ABSTRACT.** Seth's transition theory is applied to the problems of mechanical load in a thin rotating disc by finite deformation. Neither the yield criterion nor the associated flow rule is assumed here. The results obtained here are applicable to compressible materials. If the additional condition of incompressibility is imposed, then the expression for stresses corresponds to those arising from Tresca yield condition. It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. Effect of mechanical load in a rotating disc with inclusion made of isotropic material as well as transversely isotropic materials increase the values of angular speed yield at the internal surface. With the introduction of mechanical load rotating disc made of Beryl material required maximum radial stress as compare to disc made of Mg and Brass materials at the internal surface.

**Keywords:** Transversely isotropic, disc, shaft, stresses, displacement, yielding.

### INTRODUCTION

This paper is concerned with finitesimal deformation of rotating thin circular disk made of transversely isotropic material. Finitesimal deformation consideration of problems of elasticity is often not easy to solve. There are many applications of rotating disks in science and engineering. As typical examples, we mention, steam and gas turbines, rotors, and flywheels. In the design of modern structures, increasing use is being made of materials which are transversely isotropic. The analysis of stress distribution in the circular disk rotating is important for a better understanding of the behavior and optimum design of structures. In the context of small deformation theory, the solutions for this problem of rotating disks made of isotropic material can be found in the most standard text books [1-4]. The analysis of thin rotating discs made of isotropic material has been discussed extensively by TIMOSHENKO and GOODIER [5] in the elastic range and by CHAKRABARTY [6] and HEYMAN [7] for the plastic range. Their solution for the problem of fully plastic state does not involve the plane stress condition, that is to say, we can obtain the same stresses and angular velocity required by the disc to become fully plastic without using the plane stress condition (i.e.  $T_{zz}=0$ ). A. P.

AKINOLA *et al.* [8] solved large deformation of transversely isotropic elastic thin circular disk in rotation. THAKUR [9] solved problems in infinitesimal deformation in a transversely isotropic thin rotating disc with rigid shaft by using Seth's transition theory. This theory [10] does not required any assumptions like an yield condition, incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. It utilizes the concept of generalized strain measure and asymptotic solution at critical points or turning points of the differential equations defining the deformed field and has been successfully applied to a large number of problems [11-16]. SETH [10] has defined the generalized principal strain measure as:

$$e_{ii}^A = \int_0^{e_{ii}^A} \left[ 1 - 2 e_{ii}^A \right]^{\frac{n}{2}-1} d e_{ii}^A = \frac{1}{n} \left[ 1 - \left( 1 - 2 e_{ii}^A \right)^{\frac{n}{2}} \right], \quad (i = 1, 2, 3) \quad (1)$$

where  $n$  is the measure and  $e_{ii}^A$  are the Almansi finite strain components. In this research paper, we investigate the problem of elastic-plastic transitional stresses distribution and displacement for transversely isotropic circular disc with inclusion subjected to mechanical load by using Seth's transition theory. Results have been discussed numerically and depicted graphically.

## MATHEMATICAL MODEL

We consider a thin disc of constant density with central bore of radius  $a$  and external radius  $b$ . The annular disc is mounted on a rigid shaft. The disc is rotating with angular speed  $\omega$  about an axis perpendicular to its plane and passed through the center. The thickness of disc is assumed to be constant and is taken to be sufficiently small so that it is effectively in a state of plane stress, that is, the axial stress  $T_{zz}$  is zero.

### Boundary Condition:

The disk considered in the present study having constant density and mounted on rigid shaft. The inner surface of the disk is assumed to be fixed to a shaft. The outer surface of the disk is applied mechanical load. Thus, the boundary conditions of the problem are given by:

$$\begin{aligned} \text{(i)} \quad & r = a, \quad u = 0 \\ \text{(ii)} \quad & r = b, \quad T_{rr} = T_0 \end{aligned} \quad (2)$$

where  $u, T_0$  and  $T_{rr}$  denote displacement, Load and stress along the radial direction.

