Kragujevac J. Sci. 33 (2011) 5-14. UDC 532.5:536.24:537.84

HEAT AND MASS TRANSFER IN ELASTICO-VISCOUS FLUID PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE WITH ION SLIP

Hazem A. Attia

Department of Engineering Mathematics and Physics,
Faculty of Engineering, El-Fayoum University, Fayoum, Egypt
e-mail: ah1113@yahoo.com

(Received May 20, 2006)

ABSTRACT. An unsteady hydromagnetic free convection flow of elastico-viscous fluid past an infinite vertical plate is investigated when the temperature and concentration are assumed to be oscillate with time and also the ion slip effect is taken into account. Assuming constant suction at the plate, closed form solutions have been obtained for velocity, temperature and concentration distributions in terms of the elastic parameter (α), Schmidt number (Sc), Magnetic parameter (M), Hall parameter (Be), and ion slip parameter (Bi).

Key words: Ion slip effect, elastico-viscous, Heat-mass transfer.

INTRODUCTION

Heat and mass transfer from a vertical plate is encountered in various applications such as heat-exchangers, cooling system and electronic equipments. The study of convection with heat-mass transfer is very useful in the fields as Chemistry, agriculture and oceanography. few representative fields of interest in which combined heat-mass transfer play an important role are the design of chemical processing equipments, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and in drying process of paper. Heat and mass transfer from a vertical plate have been studied by several authors some of them are SOMERS [1], KHAIr and BEJAn [2], LIN and WU [3], BEHAR and STEPHAN [4], MUTHUKUMARSWAMY et al. [5] and CHIEN [6]. It is well known that a number of industrial fluid such as molten plastics, polymeric liquids, food stuffs or slurries exhibit non-Newtonian fluid behavior. Therefore, heat and mass transfer in non-Newtonian fluid is of practical importance. DAS and BISWAL [7] studied the mass transfer on visco-elastic fluid past a vertical channel. WANG [8] analyzed mixed convection from a vertical plate to non-Newtonian fluid with uniform surface heat flux. In recent years the non-Newtonian fluids in the presence of magnetic field find increasing application in many areas such as chemical engineering, electromagnetic propulsions, nuclear reactor, etc. SARPAKAYA [9] has given many possible applications of non-Newtonian fluids in various fields. The flow of viscoelastic fluids in the presence of magnetic field have been studied by SINGH and SINGH [10] and SHERIEF and EZZAT [11].

In an ionized gas where the density is low and (or) the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions; also a current is induced in a direction normal to both electric and magnetic field. This phenomenon is well known in the literature and is called the Hall effect. SATO [12] and SHERMAN and SATTON [13] were the first authors who investigated the hydromagnetic flow of ionized gas between two parallel plates taking Hall effect into account. Hall current has important engineering applications in problem of magnetohydrodynamics generators and Hall accelerators as well as in flight magnetoaerodynamics. The effect of Hall current for MHD free convection flow along a vertical surface and in the presence of transverse magnetic field with or without mass transfer have been studied by number of authors; POP [14], RAPTIS and RAM [15], HOSSAIN and RASHID [16], HOSSAIN and MOHAMMAD [17], POP and WATANABE [19], ACHARYA *et al.* [20,21], ABODELDAHAB and ELBARABARY [22] and ASGHAR *et al.* [23].

In the present analysis, it is proposed to study the effect of simultaneous heat and mass transfer on the flow of elastico-viscous fluid past an impulsively started infinite vertical plate taking Hall and ion slip effects into the account. Closed form solutions have been obtained for the velocity, temperature, and concentration distribution.

