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TRANSIENT CIRCULAR PIPE MHD FLOW OF A DUSTY FLUID CONSIDERING THE HALL EFFECT

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ABSTRACT. In this paper, the transient flow of a dusty viscous incompressible electrically conducting fluid through a circular pipe is studied taking the Hall effect into consideration. A constant pressure gradient in the axial direction and an uniform magnetic field directed perpendicular to the flow direction are applied. The particle-phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing non-linear equations using finite differences.

Keywords: hydromagnetics, computational fluid, flow in channels, circular pipe flow

INTRODUCTION

The flow of a dusty and electrically conducting fluid through a circular pipe in the presence of a transverse magnetic field has important applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle-phase viscosity. There have been many articles dealing with theoretical modeling and experimental measurements of the particle-phase viscosity in a dusty fluid [1-4].

The flow of a Newtonian conducting fluid in a circular pipe has been investigated by many authors [5-8]. GADIRAJU *et al*. [5] investigated steady two-phase vertical flow in a pipe. DUBE *et al*. [6] and RITTER *et al.* [7] reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses. CHAMKHA [8] obtained exact solutions which generalize the results reported in [6] and [7] by the inclusion of the magnetic and particle-phase viscous effects. In the above mentioned work the Hall effect was ignored in applying Ohm's law, as it has no marked effect for small and moderate values of the magnetic field. The flow of a non-Newtonian conducting fluid in a circular pipe has been done by METZNER [9], NAKAYAMA and KOYAMA [10], and ATTIA

[11,12] in a series of However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable under these conditions, and the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term [13].

In the present study, the transient flow of a dusty fluid through a circular pipe is investigated considering the Hall effect. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient while a uniform magnetic field is applied perpendicular to the flow direction. The governing nonlinear momentum equations for both the fluid and particle-phases are solved numerically using the finite difference approximations. The effect of the Hall current, and the particle-phase viscosity on the velocity of the fluid and particle-phases are reported.

Notation

a: pipe radius,
B_o: magnetic induction,
C: fluid-phase skin-friction coefficient,
C_p: particle-phase skin-friction coefficient,
Ha: Hartmann number,
m: Hall parameter,
N: momentum transfer coefficient,
P: pressure gradient,
Q: fluid-phase volumetric flow rate,
Q_p: fluid-phase volumetric flow rate,
r: distance in the radial direction,
t: time,
V: fluid-phase velocity,
V_p: particle-phase velocity,
z: axial direction,
α: inverse Stokes number,
B: viscosity ratio,
ϕ: particle-phase volume fraction,
k: particle loading,
μ: fluid-phase viscosity,
μ_p: particle-phase viscosity,
ρ: fluid-phase density,
ρ_p: fluid-phase density,
σ: fluid electrical conductivity.

GOVERNING EQUATIONS

Consider the unsteady, laminar, and axisymmetric horizontal flow of a dusty conducting fluid through an infinitely long pipe of radius a driven by a constant pressure gradient as shown in Fig. 1. A uniform magnetic field is applied perpendicular to the flow direction. The Hall current is taken into consideration and the magnetic Reynolds number is assumed to be very small, consequently the induced magnetic field is neglected [13]. We assume that both phases behave as viscous fluids and that the volume fraction of suspended particles is finite and constant [8]. Taking into account these and the previously mentioned assumptions, the governing momentum equations can be written as

$$\rho \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{\rho_p \phi}{1-\phi} N(V_p - V) - \frac{\sigma B_o^2 V}{1+m^2} \quad (1)$$

$$\rho_p \frac{\partial V_p}{\partial t} = \frac{\mu_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_p}{\partial r}\right) + \rho_p N(V - V_p) \quad (2)$$

