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ON THE EFFECTIVENESS OF POROSITY ON STEADY FLOW OF A VISCOELASTIC FLUID OVER A STRECHING SHEET WITH HEAT AND MASS TRANSFER

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ABSTRACT. The steady flow through a porous medium of a non-Newtonian viscoelastic Walter's liquid-*B* model fluid flow over a stretching sheet is studied with heat and mass transfer. Two different conditions for the temperature at the surface are considered. Analytical solution for the governing equations of momentum, mass and heat transfer are given. The influence of various physical parameters as the porosity parameter, non-Newtonian fluid characteristics on the flow, heat and mass transfer are examined.

Keywords: Fluid flow, heat transfer, mass transfer, non-Newtonian viscoelastic, stretching sheet, exact solution, porous medium.

INTRODUCTION

The flow, heat and mass transfer in laminar boundary layers above inextensible stretching sheets are of practical importance in the fields of electrochemistry polymer processing and in fiber industries, etc. [1-4]. CRANE [5] studied the boundary layer flow of a Newtonian fluid caused by an elastic sheet whose velocity varies linearly with the distance from a fixed point on the sheet. The extension of the problem to non-Newtonian Walter's liquid "B" model was given by ABEL *et al*. [6]. ABEL and VEENA [7] studied a viscoelastic fluid flow and heat transfer in a saturated porous medium over an impermeable stretching surface with frictional heating. RAJA GOPAL *et al*. [8, 9] studied the flow of a non-Newtonian fluid of second grade over a stretching sheet.

In the present study, the flow of a viscoelastic incompressible fluid (Walter's liquid B' model) past a stretching sheet through a porous medium is investigated with heat and mass transfer. The flow in the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [10]. The effects of various parameters as porosity parameter

M, visco-elastic parameter k_1, Schmidt number Sc, and Prandtl number Pr on flow, heat and mass transfer characteristics are examined.

MATHEMATICAL FORMULATION

Momentum Transfer

The fundamental equations of motion in the case of a second grade fluid is given by [8,9]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-k_o\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\nu}{K}u \tag{2}$$

Mass Transfer

The diffusion equation is written as

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2} \tag{3}$$

and the relevant boundary conditions are

$$y=0: u=u_w=\lambda x, v=0, c=c_w=c_\infty+Ax^\lambda, \tag{4a}$$

$$y\to\infty: u\to 0, u_y\to 0, c=c_\infty, \tag{4b}$$

where u, v, c, ν, ρ, D and K are the velocity components in the x-direction, in the y-direction, concentration profile, kinematic viscosity, density, diffusion coefficient, Darcy permeability. A and λ are constants. The equation of continuity is satisfied if we choose a dimensionless stream function ψ such that

$$u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x} \tag{5}$$

Introducing the similarity transformations

$$\eta=y\sqrt{\frac{\lambda}{\nu}};\ f(\eta)=\frac{\psi(x,y)}{x\sqrt{\nu\lambda}};\varphi(\eta)=\frac{c-c_\infty}{c_w-c_\infty} \tag{6}$$

transforms Eq. (2) to

$$f'^2-ff''=f'''-\gamma f'-k_0\frac{\lambda}{\nu}(2ff'''-f''^2-ff^{IV}), \tag{7}$$

where $\gamma=\nu/\lambda K$ is the porosity parameter [10]. Following the procedures discussed in [8, 9] we seek a solution of Eq. (7) in the form

$$f'(\eta)=e^{-\alpha\eta}, \alpha>0, \tag{8}$$

which is satisfied by the following boundary conditions:

$$\eta = 0 : f = 0, f' = 1, \eta \to \infty : f' \to 0, f'' \to 0. \tag{9}$$

Upon substituting (8) into (7) and using boundary conditions (5) the velocity components take the form

$$u = \lambda x e^{-\alpha\eta}, v = \frac{-\sqrt{\lambda\nu}(1 - e^{-\alpha\eta})}{\alpha}, \tag{10}$$

where $\alpha = \sqrt{\dfrac{1+\gamma}{1-k_1}}$, (11)