MATHEMATICAL FORMULATION

The constitutive equations for the rheological equation of state for an elastico-viscous fluid (Walter's liquid B') are

$$p_{ik} = -p\, g_{ik} + p'_{ik} \tag{1}$$

$$p'_{ik} = 2\int_{-\infty}^{t} \psi(t-t')\, e^{(1)}_{ik}(t')\, dt' \tag{2}$$

in which

$$\psi(t-t') = \int_0^{\infty} \frac{N(\tau)}{\tau} e^{-(t-t')\tau}\, d\tau \tag{3}$$

$N(\tau)$ is the distribution function of relaxation times. In the above equations p_{ik} is the stress tensor, p an arbitrary isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x_i and $e^{(1)}_{ik}$, the rate of strain tensor. It was shown by WALTER's [24] that equation (2) can be put in the following generalized form which is valid for all types of motion and stress

$$p'^{ik}(x,t) = 2\int_{-\infty}^{t} \psi(t-t') \frac{\partial x^i}{\partial x'^m} \frac{\partial x^k}{\partial x'^r} e^{(1)mr}(x'\,t')\, dt' \tag{4}$$

where x'^i is the position at time t' of the element that is instantaneously at the print x^i at time *t*. The fluid with equation of state (1) and (4) has been designated as liquid B'. In the case of liquids with short memories, i.e. short relaxation times, the above equation of state can be written in the following simplified form

$$p'^{ik}(x,t) = 2\eta_0\, e^{(1)ik} - 2k_0 \frac{\partial e^{(1)ik}}{\partial t}, \tag{5}$$

in which $\eta_0 = \int_0^\infty N(\tau)\, d\tau$ is limiting viscosity at small rates of shear,

$k_0 = \int_0^\infty \tau N(\tau)\, d\tau$ and $\frac{\partial}{\partial t}$ denotes the convective time derivative.

We consider the unsteady flow of a viscous incompressible and electrically conducting elastico-viscous fluid with oscillating temperature and concentration. We consider the flow along x-axis which is taken to be along the plate and y-axis is taken normal to it. The plate starts moving in its own plane with velocity U_0 (a constant velocity). A uniform magnetic field is applied normal to the plate with constant suction as shown in figure 1. The equations governing the flow of fluid together with Maxwell's electromagnetic equations are as follows

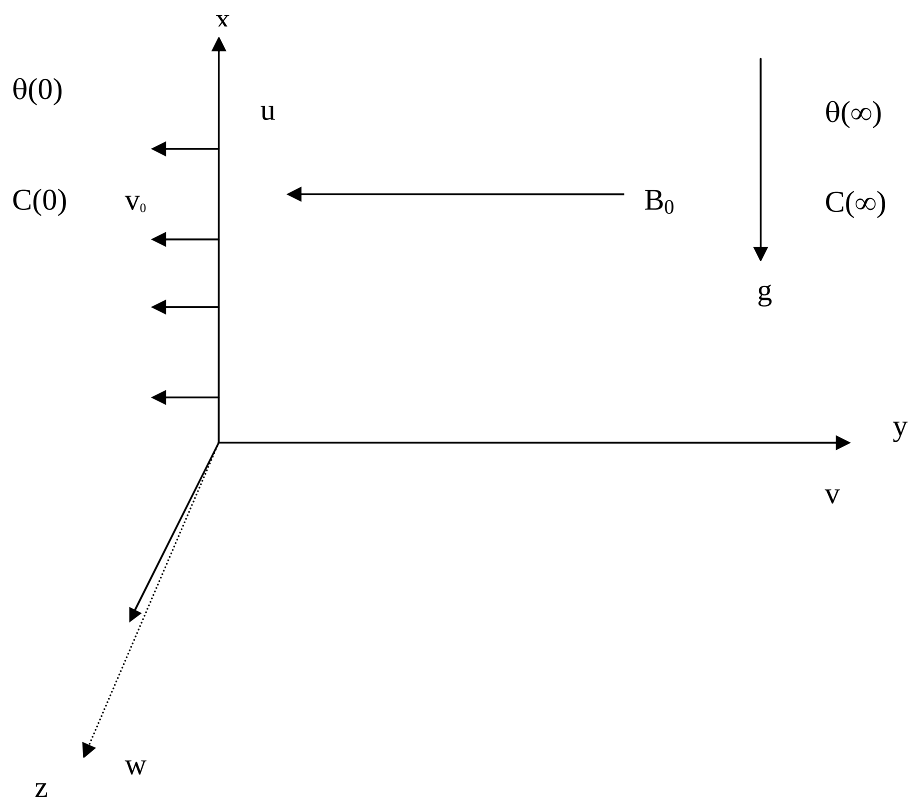

Figure 1: Physical model of the problem

Equation of Continuity

$$\nabla . V = 0 \tag{6}$$

Momentum Equation

$$\frac{\partial V}{\partial t} + (V.\nabla)\, V = -\frac{1}{\rho} \nabla P + \nabla . p_{ij} + g\, \beta (T - T_\infty) + g\, \beta^* (C - C_\infty) + \frac{1}{\rho} (J \text{ x } B) \tag{7}$$