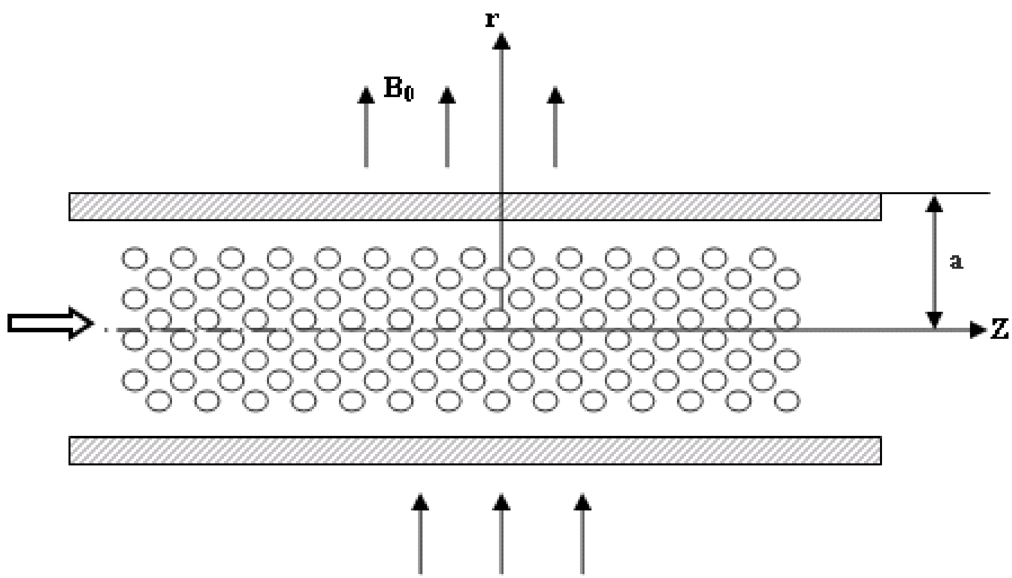


Fig. 1. - Problem definition

where $\partial P/\partial z$ is the fluid pressure gradient, N is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce e^{-1} of its original value [8], $m=\sigma\beta B_o$ is the Hall parameter, and β is the Hall factor [13]. In this work, ρ, ρ_p, μ_p, φ, and B_o are all constant. It should be pointed out that the particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid-phase.

The initial and boundary conditions of the problem are given as

$$V(r,0) = 0, V_p(r,0) = 0, \quad (3a)$$

$$\frac{\partial V(0,t)}{\partial r} = 0, \frac{\partial V_p(0,t)}{\partial r} = 0, V(a,t) = 0,\ V_p(a,t) = 0 \quad (3b)$$

where a is the pipe radius.

Equations (1)-(3) constitute a nonlinear initial-value problem which can be made dimensionless by introducing the following dimensionless variables and parameters

$$\bar{r} = \frac{r}{a}, \bar{t} = \frac{t\mu}{\rho a^2}, G_o = -\frac{\partial P}{\partial z}, k = \frac{\rho_p \phi}{\rho(1-\phi)},$$

$$\overline{V}(r,t)=\frac{\mu V(r,t)}{G_o a^2},\overline{V}_p(r,t)=\frac{\mu V_p(r,t)}{G_o a^2},$$

$\alpha = Nd^2\rho/\mu$ is the inverse Stoke's number,

$B=\mu_p/\mu$ is the viscosity ratio,

$Ha = B_o a\sqrt{\sigma/\mu}$ is the Hartmann number [13].

By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs. (1)-(3) can be written as (the bars are dropped),

$$\frac{\partial V}{\partial t}=1+\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+k\alpha(V_p-V)-\frac{Ha^2 V}{1+m^2} \tag{4}$$

$$\frac{\partial V_p}{\partial t}=B\left(\frac{\partial^2 V_p}{\partial r^2}+\frac{1}{r}\frac{\partial V_p}{\partial r}\right)+\alpha(V-V_p) \tag{5}$$

$$V(r,0)=0,V_p(r,0)=0, \tag{6a}$$

$$\frac{\partial V(0,t)}{\partial r}=0,\frac{\partial V_p(0,t)}{\partial r}=0,V(1,t)=0,\ V_p(1,t)=0 \tag{6b}$$

The volumetric flow rates and skin-friction coefficients for both the fluid and particle phases are defined, respectively, as [8]

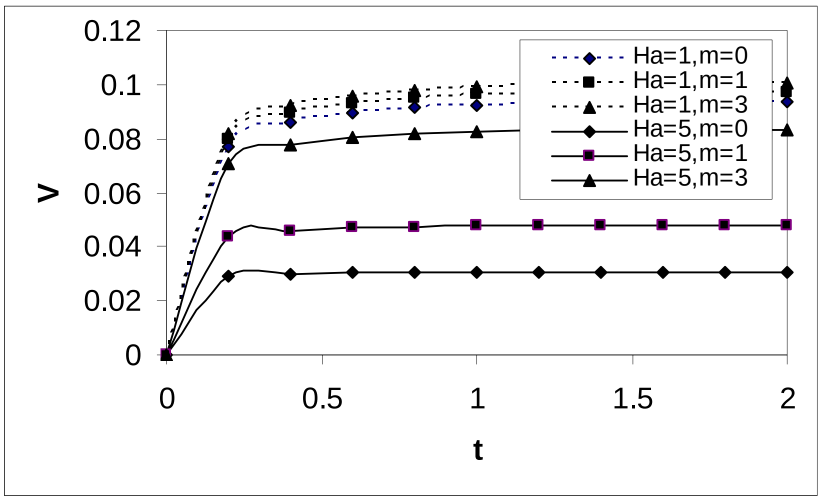
$$Q=2\pi\int_0^1 rV(r,t)dr, Q_p=2\pi\int_0^1 rV_p(r,t)dr,\ C=-\frac{\partial V(1,t)}{\partial r},C_p=-Bk\frac{\partial V_p(1,t)}{\partial r} \tag{7}$$