$k_1 = k_0\lambda/\nu$ is the visco-elastic parameter, consequently the diffusion equation takes the form

$$\varphi'' + Sc\frac{(1 - e^{-\alpha\eta})}{\alpha}\varphi' - Sc\lambda e^{-\alpha\eta} = 0, \tag{12}$$

where Schmidt number $Sc = \nu/D$ and the boundary conditions are

$$\eta = 0 : \varphi = 1, \eta \to \infty : \varphi \to 0. \tag{13}$$

Now to seek the solution of (12) we introduce the change of variable

$$\frac{Sc}{\alpha^2}e^{-\alpha\eta} = \xi, \tag{14}$$

then (12) is written as

$$\xi\varphi'' + (1 - \frac{Sc}{\alpha^2} + \xi)\varphi' - \lambda\varphi = 0, \tag{15}$$

with the boundary conditions

$$\varphi(\xi = \frac{Sc}{\alpha^2}) = 1 \text{ and } \varphi(\infty) = 0. \tag{16}$$

Now the solution of (15) satisfying (16) in terms of Kummer's function *M* is given by

$$\varphi(\xi) = (\frac{\alpha^2}{Sc}\xi)^{Sc/\alpha^2}\frac{M((Sc/\alpha^2) - \lambda, (Sc/\alpha^2) + 1, -\xi)}{M((Sc/\alpha^2) - \lambda, (Sc/\alpha^2) + 1, -Sc/\alpha^2)} \tag{17}$$

Equation (17) in terms of η is written as

$$\varphi(\eta) = e^{-(Sc/\alpha)\eta}\frac{M((Sc/\alpha^2) - \lambda, (Sc/\alpha^2) + 1, -Sc/\alpha^2 e^{-\alpha\eta})}{M((Sc/\alpha^2) - \lambda, (Sc/\alpha^2) + 1, -Sc/\alpha^2)}$$

Thus, the local wall mass flux can be expressed as

$J_w = -\rho D(\partial c / \partial y) A x^{\lambda} \varphi'(0)$.

Heat transfer

The governing boundary layer equation with temperature-dependent heat generation (absorption) is

$$\rho C_p (u \frac{\partial T}{\partial X} + v \frac{\partial T}{\partial Y}) = k \frac{\partial^2 T}{\partial Y^2} + Q(T - T_\infty). \qquad (18)$$

The thermal boundary conditions depend on the type of heating process under consideration. Here we consider two different heating processes, namely:
(i) prescribed surface temperature (PST) and
(ii) prescribed wall heat flux (PHF).

Prescribed surface temperature (PST case)

In this case, the boundary conditions are

$$y = 0 : T = T_w (= T_\infty + AX^r), y \to \infty : T \to T_\infty, \qquad (19)$$

where *r* is the wall temperature parameter; when *r*=0, the thermal boundary conditions become isothermal. Defining the non-dimensional temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \qquad (20)$$

Using relation (10), (11), (20), Eq. (18) and boundary conditions (19) can be written as

$$\theta'' + \Pr \frac{(1 - e^{-\alpha\eta})}{\alpha} \theta' - (\Pr e^{-\alpha\eta} - \beta)\theta = 0, \qquad (21)$$

$$\theta(0) = 1 \text{ and } \theta(\infty) = 0, \qquad (22)$$

where $\Pr = \mu C_p / k$, the Prandtl number and $\beta = Q\nu / Ck$, the heat source/sink parameter. Introducing the change of variable

$$\frac{\Pr}{\alpha^2} e^{-\alpha\eta} = \xi \qquad (23)$$

and inserting (23) in (21) we obtain

$$\xi\theta'' + (1 - \frac{\Pr}{\alpha^2} + \xi)\theta' - (r - \frac{\beta}{\alpha^2} \xi^{-1})\theta = 0, \qquad (24)$$

and (22) transforms to

$$\theta(\xi = \frac{\Pr}{\alpha^2}) = 1 \text{ and } \theta(\infty) = 0. \qquad (25)$$

