UNSTEADY MHD COUETTE FLOW
OF A VISCOELASTIC FLUID WITH HEAT TRANSFER

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ABSTRACT. The unsteady magnetohydromagnetic Couette flow of an electrically conducting incompressible non-Newtonian viscoelastic fluid between two parallel horizontal non-conducting porous plates is studied with heat transfer. A sudden uniform and a constant pressure gradient, an external uniform magnetic field that is perpendicular to the plates and uniform suction and injection through the surface of the plates are applied. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are taken into consideration. Numerical solutions for the governing momentum and energy equations are obtained using finite difference approximations. The effect of the magnetic field, the parameter describing the non-Newtonian behavior, and the velocity of suction and injection on both the velocity and temperature distributions is examined.

INTRODUCTION

The flow of an electrically conducting viscous fluid between two parallel plates in the presence of a transversely applied magnetic field has applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays. Hartmann flow of a Newtonian fluid with heat transfer, subjected to different physical effects, have been studied by many authors [1-9]. These results are important for the design of the duct wall and the cooling arrangements. The rectangular channel problem has later been extended also to fluids obeying non-Newtonian constitutive equations. The hydrodynamic flow of a viscoelastic fluid has attracted the attention of many authors [10-13] due to its important industrial applications [11]. In most cases the Hall term was ignored in applying Ohm's law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [4]. Under these conditions, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term.

Attia [8, 14] has studied the influence of the magnetic field on the velocity and temperature fields of an unsteady Hartmann flow of a conducting Newtonian fluid between two infinite non-

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conducting horizontal parallel fixed and porous plates. In the present work the MHD Couette flow of a conducting non-Newtonian viscoelastic fluid is studied with heat transfer in the present of a uniform magnetic field and an exponential decaying pressure gradient. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The flow is subjected to a uniform suction from above and a uniform injection from below, and an external uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The two plates are kept at two different but constant temperatures. The Joule and viscous dissipations are taken into consideration in the energy equation. The governing momentum and energy equations are solved numerically using the finite difference approximations. The inclusion of the magnetic field, the non-Newtonian fluid characteristics as well as the velocity of suction and injection leads to some interesting effects, on both the velocity and temperature fields.

FORMULATION OF THE PROBLEM

The fluid is assumed to be incompressible, viscoelastic and flows between two infinite horizontal parallel non-conducting plates located at the $y=\pm h$ planes and extend from $x= -\infty$ to ∞ and from $z=-\infty$ to ∞. The upper plate is moving with a constant velocity U_o while the lower plate is kept stationary. The upper and lower plates are kept at two constant temperatures T_2 and T_1 respectively, with $T_2>T_1$. The flow is driven by a uniform and a constant pressure gradient dP/dx in the x-direction, and a uniform suction from the above and injection from below which are applied at $t=0$. A uniform magnetic field with magnetic flux density vector $\boldsymbol{B}_o$ is applied in the positive y-direction which is assumed to be also the total magnetic field, as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The plates are assumed to be infinite in the x and z- directions which makes the physical quantities do not change in these directions. Thus, the velocity vector of the fluid, in general, is given by

$\boldsymbol{v}(y,t)=u(y,t)\boldsymbol{i} + v_o\,\boldsymbol{j}$

It is because of the conservation of mass, i.e., *div v=0* and due to the uniform suction the velocity component $v(y,t)$ is assumed to have a constant value v_o.

The fluid motion starts from rest at $t=0$, that is $u=0$ for $t\leq0$. The no-slip condition at the plates implies that $u=0$ at $y=-h$ and $u=U_o$ at $y=h$. It is also assumed that the initial temperature of the fluid is T_1, thus the initial and boundary conditions of temperature are $T= T_1$ at $t=0$, $T=T_1$, at $y=-h$, $t>0$ and $T=T_2$, at $y=h$, $t>0$. The fluid motion is governed by the momentum equations [16]

$$\rho\left(\frac{\partial u}{\partial t}+v_o\,\frac{\partial u}{\partial y}\right)=-\frac{dP}{dx}-\sigma B_o^2 u +\frac{\partial \tau_{xy}}{\partial y} \qquad (1)$$

where ρ is the density of the fluid, σ is the electric conductivity of the fluid, the second term in the right-hand side represents electromagnetic force, and τ_{xy} is the component of the shear stress of the viscoelastic fluid given respectively as [10]

$$\tau_{xy} = \mu\frac{\partial u}{\partial y}-\frac{\mu}{\alpha}\frac{\partial \tau_{xy}}{\partial t} \qquad (2)$$

where μ is the coefficient of viscosity and α is the modulus of rigidity. In the limit α tends to infinity or at steady state, the fluid behaves like a viscous fluid without elasticity. Solving Eq. (2) for τ_{xy} in terms of the velocity component u we obtain

$$\frac{\partial \tau_{xy}}{\partial t} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) - \frac{1}{\alpha}\frac{\partial}{\partial y}\left(\mu \frac{\partial}{\partial t}\left(\mu \frac{\partial u}{\partial y}\right)\right), \tag{3}$$

where the term $(1/\alpha^2)\frac{\partial}{\partial y}(\mu \frac{\partial}{\partial t}(\mu \frac{\partial \tau_{xy}}{\partial t}))$, which is proportional to $(1/\alpha^2)$ have been neglected. Substituting Eq. (3) in the momentum Eq. (1) yields

$$\rho\left(\frac{\partial u}{\partial t} + v_o \frac{\partial u}{\partial y}\right) = -\frac{dP}{dx} - \sigma B_o^2 u + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu^2}{\alpha}\frac{\partial^3 u}{\partial t \partial y^2} \tag{4}$$

