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A BRIEF DESCRIPTION OF DEVELOPMENTS AND APPLICATIONS OF DIRECT BOUNDARY ELEMENT METHOD FOR COMPRESSIBLE FLUID FLOW PROBLEMS

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ABSTRACT. The aim of this paper is to present a brief description of developments and applications of direct boundary element method (DBEM) for compressible steady fluid flow problems. The direct boundary element method (DBEM) is expressed in an integral equation form which relates the value of the potential at any point within the flow field with the values of the potential and potential derivatives over the surface of the body and thus the unknowns are calculated in the form of potential and potential derivatives over the surface of body. When the analytical solution of a problem is not possible or difficult to find out, then the importance of its numerical solution increases. The numerical solutions of compressible flows play significant role in computational fluid dynamics (CFD). Some contributions of different researchers in this field are also highlighted.

Keywords: DBEM, Developments and Applications, Compressible flow, Steady flow, CFD.

INTRODUCTION

A compressible flow is different from an incompressible one in that the density of fluid does not remain the same. In compressible flow, the equation governing the in viscid compressible flow of a homogeneous fluid was firstly derived by EULER. Euler considered all the characteristics of the fluid to be continuous functions of time and space. The approach taken by Euler assumes that the fluid is a continuum. In applying the continuum assumption, the care must be taken that the average distance between molecules is small as compared to the scale of the problem under consideration. From the time of fluid flow modeling, it had been struggled to find the solution of a complicated system of partial differential equations (PDE) for the fluid flows which needed more efficient numerical methods. From time to time, many numerical techniques such as finite difference method, finite element method, finite volume method and boundary element method etc. came into beings which made possible the calculation of practical flows. Due to discovery of new algorithms and faster computers, these methods were evolved in all areas in the past such as stress analysis, heat transfer and electromagnetic theory, potential theory, fracture mechanics, fluid mechanics, elasticity, elastostatics and elastodynamics, biological and biomedical problems ,etc. These methods are CPU time and storage hungry. Boundary element method originated within the Department of civil engineering at Southampton University, U.K. These methods existed under different names such as ‘Panel Method’, ‘Surface singularity methods’, ‘Boundary integral equation methods’, or ‘Boundary integral solutions’. Nowadays, the boundary element method is successfully applied by numerical community. One of the advantages is that with boundary elements one has to discretize the entire surface of the body, whereas with domain methods it is essential to discretize the entire region of the flow field. The most important characteristics of boundary element methods are the much smaller system of equations and considerable reduction in data which is prerequisite to run a computer program efficiently. Moreover, this method is well–suited to problems with an infinite domain. From above discussion, it is concluded that boundary element method is a time saving, accurate and efficient numerical technique as compared to other numerical techniques which can be classified into direct boundary element method and indirect boundary element method. The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data. Whereas the indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation, the equation of direct boundary element method (DBEM) can be formulated using either as an approach based on Green’s theorem or a particular case of the weighted residual methods. Now the direct boundary element method (DBEM) is being used for the solution of compressible flows around complex configurations .Thus it can be said that the direct boundary element method (DBEM) is a powerful numerical technique receiving much attention from researchers, engineering community and is becoming popular technique in the computational solution of a number of physical problems.

MATHEMATICAL FORMULATION OF DIRECT BOUNDARY ELEMENT METHOD

Consider a close body having surface S enclosing a region R and $\hat{n}$ is the outward drawn unit normal at any point on the surface S as shown in figure (1).

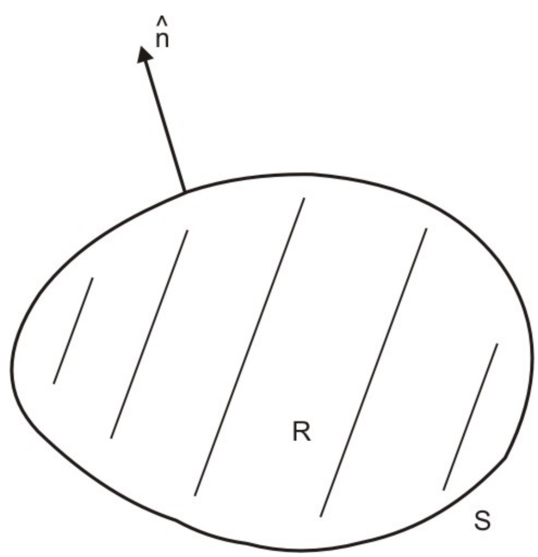

Fig. 1. – Closed body having surface S enclosing a region R

Then the mathematical formulation of direct boundary element method (DBEM) is given by