RESULTS AND DISCUSSION

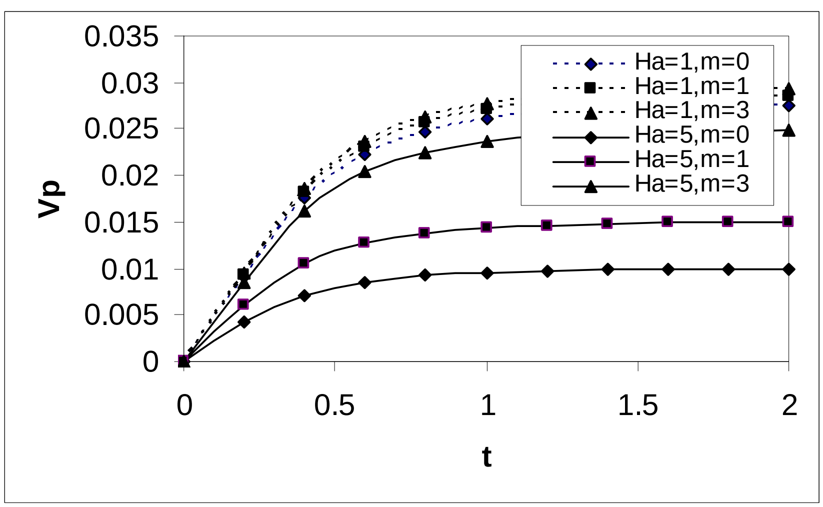
Equations (4) and (5) represent a coupled system of partial differential equations which are solved numerically under the initial and boundary conditions (6), using the finite difference approximations. The Crank-Nicolson implicit method [14,15] is used at two successive time levels. The resulting block tri-diagonal system is solved using the generalized Thomas algorithm [14,15]. Computations have been made for α=1 and k=10. Grid-independence studies show that the computational domain $0<t<\infty$ and $0<r<1$ can be divided into intervals with step sizes Δt=0.0001 and Δr=0.005 for time and space respectively. It should be mentioned that the results obtained herein reduce to those reported by [6] and [8] for the cases of non-magnetic, inviscid particle-phase (B=0), and when neglecting the Hall effect (m=0). These comparisons lend confidence in the accuracy and correctness of the solutions.

Imposing of a magnetic field normal to the flow direction gives rise to a drag-like or resistive force and it has the tendency to slow down or suppress the movement of the fluid in the pipe, which in turn, reduces the motion of the suspended particle-phase. This is translated into reductions in the average velocities of both the fluid- and the particle-phases and, consequently, in their flow rates. In addition, the reduced motion of the particulate suspension in the pipe as a result of increasing the strength of the magnetic field causes lower velocity gradients at the wall. This has the direct effect of reducing the skin-friction coefficients of both phases.

Figure 2 presents the time evolution of the profiles of the velocity of the fluid V and dust particles V_p respectively for various values of the Hartmann number Ha and the Hall parameter m and for B=0.5. Both V and V_p increase with time and V reaches the steady-state faster than V_p for all values of m or Ha. It is clear from Fig. 2 that increasing m decreases both V and V_p and their steady-state times as a result of increasing the effective conductivity ($\sigma/(1+m^2)$) which decreases the damping force on V and then increases V. On the other hand, increasing Ha decreases V and V_p since the damping force on V is proportional to Ha^2 and then increasing Ha increases the damping force which increases V. It is shown in Fig. 2 that the effect of m on V or V_p becomes more pronounced for higher values of Ha. It is also clear that the effect of m on V and V_p becomes more pronounced for higher values of Ha. This is expected since the importance of the Hall term is more clear in the case of high magnetic fields.