Maxwell Equations

$$\nabla . B = 0,\ \nabla \times B = \mu_m J,\ \nabla \times E = -\frac{\partial B}{\partial t} \tag{8}$$

Generalized Ohm's Law [13,23] (where the Hall and ion slip terms are retained)

$$J = \sigma\left\{E + VxB_o - \bar{\beta}(JxB_o) + \frac{\bar{\beta}Bi}{B_o}(JxB_o)xB_o\right\}, \tag{9}$$

where V = (u, v,w) is the velocity field, P is the pressure field, g is acceleration due to gravity, β the volumetric coefficient of the thermal expansion, β^* the volumetric coefficient of expansion with concentration, ρ the density of the fluid, J is the current density, B is the magnetic field, E is the electric field, μ_m is the magnetic permeability, p_{ij} is stress tensor, $\bar{\beta}$ is the Hall factor, Be is the Hall parameter, Bi is the ion slip parameter and σ is the electrical conductivity. Therefore the flow becomes three dimensional. It is assumed that there is no applied or polarization voltage so that E = 0 and the induced magnetic field is negligible so that the total magnetic field B = $(0,B_0, 0)$ where B_0 is the applied magnetic field parallel to y-axis. This assumption is justified when the magnetic Reynolds number (The ratio of the moduli of the convection term and diffusive term. This number is non-dimensional and strictly analogous in the properties and uses to the Reynolds number) is very small. Then equation (9) reduces to

$$J_x = \frac{\sigma B_0}{(1+BeBi)^2 + Be^2}\,(Beu - (1+BeBi)w) \tag{10}$$

$$J_z = \frac{\sigma B_0}{(1+BeBi)^2 + Be^2}\,((1+BeBi)u + Bew) \tag{11}$$

Under this condition the Boussinesq approximation equations governing the flows are as follows

Equation of Continuity

$$\frac{\partial v}{\partial y} = 0 \tag{12}$$

$\Rightarrow$ v = - v_0 where v_0 is constant suction velocity.

Momentum Equation

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\,\frac{\partial^2 u}{\partial y^2} - k_0\,\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma B_0^2((1+BeBi)u + Be\,w)}{\rho((1+BeBi)^2 + Be^2)} + g\,\beta\theta + g\,\beta^* C^* \tag{13}$$

$$\frac{\partial w}{\partial t} + v\,\frac{\partial w}{\partial y} = \nu\,\frac{\partial^2 w}{\partial y^2} - k_0\,\frac{\partial^3 w}{\partial y^2 \partial t} - \frac{\sigma B_0^2\,((1+BeBi)w - Beu)}{\rho((1+BeBi)^2 + Be^2)} \tag{14}$$

Energy Equation

$$\frac{\partial \theta}{\partial t} + v\,\frac{\partial \theta}{\partial y} = \frac{K}{\rho C_p}\,\frac{\partial^2 \theta}{\partial y^2} \tag{15}$$

Concentration Equation

$$\frac{\partial C^*}{\partial t} + v\,\frac{\partial C^*}{\partial y} = D\,\frac{\partial^2 C^*}{\partial y^2} \tag{16}$$

where

$$T(y,t) - T_\infty = \theta(y,t)\,,\; C(y,t) - C_\infty = C^*(y,t) \tag{17}$$

ρ is the density of the fluid, ν is the kinematic viscosity, k_0 the elastic parameter, K the thermal conductivity, C_p is the specific heat of the fluid, D the chemical molecular diffusivity and g is the acceleration due to gravity. In equation (15) the terms due to viscous dissipation are neglected and in equation (16) the term due to chemical reason is assumed to be absent.