$$c_i \phi_i = \phi_\infty - \frac{1}{4\pi} \iint_S \frac{1}{r} \frac{\partial \phi}{\partial n} dS + \frac{1}{4\pi} \iint_{S-i} \phi \frac{\partial}{\partial n}\left(\frac{1}{r}\right) dS$$

$$\text{where } c_i = \begin{cases} 0 & \text{when 'i' is exterior to } R \\ 1 & \text{when 'i' is interior to } R \\ \frac{1}{2} & \text{when 'i' lies on } S \text{ and } S \text{ is smooth} \end{cases}$$

DEVELOMENTS AND APPLICATIONS OF DIRECT BOUNDARY ELEMENT METHOD

In past, a few struggles had been made to calculate the flow field around complex bodies with the help of theoretical methods. It therefore appears that the theoretical methods will be of great importance for calculating the flow field around complicated bodies. Recently the available theoretical methods for the flow field calculations are the finite difference, finite element, finite volume and boundary element methods. Boundary element methods have many advantages over the ‘domain’ type methods such as finite elements or finite differences. One of such advantages is that with boundary elements one only has to define the surface of the body, whereas with field methods, it is necessary to mesh the entire flow field. The amount of input data for a boundary element method is therefore significantly less than for a field method, which is a very important advantage in practice, as many hours can be spent in preparing checking the data for finite element or finite difference programs. Also, for a given problem the boundary element method will have a much smaller system size than a field method, and will therefore be computationally more efficient. Boundary element methods are also well–suited to solve problems where some boundary conditions are applied at infinity, as is the case for exterior vehicle aerodynamics. Over the many past years the importances of boundary element methods have been widely recognized and numerous papers and other related works have been published in this field. Boundary element methods can be formulated using two different approaches called the ‘direct’ method and the ‘indirect’ method. For potential flow problems, the direct method can be expressed in the form of integral equation which relates the value of potential at any point within the flow field with the values of the potential and potential derivatives over the surface of the body and thus the unknowns are calculated in the form of potential and potential derivatives over the body surface. The direct boundary element method (DBEM) had been first applied by MORINO *et al.* for calculating the flow field around aircraft. Thus the need arose to apply the direct boundary element method (DBEM) for the calculation of in viscid flow around different types of vehicles. Finally, the ‘direct’ boundary element method (DBEM) has been applied to calculate the flow around road vehicles and other complicated body shapes. Thus direct boundary element method (DBEM) is the best approached more formally and this formulation was later used by BUTTERFIELD, CRUSE & RIZZO, JASWAN & SYMM and TOMLIN for the numerical solution of linear boundary value problems involving homogeneous and piece-wise homogeneous bodies. Water wave problems are still another class of problems faced by this formulation. The direct boundary element method (DBEM) was also applied by YOUNGREN & ACRIVOS to find the Stokes flow of a viscous fluid in the presence of a rigid particle of arbitrary shape. Later RALLISON & ACRIVOS used this method to a Stokes flow in the presence of a viscous drop. The method of YOUNGREN & ACRIVOS has been applied for the numerical solution of some problems with the iteration of solid particles, motion of a particle near a fluid interface or a rigid wall, etc.

CONCLUSION

This paper gives a brief description of developments and applications of Direct Boundary Element Method (DBEM) for Compressible Steady In viscid fluid flow problems is presented. Although Direct Boundary Element Method (DBEM) is popular amongst researchers and engineering community due its accuracy and efficiency in the field of compressible fluid flow problems, yet there is a serious need to do more in this field of interest to establish that this method would be very powerful and useful for such problems in the time to come.

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