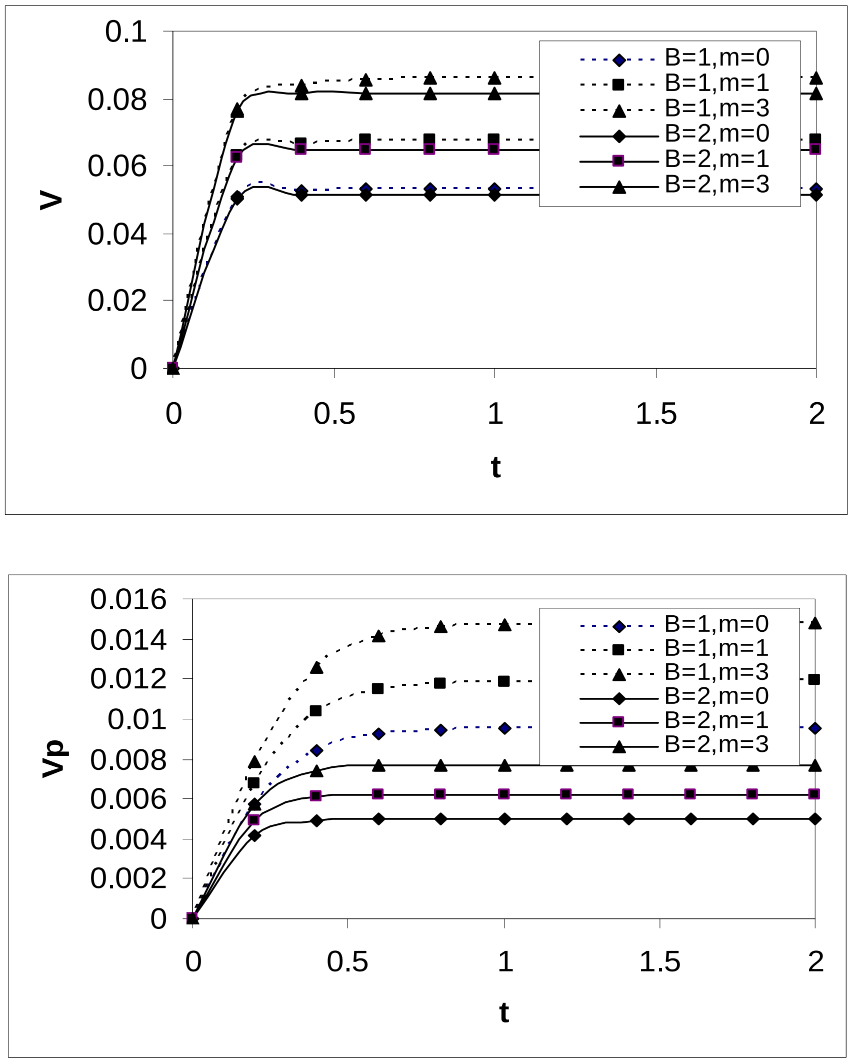
(a)



(b)

Fig. 2. - Effect of the parameters Ha and m on the time development of:
(a) V at r=0 and (b) V_p at r=0. (β=0.5)

Figure 3 presents the time evolution of the profiles of the velocity of the fluid V and dust particles V_p, respectively, for various values of the particle-phase viscosity B and the Hall parameter m and for Ha=3.Figure 3 shows that increasing the parameter B decreases V and V_p for all values of m, as a result of increasing the viscosity which increases the resistive viscous forces and then decreases the velocities of both phases. The effect of B on V_p is more pronounced than its effect on V.



(a)

(b)

Fig. 3. - Effect of the parameters β and m on the time development of:
(a) V at r=0 and (b) V_p at r=0. (Ha=3)

Tables 1 and 2 present the steady state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters B and m and for Ha=1 and 5, respectively. It is clear that increasing the parameter m increases Q, Q_p, C, and C_p for all values of B and Ha. This comes from the effect of increasing m in increasing the velocities and their gradients which increases the average velocities of both the fluid- and the particle-phases and, consequently, increases their flow rates and skin-friction coefficients of both phases. It is also shown that increasing the parameter B decreases the quantities Q, Q_p, and C, but increases C_p for all values of m and Ha. This can be attributed to the fact that increasing B increases viscosity and therefore the flow rates of both phases as well as the fluid-phase wall friction decreases considerably. However, since C_p is defined as directly proportional to B, it increases as B increases at all times.