### Formulation of Problem:

The components of displacement in cylindrical polar co-ordinates are given by [11]:

$$u = r(1 - \beta); \quad v = 0; \quad w = dz \quad (3)$$

where  $\beta$  is position function, depending on  $r = \sqrt{x^2 + y^2}$  only, and  $d$  is a constant. The finite strain components are given by [10] as:

$$e_{rr}^A = \frac{1}{2} \left[ 1 - (\beta + r\beta')^2 \right], \quad e_{\theta\theta}^A = \frac{1}{2} \left[ 1 - \beta^2 \right], \quad e_{zz}^A = \frac{1}{2} \left[ 1 - (1-d)^2 \right], \quad e_{r\theta}^A = e_{\theta z}^A = e_{zr}^A = 0. \quad (4)$$

where  $\beta' = d\beta/dr$  and meaning of superscripts "A" is Almansi. By substituting equation (4) into equation (1), the generalized components of strain are:

$$e_{rr} = \frac{1}{n} \left[ 1 - (\beta + r\beta')^n \right], e_{\theta\theta} = \frac{1}{n} \left[ 1 - \beta^n \right], e_{zz} = \frac{1}{n} \left[ 1 - (1-d)^n \right], e_{r\theta} = e_{\theta z} = e_{zr} = 0. \quad (5)$$

The stress-strain relations for transversely isotropic material are:

$$\begin{aligned} T_{rr} &= C_{11}e_{rr} + (C_{11} - 2C_{66})e_{\theta\theta} + C_{13}e_{zz}, T_{\theta\theta} = (C_{11} - 2C_{66})e_{rr} + C_{11}e_{\theta\theta} + C_{13}e_{zz}, \\ T_{zz} &= C_{13}e_{rr} + C_{13}e_{\theta\theta} + C_{33}e_{zz} = 0, T_{zr} = T_{\theta z} = T_{r\theta} = 0. \end{aligned} \quad (6)$$

Using equation (4) in equation (6), the strain components in terms of stresses are obtained as:

$$\begin{aligned} e_{rr} &\equiv \frac{\partial u}{\partial r} - \frac{1}{2} \left( \frac{\partial u}{\partial r} \right)^2 = \frac{1}{2} \left[ 1 - (r\beta' + \beta)^2 \right] = \frac{1}{E} \left[ T_{rr} - \left( \frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2} \right) T_{\theta\theta} \right], \\ e_{\theta\theta} &\equiv \frac{u}{r} - \frac{u^2}{2r^2} = \frac{1}{2} \left[ 1 - \beta^2 \right] = \frac{1}{E} \left[ T_{\theta\theta} - \left( \frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2} \right) T_{rr} \right], \\ e_{zz} &\equiv \frac{\partial w}{\partial z} - \frac{1}{2} \left( \frac{\partial w}{\partial z} \right)^2 = \frac{1}{2} \left[ 1 - (1-d)^2 \right] = -\frac{1}{E} \left( \frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2} \right) [T_{rr} - T_{\theta\theta}], \\ e_{r\theta} &= e_{\theta z} = e_{zr} = 0, \end{aligned} \quad (7)$$

where  $E = 4C_{66} \left( \frac{C_{11}C_{33} - C_{13}^2 - C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2} \right)$  is *Young's modulus*. By substituting equations (5) into equations (6), we get:

$$\begin{aligned} T_{rr} &= \frac{A}{n} \left[ 2 - \beta^n \left\{ 1 + (1+P)^n \right\} \right] - 2 \frac{C_{66}}{n} \left[ 1 - \beta^n \right], T_{\theta\theta} = \frac{A}{n} \left[ 2 - \beta^n \left\{ \frac{1 + (1+P)^n}{(1+P)^n} \right\} \right] - 2 \frac{C_{66}}{n} \left[ \frac{1 - \beta^n}{(1+P)^n} \right], \\ T_{r\theta} &= T_{\theta z} = T_{zr} = T_{zz} = 0. \end{aligned} \quad (8)$$

where  $A = C_{11} - (C_{13}^2/C_{33})$ .

Equations of equilibrium are all satisfied except:

$$\frac{d}{dr} ( rT_{rr} ) - T_{\theta\theta} + \rho\omega^2 r^2 = 0 \quad (9)$$

where  $\rho$  is the density of material. By substituting equations (8) into equation (9), we get a non-linear differential equation with respect to  $\beta$ :

$$\beta^{n+1} (1+P)^{n-1} \frac{dP}{d\beta} = \left[ \frac{\rho\omega^2 r^2}{A} + \beta^n \left\{ \frac{2C_{66}}{nA} \left[ 1 + nP - (1+P)^n - P \left\{ 1 + (1+P)^n \right\} \right] \right\} \right] \quad (10)$$

where  $r\beta' = \beta P$  ( $P$  is function of  $\beta$  and  $\beta$  is function of  $r$ ). The transition points of  $\beta$  in equation (10) are  $P \rightarrow -1$  and  $P \rightarrow \pm\infty$ .