The initial boundary conditions are

$$t \le 0,\; u(y,t) = w(y,t) = 0,\; \theta = 0,\; C^* = 0 \;\text{ for all } y$$

$$t \ge 0 \begin{cases} u(0,t) = U_0,\; w(0,t) = 0,\; \theta(0,t) = a\, e^{i\omega t} \\ C^*(0,t) = b\, e^{i\omega t},\; \text{at } y = 0 \\ u(\infty,t) = w(\infty,t),\; \theta(\infty,t) = C^*(\infty,t) = 0 \;\text{ as } y \to \infty \end{cases} \tag{18}$$

where ω is frequency of oscillation, a and b are constant and subscript ∞ denotes the physical quantity in the free stream.

We introduce the following non-dimensional parameters

$$\eta = \frac{v_0 y}{\nu},\; t' = \frac{v_0^2 t}{4\nu},\; u' = \frac{u}{U_0},\; w' = \frac{w}{U_0}$$

$$\theta' = \frac{\theta}{a},\; C' = \frac{C^*}{b},\; G = \frac{4 g \beta \nu a}{v_0^2 U_0}$$

$$Gc = \frac{4 g \beta^* \nu b}{v_0^2 U_0},\; M = \frac{4 B_0^2 \sigma \nu}{\rho v_0^2 U_0}$$

$$Pr = \frac{\nu \rho C_p}{K},\; \alpha = \frac{k_0 v_0^2}{\nu^2},\; Sc = \frac{\nu}{D} \tag{19}$$

Substituting equation (19) in (14) – (17) and (18) and dropping the dashes we get

$$\frac{\partial u}{\partial t} - \frac{4\partial u}{\partial \eta} = \frac{4\partial^2 u}{\partial \eta^2} - \alpha \frac{\partial^2 u}{\partial \eta^2 \partial t} - \frac{M}{(1+\mathrm{BeBi})^2 + Be^2}\left(Be\,w + (1 + BeBi)u\right) + G\theta + Gc\,C \tag{20}$$

$$\frac{\partial w}{\partial t} - \frac{4\partial w}{\partial \eta} = \frac{4\partial^2 w}{\partial \eta^2} - \alpha \frac{\partial^2 w}{\partial \eta^2 \partial t} - \frac{M}{(1+BeBi)^2 + Be^2}\left((1 + BeBi)w - Be\,u\right) \tag{21}$$

$$\frac{\partial \theta}{\partial t} - \frac{4\partial \theta}{\partial \eta} = \frac{4}{Pr}\frac{\partial^2 \theta}{\partial \eta^2} \tag{22}$$

$$\frac{\partial C}{\partial t} - \frac{4\partial C}{\partial \eta} = \frac{4}{Sc}\frac{\partial^2 C}{\partial \eta^2} \tag{23}$$

and the boundary conditions for equation (20) –(23) are

$$t \le 0,\; u(\eta,t) = w(\eta,t) = \theta(\eta,t) = C(\eta,t) = 0 \;\forall\; \eta$$

$$t \geq 0 \begin{cases} u(0,t) = 1,\ w(0,t) = 0,\ \theta(0,t) = e^{i\omega t}, C(0,t) = e^{i\omega t} & \text{at } \eta = 0 \\ u(\infty,t) = w(\infty,t) = 0,\quad \theta(\infty,t) = C(\infty,t) = 0 & \text{as } \eta \to \infty \end{cases} \tag{24}$$

SOLUTION

The equation (20) and (21) can be combined using the complex variable

$$\psi = u + iw \tag{25}$$

Equations (20) –(21) give

$$\frac{\partial^2 \psi}{\partial \eta^2} - \alpha \frac{\partial^3 \psi}{\partial \eta^2 \partial t} - \frac{1}{4}\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial \eta} - \frac{M((1+BeBi) - iBe)}{4((1+BeBi)+Be^2)}\psi = -\frac{G\theta}{4} - \frac{Gc\,C}{4} \tag{26}$$

Introducing $\Omega = \dfrac{4\omega}{v_0^2}$ where Ω is non-dimensional frequency of oscillation and using Eq. (25), we get boundary conditions as

$$\begin{aligned} &\psi(0,t) = 1,\ \psi(\infty,t) = 0,\ C(0,t) = e^{i\Omega t} \\ &\theta(0,t) = e^{i\Omega t},\ \theta(\infty,t) = 0,\ C(\infty,t) = 0 \end{aligned} \tag{27}$$