Table 1. - The steady state values of Q, Q_p, C, C_p for various values of m and B and for Ha=1

B=0	m=0	m=1	m=3
Q	0.2471	0.2589	0.2692
Q_p	0.1855	0.1933	0.2000
C	0.3579	0.3697	0.3798
C_p	0	0	0
B=0.5	m=0	m=1	m=3
Q	0.1675	0.1732	0.1781
Q_p	0.0403	0.0418	0.0429
C	0.2726	0.2786	0.2836
C_p	0.2003	0.2070	0.2128
B=1	m=0	m=1	m=3
Q	0.1564	0.1614	0.1656
Q_p	0.0215	0.0222	0.0228
C	0.2622	0.2675	0.2719
C_p	0.2129	0.2196	0.2253

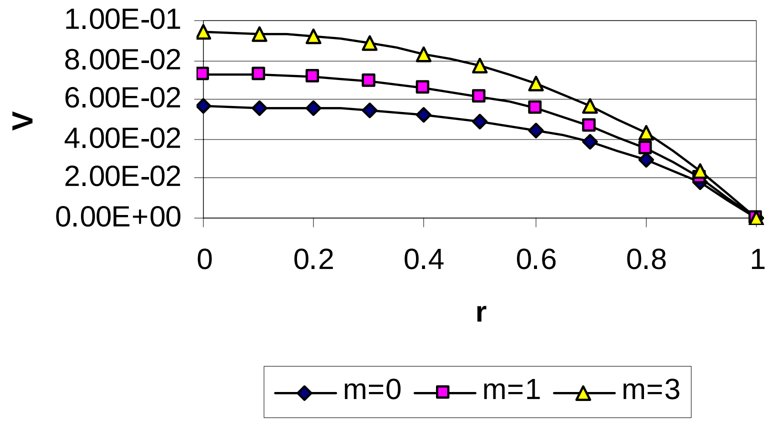
Table 2. - The steady state values of Q, Q_p, C, C_p for various values of m and B and for Ha=5

B=0	m=0	m=1	m=3
Q	0.1422	0.1869	0.2494
Q_p	0.1126	0.1445	0.187
C	0.2494	0.2969	0.3602
C_p	0	0	0
B=0.5	m=0	m=1	m=3
Q	0.11	0.1361	0.1686
Q_p	0.0259	0.0325	0.0406
C	0.2107	0.2395	0.2738
C_p	0.1326	0.1634	0.2016
B=1	m=0	m=1	m=3
Q	0.1053	0.1288	0.1574
Q_p	0.0142	0.0175	0.0216
C	0.2062	0.2326	0.2632
C_p	0.1439	0.1757	0.2142

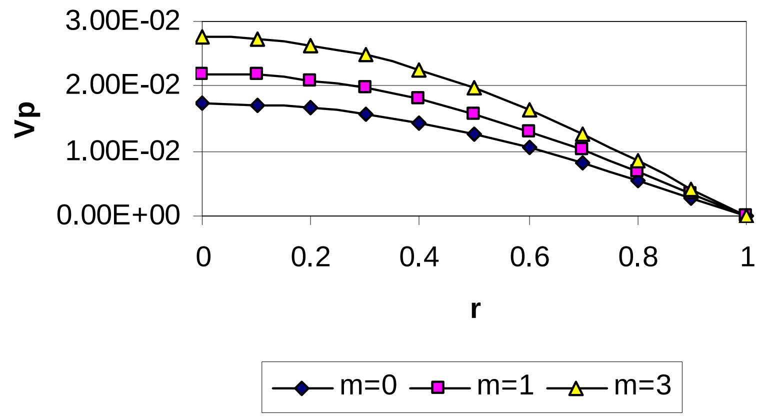
Figure 4 presents the steady state profiles of the velocities V and V_p, respectively, for various values of the Hall parameter m. In this figure, Ha=3 and B=0.5. Due to symmetry, only half of each profile is presented. The figure indicates the effect of the Hall parameter m, discussed above, in increasing the velocity V and V_p for all distances r from the central line of the pipe.

CONCLUSION

The transient MHD flow of a particulate suspension in an electrically conducting fluid in a circular pipe is studied considering the Hall effect. The governing partial differential equations are solved numerically using finite differences. The effect of the magnetic field parameter Ha, the Hall parameter m, and the particle-phase viscosity *B* on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases is studied. It is shown that increasing the magnetic field decreases the fluid and particle velocities, while increasing the Hall parameter increases both velocities. It is found that increasing the parameter m increases Q, Q_p, C, and C_p for all values of Ha and B. The effect of the Hall parameter on the quantities Q, Q_p, C, and C_p becomes more pronounced for higher values of Ha or smaller values of B.



(a)



(b)

Fig. 4. - Effect of the parameter m on the steady state profiles of:
(a) V and (b) V_p. (Ha=3 and β=0.5)

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