## SOLUTION THROUGH PROBLEM

It has been shown that the asymptotic solution through the principal stress [11-23] leads from elastic to plastic state at the transition point  $P \rightarrow \pm\infty$ . If the transition function  $R$  is defined as:

$$R \equiv T_{\theta\theta} = \frac{A}{n} \left[ 2 - \beta^n \{1 + (1+P)^n\} \right] - 2 \frac{C_{66}}{n} \left[ 1 - \beta^n (1+P)^n \right] \quad (11)$$

Taking the logarithmic differentiation and substitute the value of  $dP/d\beta$  from equation (10) in equation (11), we get:

$$\frac{d}{dr} (\log R) = -\frac{A}{rR} \left( \begin{aligned} & -\beta^n P \{1 + (1+P)^n\} + \frac{2C_{66}}{nA} \beta^n - \frac{2C_{66}}{nA} \beta^n (1+P)^n + \frac{4C_{66}}{A} P \beta^n \\ & + \frac{\rho\omega^2 r^2}{A} - \frac{4C_{66}^2}{nA^2} \beta^n \{1 - (1+P)^n\} - \frac{4C_{66}^2}{A^2} P \beta^n - \frac{2C_{66}}{A^2} \rho\omega^2 r^2 \end{aligned} \right) \quad (12)$$

Asymptotic value of equation (12) as  $P \rightarrow \pm\infty$  and integrating, we get

$$R = K_1 r^{-C_2} \quad (13)$$

where  $C_2 = 2C_{66}/A$ ,  $A = C_{11} - (C_{13}^2/C_{33})$  and  $K_1$  is a constant of integration, which can be determined by the boundary condition.

Using equation (13) in equation (11), we have

$$T_{\theta\theta} = K_1 r^{-C_2} \quad (14)$$

By substituting equation (14) into equation (9), one gets:

$$T_{rr} = \frac{K_2}{r} + K_1 \frac{r^{-C_2}}{1-C_2} - \frac{\rho\omega^2 r^2}{3} \quad (15)$$

where  $K_2$  is a constant of integration, which can be determined by the boundary condition.

Substituting equation (14) and (15) in second equation of (7), we get:

$$\beta = \sqrt{1 - \frac{2(1-C_2)}{E} \left[ \frac{\rho\omega^2 r^2}{3} - \frac{K_2}{r} \right]} \quad (16)$$

Substituting equation (16) in equation (3), we get

$$u = r - r \sqrt{1 - \frac{2(1-C_2)}{E} \left[ \frac{\rho\omega^2 r^2}{3} - \frac{K_2}{r} \right]}, \quad (17)$$

where  $1-C_2 = \left( \frac{C_{11}C_{33} - C_{13}^2 - 2C_{66}C_{33}}{C_{11}C_{33} - C_{13}^2} \right)$ ,  $E = 2C_{66}(2-C_2)$ . is the *Young's modulus*. By

applying boundary conditions (2) in equations (15) and (17), we gets:  $K_2 = \rho\omega^2 a^3 / 3$ ,

$K_1 = \frac{1-C_2}{b^{-C_2}} \left[ T_0 + \frac{\rho\omega^2}{3b} (b^3 - a^3) \right]$ . Substituting values of constants  $K_1$  and  $K_2$  in equations

(14), (15), and (17) respectively, we get the transitional stresses and displacement as:

$$T_{\theta\theta} = (1-C_2) \left[ T_0 + \frac{\rho\omega^2}{3b} (b^3 - a^3) \right] \left( \frac{r}{b} \right)^{-C_2} \quad (18)$$

$$T_{rr} = \frac{\rho\omega^2}{3r} \left[ (b^3 - a^3) \left( \frac{r}{b} \right)^{1-C_2} + a^3 - r^3 \right] + T_0 \left( \frac{r}{b} \right)^{-C_2} \quad (19)$$

$$u = r - r \sqrt{1 - \frac{2(1-C_2)\rho\omega^2(r^3 - a^3)}{3Er}}. \quad (20)$$

From equations (18) and (19), we get

$$T_{rr} - T_{\theta\theta} = \frac{\rho\omega^2}{3r} \left[ (b^3 - a^3) \left( \frac{r}{b} \right)^{1-C_2} C_2 + a^3 - r^3 \right] + T_0 \left( \frac{r}{b} \right)^{-C_2} C_2 \quad (21)$$