Thus, the solution of Eq. (24) satisfying (25) is given by

$$\theta(\xi) = (\frac{\alpha^2}{\Pr}\xi)^{a+b}\frac{M((a+b)-r,1+2b,-\xi)}{M((a+b)-r,1+2b,-\Pr/\alpha^2)} \tag{26}$$

where $a = \Pr/2\alpha^2$ and $b = (\Pr^2 - 4\beta\alpha^2)^{1/2}/2\alpha^2$. The solution of (26) in terms of η is:

$$\theta(\eta) = e^{-\alpha(a+b)\eta}\frac{M((a+b)-r,1+2b,-(\Pr/\alpha^2)e^{-\alpha\eta})}{M((a+b)-r,1+2b,-\Pr/\alpha^2)} \tag{27}$$

The non-dimensional wall temperature gradient derived from (27) is

$$\theta'(0) = \frac{(\Pr/\alpha)((a+b)-r)/(1+2b)M((a+b)-r+1,2+2b-\Pr/\alpha^2)}{M((a+b)-r,1+2b,-\Pr/\alpha^2)} - \alpha(a+b) \tag{28}$$

and the local wall heat flux can be expressed as

$$q_w = -k(\frac{\partial T}{\partial Y})_w = -kA(\frac{\lambda}{\nu})^{1/2}x^r\theta'(0). \tag{29}$$

Closed-form solutions are obtained for several sets of values of a, b and r. Also, the expressions in Eqs. (28) and (29) are evaluated for several sets of values of the parameters γ, Pr, α, r and β.

Prescribed wall heat flux (PHF case)

Here the boundary conditions are

$$y = 0: -k(\frac{\partial T}{\partial Y}) = q_w = Dx^s, y \to \infty: T \to T_\infty, \tag{30}$$

where s is the wall heat flux parameter. For s=0, the stretching sheet is subjected to uniform heat flux. Defining

$$T - T_\infty = \frac{Dx^s}{k(\nu/\lambda)^{1/2}}g(\eta) \tag{31}$$

and inserting (6), (7), and (31) in (18) and (30) we obtain

$$g''(\eta) + \Pr\frac{(1-e^{-\alpha\eta})}{\alpha}g'(\eta) - (\Pr se^{-\alpha\eta} - \beta)g(\eta) = 0, \tag{32}$$

$$g'(0) = -1 \text{ and } g(\infty) = 0, \tag{33}$$

Using transformation (23), Eq. (32) and (33) reduce to

$$\xi g''(\xi) + (1 - \frac{\Pr}{\alpha^2} + \xi)g'(\xi) - (s - \frac{\beta}{\alpha^2}\xi^{-1})g(\xi) = 0, \tag{34}$$

$$g'(\xi = \Pr/\alpha^2) = \alpha/\Pr \text{ and } g(\infty) = 0. \tag{35}$$

Now the solution of (34) satisfying (35) is obtained as

$$g(\xi) = C_0(\frac{\alpha^2}{\Pr}\xi)^{a+b}M((a+b)-s,1+2b,-\xi), \tag{36}$$

where $C_0 = \frac{1}{\alpha}((a+b)M((a+b)-s,1+2b,-\Pr/\alpha^2) - \frac{\Pr}{\alpha^2}M'((a+b)-s,1+2b,-\Pr/\alpha^2))^{-1}$.

Equation (36) in terms of η is written as

$$g(\eta) = C_0e^{-\alpha(a+b)\eta}M((a+b)-s,1+2b,-(\Pr/\alpha^2)e^{-\alpha\eta}) \tag{37}$$

where C_0 is the same as given above.

The wall temperature gradient T_w is obtained from (34) as

$$T_w - T_\infty = \frac{Dx^s}{k} (\frac{\nu}{\lambda})^{1/2} g(0) \cdot \tag{38}$$

Several closed-form solutions are obtained from Eq. (37), also numerical values of $g(0)$ for several sets of values of the parameters γ, Pr, s, α and β can be are estimated.

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