Putting $\theta(\eta,t) = e^{i\Omega t} f(\eta)$ in equation (22), we get

$$f''(\eta) + \Pr f'(\eta) - \frac{i\Omega \Pr}{4} f(\eta) = 0 \tag{28}$$

which has to be solved under the boundary conditions

$$\begin{aligned} &f(0) = 1 \\ &f(\infty) = 0 \end{aligned} \tag{29}$$

Hence $f(\eta) = e^{1/2[-\Pr - \sqrt{\Pr^2 + i\Omega \Pr}\,]\eta}$

$$\Rightarrow \theta(\eta,t) = e^{i\Omega t - \frac{\eta}{2}[\Pr + \sqrt{\Pr^2 + i\Omega\Pr}\,]}$$

Separating real and imaginary part, the real part is given by

$$\theta_r(\eta,t) = \cos\left\{\Omega t - \frac{\eta}{2} R_1 \sin\frac{\beta_1}{2}\right\} e^{-\frac{\eta}{2}(\Pr + R_1 \cos\frac{\beta_1}{2})} \tag{30}$$

where $R_1 = \Pr^{1/2}(\Pr^2 + \Omega^2)^{1/4}$

$$\beta_1 = \tan^{-1}\left(\frac{\Omega}{\Pr}\right) \tag{31}$$

Putting $C(\eta,t) = e^{i\Omega t} g(\eta)$ in equation (23), we get

$$g''(\eta) + Sc\, g'(\eta) - \frac{i\Omega Sc}{4} g(\eta) = 0 \tag{32}$$

which can be solved under the boundary conditions

$g(0) = 1,\ g(\infty) = 0$

Hence $g(\eta) = e^{1/2[-Sc - \sqrt{Sc^2 + i\Omega Sc}\,]\eta}$

$$\Rightarrow C(\eta,t) = e^{i\Omega t - [Sc + \sqrt{Sc^2 + i\Omega Sc}\,]\frac{\eta}{2}} \tag{33}$$

Separating real and imaginary part, the real part is given by

$$C_r(\eta, t) = \cos\left\{\Omega t - \frac{\eta}{2} R_2 \sin\frac{\beta_2}{2}\right\} e^{-\frac{\eta}{2}(Sc + R_2 \cos\frac{\beta_2}{2})}$$

where

$$\left.\begin{aligned} R_2 &= Sc^{1/2}(Sc^2 + \Omega^2)^{1/4} \\ \beta_2 &= \tan^{-1}\left(\frac{\Omega}{Sc}\right) \end{aligned}\right\} \tag{34}$$

In order to solve equation (26), substituting $\psi = e^{i\Omega t} F(\eta)$ and using boundary conditions

$$\left.\begin{aligned} F(0) &= e^{-i\Omega t} \\ F(\infty) &= 0 \end{aligned}\right\} \tag{35}$$

Separating real and imaginary part, we get

$$\begin{aligned} u = {} & e^{-\eta a_4}[\{\cos \eta a_5 + (A_9 A_7 + A_{10} A_{12}) \cos(\Omega t - \eta a_5)\} \\ & + \{(A_8 A_9 + A_{11} A_{12}) \sin(\Omega t - \eta a_5)\}] \\ & - e^{\eta a_6}[A_9 A_7 \cos(\Omega t - \eta a_7) + A_9 A_8 \sin(\Omega t - \eta a_7)] \\ & - e^{-\eta a_8}[A_{10} A_{12} \cos(\Omega t - \eta a_9) + A_{11} A_{12} \sin(\Omega t - \eta a_9)] \end{aligned} \tag{36}$$

and

$$\begin{aligned} w = {} & e^{-\eta a_4}[\{\sin \eta a_5 + (A_9 A_7 + A_{10} A_{12}) \sin(\eta t - \eta a_5\} \\ & \quad - \{(A_8 A_9 + A_{11} + A_{12}) \cos(\Omega t - \eta a_5)\}] \\ & - e^{-\eta a_6}[A_9 A_7 \sin(\Omega t - \eta a_7) - A_9 A_8 \cos(\Omega t - \eta a_7)] \\ & - e^{-\eta a_8}[A_{10} A_{12} \sin(\Omega t - \eta a_9) - A_{11} A_{12} \cos(\Omega t - \eta a_9)] \end{aligned} \tag{37}$$