**Initial Yielding.** From equation (21), it is seen that  $|T_{rr} - T_{\theta\theta}|$  is maximum at the internal surface (that is at  $r = a$ ), therefore yielding will take place at the internal surface of the disc and equation (21) gives:

$$|T_{rr} - T_{\theta\theta}|_{r=a} = \left| \frac{\rho\omega^2}{3a} (b^3 - a^3) \left( \frac{a}{b} \right)^{1-C_2} C_2 + T_0 \left( \frac{a}{b} \right)^{-C_2} C_2 \right| \equiv Y(\text{say})$$

where  $Y$  is the yielding stress. Angular velocity  $\omega_i$  required for initial yielding is given by:

$$\Omega_i^2 = \frac{\rho\omega_i^2 b^2}{Y} = \left| \frac{3ab^2}{(b^3 - a^3)C_2 (a/b)^{1-C_2}} - \left( \frac{T_0}{Y} \right) \frac{3ab^2}{(b^3 - a^3)(a/b)} \right| \quad (22)$$

$$\text{and } \omega_i = \frac{\Omega_i}{b} \sqrt{\frac{Y}{\rho}}.$$

**Fully-plastic state.** The disc become fully plastic state ( $C_2 \rightarrow 1/2; C \rightarrow 0$ ) at the external surface and equations (21) becomes:

$$|T_{rr} - T_{\theta\theta}|_{r=b} = \left| \frac{\rho\omega^2}{6b} (a^3 - b^3) + \frac{T_0}{2} \right| \equiv Y^*(\text{say})$$

where  $Y^*$  is the yielding stress. The angular velocity  $\omega_f$  for fully-plastic state is given by:

$$\Omega_f^2 = \frac{\rho\omega_f^2 b^2}{Y^*} = \left| \frac{6b^3}{(a^3 - b^3)} \left[ 1 - \frac{1}{2} \left( \frac{T_0}{Y^*} \right) \right] \right| \quad (23)$$

where  $\omega_f = \frac{\Omega_f}{b} \sqrt{\frac{Y^*}{\rho}}$ . We introduce the following non-dimensional components as:

$R = r/b$ ,  $R_0 = a/b$        $\sigma_r = T_{rr}/Y$ ,  $\sigma_\theta = T_{\theta\theta}/Y$ ,  $H^* = Y^*/E$ ,  $H = Y/E$ ,  $\sigma_0 = T_0/Y$  and  $U = u/b$ . Elastic-plastic transitional stresses, displacement and angular speed from equations (18),(19), (20) and (22) in non-dimensional form become:

$$\sigma_\theta = (1 - C_2) \left[ \sigma_0 + \frac{\Omega_i^2}{3} (1 - R_0^3) \right] R^{-C_2}, \sigma_r = \frac{\Omega_i^2}{3R} \left[ (1 - R_0^3) R^{1-C_2} + R_0^3 - R^3 \right] + \sigma_0 R^{-C_2},$$

$$U = R - R \sqrt{1 - \frac{2(1 - C_2)H\Omega_i^2(R^3 - R_0^3)}{3R}} \quad (24)$$

$$\text{and } \Omega_i^2 = \frac{\rho \omega_i^2 b^2}{Y} = \left| \frac{3R_0^{C_2}}{(1-R_0^3)C_2} - \frac{3\sigma_0}{(1-R_0^3)} \right| \quad (25)$$

Stresses, displacement and angular speed for fully- plastic state ( $C_2 \rightarrow 1/2; C \rightarrow 0$ ) are obtained from equation (24) and (23) become:

$$\sigma_\theta = \frac{1}{2\sqrt{R}} \left[ \sigma_0 + \frac{\Omega_f^2}{3} (1-R_0^3) \right], \sigma_r = \frac{\Omega_f^2}{3R} \left[ (1-R_0^3)\sqrt{R} + R_0^3 - R^3 \right] + \frac{\sigma_0}{\sqrt{R}},$$

$$U_f = R - R \sqrt{1 - \frac{H\Omega_f^2(R^3 - R_0^3)}{3R}}$$

$$\text{and } \Omega_f^2 = \frac{\rho \omega_f^2 b^2}{Y^*} = \left| \frac{6}{(1-R_0^3)} \left( 1 - \frac{1}{2} \sigma_0 \right) \right| \quad (26)$$

**Isotropic Case.** For isotropic materials, the material constants reduce to two only, *i.e.*  $C_{11} = C_{22} = C_{33}$ ,  $C_{12} = C_{21} = C_{13} = C_{31} = C_{23} = C_{32} = (C_{11} - 2C_{66})$ , and  $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ . In term of constants  $\lambda$  and  $\mu$ , these can be written as:  $C_{12} = \lambda$ ,  $C_{11} = \lambda + 2\mu$  and

$$C_{66} = \frac{1}{2}(C_{11} - C_{12}) \equiv \mu \quad (27)$$

Elastic-plastic transitional stresses are obtained by using equation (27) in equations (18) - (20) and (22) in non dimensional form become:

$$\sigma_\theta = \frac{(1-C)}{(2-C)} \left[ \sigma_0 + \frac{\Omega_i^2}{3} (1-R_0^3) \right] R^{-1/(2-C)}, \sigma_r = \frac{\Omega_i^2}{3R} \left[ (1-R_0^3)R^{1-C/2-C} + R_0^3 - R^3 \right] + \sigma_0 R^{-1/(2-C)},$$

$$U = R - R \sqrt{1 - \frac{2(1-C)H\Omega_i^2(R^3 - R_0^3)}{3(2-C)R}} \quad (28)$$

$$\text{and } \Omega_i^2 = \frac{\rho \omega_i^2 b^2}{Y} = \left| \frac{3(2-C)R_0^{1/(2-C)}}{(1-R_0^3)} - \frac{3\sigma_0}{(1-R_0^3)} \right| \quad (29)$$

where  $C = 2\mu/(\lambda + 2\mu)$ ,  $1 - C_2 = (1 - C/2 - C)$ .

**Fully-plastic state (isotropic case).** For fully plastic state ( $C \rightarrow 0$ ), equations (28) and (29) become:

$$\sigma_\theta = \frac{1}{2} \left[ \sigma_0 + \frac{\Omega_i^2}{3} (1-R_0^3) \right] R^{-1/2}, \sigma_r = \frac{\Omega_i^2}{3R} \left[ (1-R_0^3)R^{1/2} + R_0^3 - R^3 \right] + \sigma_0 R^{-1/2}, \quad (30)$$

$$U_f = R - R \sqrt{1 - \frac{H\Omega_i^2(R^3 - R_0^3)}{3R}} \quad (31)$$

The disc become fully plastic state ( $C_2 \rightarrow 1/2$  or  $C \rightarrow 0$ ) at the external surface and equations (21) becomes:

$$\Omega_f^2 = \frac{\rho \omega_f^2 b^2}{Y^*} = \left| \frac{6b^3}{(a^3 - b^3)} \left[ 1 - \frac{1}{2} \left( \frac{T_0}{Y^*} \right) \right] \right|$$

where  $R_0 = a/b$  and  $\omega_f = \frac{\Omega_f}{b} \sqrt{\frac{Y^*}{\rho}}$ .

Table 1. Elastic constants  $C_{ij}$  (in units of  $10^{10} N/m^2$ )

Materials	$C_{44}$	$C_{11}$	$C_{12}$	$C_{13}$	$C_{33}$
Transversely Isotropic Material ( $C_2=0.69$ , Beryl)	0.883	2.746	0.980	0.674	4.69
Isotropic Material ( $\sigma=0.33$ , $C_2 = 0.50$ , Brass)	0.999997	3.0	1.0	1.0	3.0
Transversely Isotropic Material ( $C_2=0.64$ , Magnesium)	1.64	5.97	2.62	2.17	6.17

Table 2. Angular speed required for initial yielding and fully plastic state

	Materials $C_2$	Load $\sigma_0$	Angular Speed required for initial yielding $\Omega_i^2$	Angular Speed required for fully-plastic state $\Omega_f^2$	Percentage increase in Angular speed $\left( \sqrt{\frac{\Omega_f^2}{\Omega_i^2}} - 1 \right) \times 100$	
0.5 < R < 1	Magnesium $C_2 = 0.64$	Transversely Isotropic Material	0	3.437748	6.857143	41.23248 %
			1	6.8663194	3.428571	-29.3366 %
			1.9	9.952034	0.342857	-81.439 %
	Beryl $C_2 = 0.69$	Transversely Isotropic Material	0	3.080019	6.857143	49.20896
			1	6.508591	3.428571	-27.4206
			1.9	9.594305	0.342857	-81.0962
	Brass $C_2 = 0.5$	Isotropic Material	0	4.848732	6.857143	18.92072%
			1	8.277304	3.428571	-35.6406 %
			1.9	11.36303	0.342857	-82.6296 %