where

$$a_1 = \frac{\alpha}{4},\ a_2 = \frac{M(1+BeBi)}{4((1+BeBi)^2 + Be^2)},\ a_3 = \frac{\Omega}{4} - \frac{M\,Be}{4((1+BeBi)^2 + Be^2)}$$

$$A_1 = 4(a_2 + a_1 a_3),\ A_2 = 4(a_3 - a_1 a_2)$$

$$A_3 = 1 + r^{1/4} \cos\gamma/2,\ A_4 = r^{1/4} \sin\gamma/2$$

$$r = (1 + A_1^2)^2 + A_2^2,\ \gamma = \tan^{-1}\frac{A_2}{1 + 4A_1}$$

$$A_4 = \frac{(A_3 - A_4)}{2(1 + a_1^2)},\ a_5 = \frac{(a_1 A_3 + A_4)}{2(1 + a_1^2)}$$

$$a_6 = \frac{1}{2}(Pr + R_1 \cos\frac{\beta_1}{2}),\ a_8 = \frac{1}{2}(Sc + R_2 \sin\beta_2/2)$$

$$a_7 = \frac{1}{2} R_1 \sin\frac{\beta_1}{2},\ a_9 = \frac{1}{2}(R_2 \cos\beta_2/2),\ R_2 = Sc^{1/2}(Sc^2 + \Omega^2)^{1/4}$$

$$R_1 = Pr^{1/2}(Pr^2 + \Omega^2)^{1/4},\ \beta_1 = \tan^{-1}\left(\frac{\Omega}{Pr}\right)\ \beta_2 = \tan^{-1}\left(\frac{\Omega}{Sc}\right)$$

$$A_5 = a_6^2 - a_7^2 + 2a_1a_6a_7 - a_6 - a_2$$

$$A_6 = 2a_6a_7 - a_1(a_6^2 - a_7^2) - a_7 - a_3$$

$$A_7 = \frac{G}{4(A_7^2 + A_8^2)},$$

$$A_8 = a_8^2 - a_9^2 + 2a_1a_8a_9 - a_8 - a_2$$

$$A_9 = 2a_8a_9 - a_1(a_8^2 - a_9^2) - a_9 - a_3,\ A_{10} = \frac{Gc}{4(A_8^2 + A_9^2)}$$

Knowing the velocity field it is important from a practical point of view to know the effect of physical parameters, Sc, M, m and α on skin friction. We now calculate the skin friction from these relations

$$\tau_{xw} = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

in non-dimensional form it takes

$$\tau_1 = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0}$$ where τ_1 is the x-component of skin friction.

Similarly z-component of skin friction τ_2 is given as

$$\tau_2 = \left[\frac{\partial w}{\partial \eta}\right]_{\eta=0}$$ (In non-dimensional form)

The rate of heat transfer in terms of Nusselt number is given by

$$Nu = \frac{q\nu}{v_0 K}(T_w - T_\infty)$$

where $q = -K \left.\frac{\partial T}{\partial y}\right|_{y=0}$

In non-dimensional form it is given by

$$Nu = -\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}$$

$$= \frac{1}{2}\left[Pr\cos\Omega t + R_1 \cos\left(\Omega t + \frac{\beta_1}{2}\right)\right] \qquad (38)$$

The rate of mass transfer is given by

$$J^*\,(\text{Diffusion flux}) = -\rho D \left.\frac{\partial C^*}{\partial y}\right|_{y=0}$$

The coefficient of mass transfer which is generally known as Sherwood number S_h is given by

$$S_h = \frac{J^* \nu}{v_0 \rho D (C_w - C_\infty)} = \left.\frac{\partial C}{\partial \eta}\right|_{y=0}$$
$$= \frac{1}{2}\left[Sc \cos \Omega t + R_2 \cos\left(\Omega t + \frac{\beta_2}{2} \right) \right] \tag{39}$$

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