## NUMERICAL ILLUSTRATION AND DISCUSSION

As a numerical example, elastic constants  $C_{ij}$  have been given in Table 1 for transversely isotropic materials (Magnesium and Beryl) [19] and isotropic material [20] (Brass,  $\sigma=0.33$ ). Curves have been drawn in figure 1 between angular speed  $\Omega_i^2$  required for initial yielding along the radii ratios  $R_0 = a/b$ . It has been observed that rotating disc made of isotropic material required higher angular speed to yield at the internal surface as compared to disc made of transversely isotropic materials. Effect of mechanical load in a rotating disc with inclusion made of isotropic material as well as transversely isotropic materials increase the values of angular speed yield at the internal surface. It can also be seen from Table 2, that rotating disc made of transversely isotropic materials (i.e. Mg and Beryl) required high

percentage increase in angular speed to become fully plastic as compared to disc made of isotropic material.

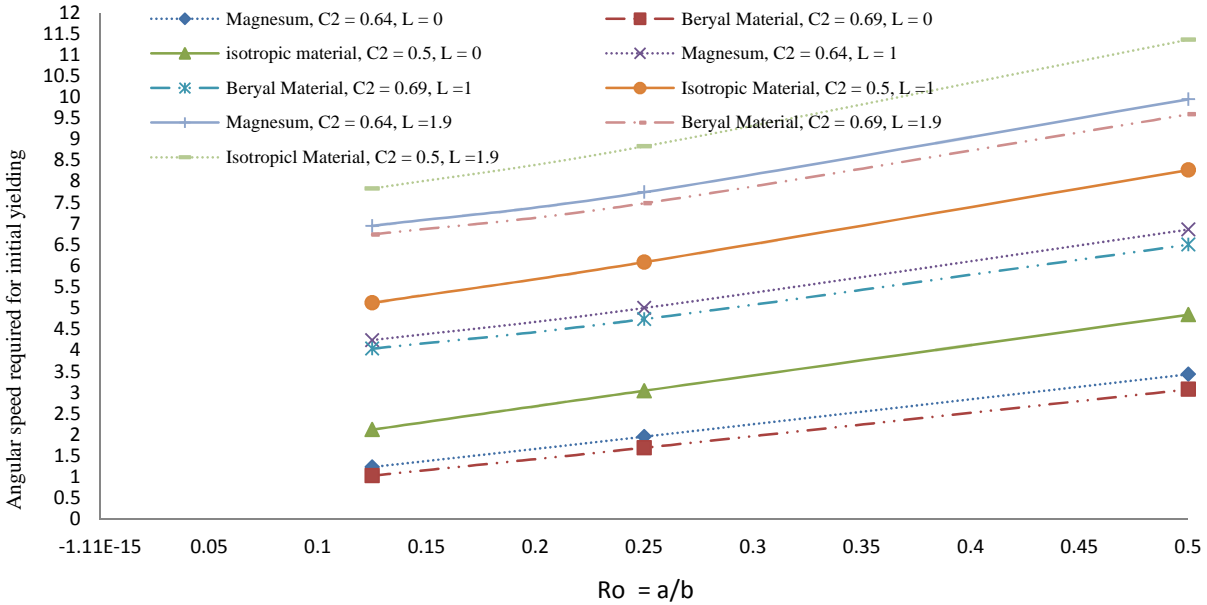


Figure 1. Angular speed required for initial yielding along the radii ratio  $R_0 = a/b$ .

Meaning of: Sigma r =  $\sigma_r$ , Sigma theta =  $\sigma_\theta$ , Displacement = U  
 $\sigma_0 = 0$  (without load)                       $\sigma_0 = 1$

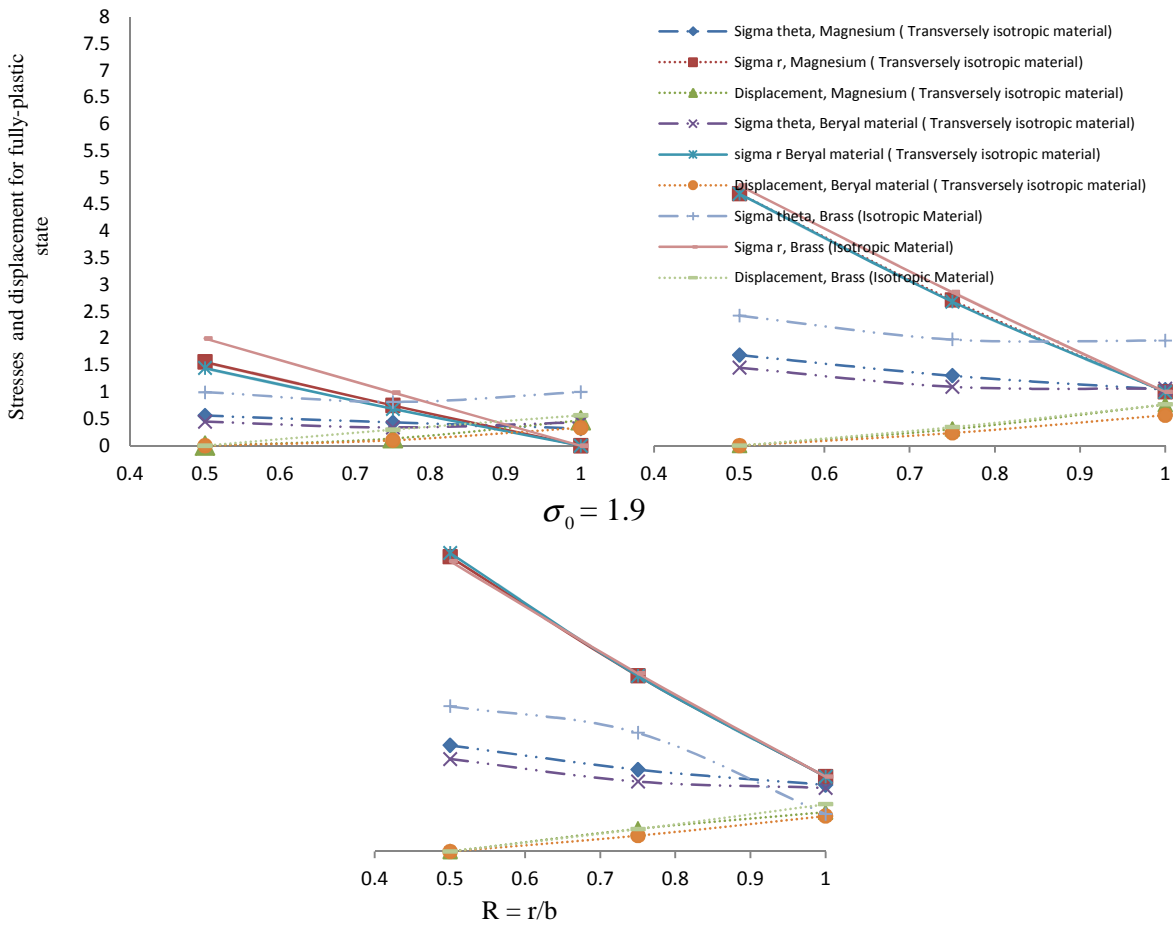


Figure 2. Stresses distribution and displacement for initial yielding along the radius ratio  $R = r/b$ .

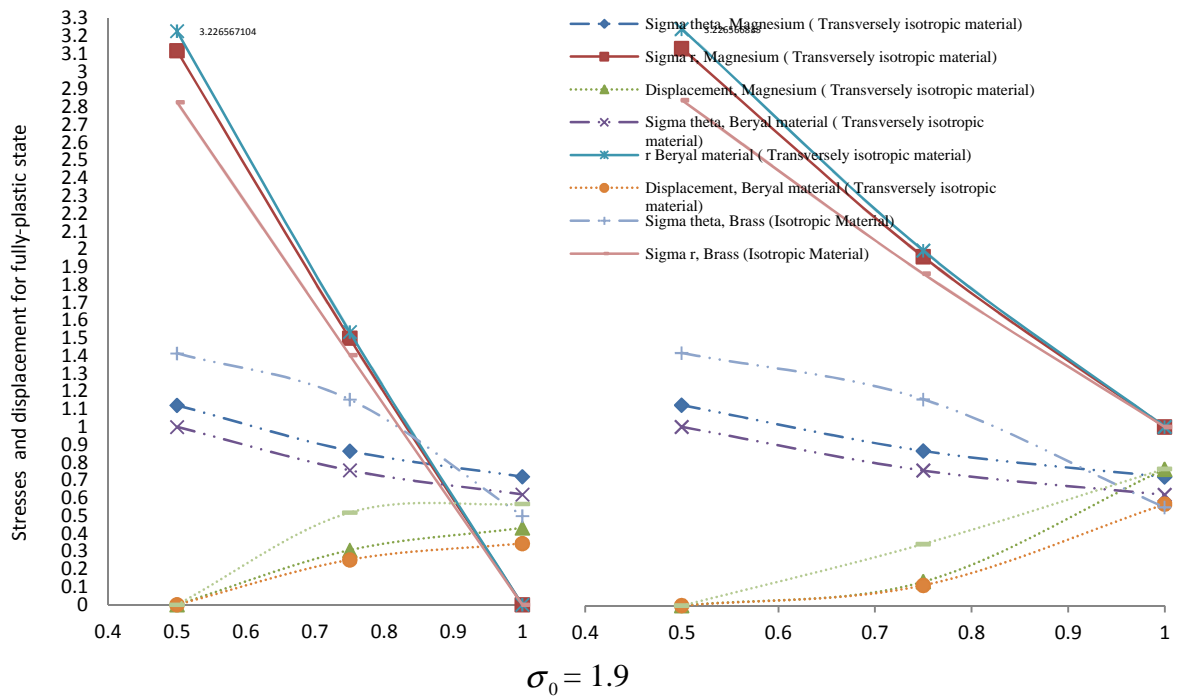


In figures 2 and 3, curves have been drawn between stresses distribution and displacement for initial yielding and fully plastic state along the radius ratio  $R = r / b$ . From fig. 2, it has been observed that rotating disc made of Brass, Mg and Beryl materials required maximum radial at the internal surface. With the introduction of mechanical load rotating disc made of Beryl material required maximum radial stress as compare to disc made of Mg and Brass materials at the internal surface. In figure 3, it is seen that radial stresses maximum for Beryl material at the internal surface. Therefore, rotating disc made of isotropic material is on the safer side of the design as compared to disc made transversely isotropic material.

Meaning of: Sigma r= $\sigma_r$ , Sigma theta = $\sigma_\theta$ , Displacement = U

$\sigma_0 = 0$ (without load)

$\sigma_0 = 1$



$\sigma_0 = 1.9$

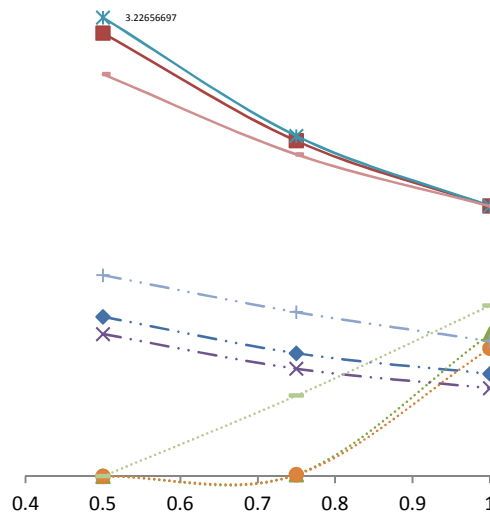


Figure 3. Stresses distribution and displacement for fully-plastic state along the radius ratio  $R = r/b$ .

### Acknowledgement

The authors gratefully acknowledge UGC, New Delhi for providing financial support to carry out this research work under UGC-Major Research Project Scheme (MRP-MAJOR-MATH-2013-41603).

### Nomenclature

$a, b$	- Inner and outer radii of the disc
$\omega$	- Angular velocity of rotation
$u, v, w$	- displacement components
$\rho$	- Density of material
$T_{ij}, e_{ij}$	- Stress and Strain rate tensor
$Y$	- Yield stress
$\sigma_0$	- Load
$K_1, K_2, d$	- constants

### Non dimensional quantities:

$R = r / b$ ;  $R_0 = a / b$  -Radii ratio;  $\sigma_0 = T_0 / Y$  - load ;  $\Omega^2 = \rho \omega^2 b^2 / Y$  -angular speed ;  $\sigma_r = T_{rr} / Y$  - Radial stress component ;  $\sigma_\theta = T_{\theta\theta} / Y$  - Circumferential stress component .

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