The temperature distribution is governed by the energy equation [16]

$$\rho c_p\left(\frac{\partial T}{\partial t} + v_o \frac{\partial T}{\partial y}\right) = k \frac{\partial^2 T}{\partial y^2} + \sigma B_o^2 u^2 + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{5}$$

where c_p and k are, respectively, the specific heat capacity at constant pressure and the thermal conductivity of the fluid. The second and third terms on the right-hand side represent the Joule and viscous dissipations respectively. Equations (4) and (5) can be made dimensionless by introducing the following dimensionless variables and parameters

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{u} = \frac{u}{U_o}, \hat{P} = \frac{P}{\rho U_o^2}, t = \frac{tU_o}{h}, \hat{T} = \frac{T - T_1}{T_2 - T_1},$$

$Re = \rho h U_o / \mu$, is the Reynolds number,

$S = v_o / U_o$, is the suction parameter,

$Pr = \mu c / k$ is the Prandtl number,

$Ec = U_o^2 / c(T_2 - T_1)$ is the Eckert number,

$Ha^2 = \sigma B_o^2 h^2 / \mu$ where Ha is the Hartmann number,

$K = \rho U_o / \alpha$, the viscoelastic parameter,

In terms of these dimensionless quantities, Eqs. (4) and (5) may be written, after dropping all hats for convenience, as

$$\frac{\partial u}{\partial t} + S \frac{\partial u}{\partial y} = -\frac{dP}{dx} - \frac{1}{Re} Ha^2 u + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - K \frac{\partial^3 u}{\partial t \partial y^2} \tag{6}$$

$$\frac{\partial T}{\partial t} + S \frac{\partial T}{\partial y} = \frac{1}{Re\,Pr}\frac{\partial^2 T}{\partial y^2} + \frac{Ha^2 E_c}{Re} u^2 + \frac{E_c}{Re}\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

The initial and boundary conditions for the velocity and temperature in the dimensionless form are written as

$$u(y,0) = 0, u(-1,t) = 0, u(1,t) = 1 \tag{8}$$

$$T(y,0) = 0, T(-1,t) = 0, T(1,t) = 1 \tag{9}$$

Equations (6) and (7) represent a system of partial differential equations which is solved numerically under the initial and boundary conditions (8) and (9), using the finite difference approximation. The Crank-Nicolson implicit method [17] is used at two successive time levels. Finite difference equations relating the variables are obtained by writing the equations at the mid

point of the computational cell and then replacing the different terms by their second order central difference approximation in the *y*-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. Finally, the resulting block tridiagonal system is solved using the generalized Thomas-algorithm [17]. Grid-independence studies show that the computational domain $0<t<\infty$ and $-1<y<1$ can be divided into intervals with step sizes $\Delta t=0.0001$ and $\Delta y=0.005$ for time and space, respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when every one of *u, T,* and their gradients for the last two approximations differs from unity by less than 10^{-6} for all values of *y* in -$1<y<1$ at every time step. Computations have been made for *dP/dx=5, Pr=1, Re=1* and *E_c=0.2*.

RESULTS AND DISCUSSION

Figures 1-2 show the time development of the profiles of the velocity *u* and the temperature T respectively for various values of time *t* and for *K=0*, 0.5, and 1. The figures are evaluated for *Ha=1, S=1*. Figures 1-2 indicate that increasing *K* decreases *u* and *T* for all values of time. Also, it is shown that, for all values of K, *u* and *T* reach the steady state monotonically with time. Figure 2b and 2c indicate the influence of *K* in decreasing the dissipations as a result of decreasing the velocity *u* and its gradient.

Figures 3-4 show the time evolution of *u* and *T* at the centre of the channel (*y=0*), respectively for various values of the parameter *Ha* and for *K=* 0, 0.5, and 1. In these figures *S=0*. It is clear from Fig. 3 that increasing the parameter *Ha* decreases *u* and its steady state time for all values of *K*. This is because the magnetic damping force on *u* increases with increasing *Ha*. It is clear from Fig. 3 that the parameter *K* has a great effect on the steady state time of *u* and this effect is more clear for smaller values of *Ha*.

Figure 4 indicates that, for small values of *Ha*, increasing *Ha* decreases *T* for all time due to decreasing the velocity u and its gradient which decreases the dissipations. On the other hand, for higher values of *Ha*, the effect of the parameter *Ha* on *T* depends on time. Increasing *Ha* increases *T* at small times but increases it at large times. This can be attributed to the fact that, for small times, an increase in *Ha* increases the Joule dissipation with is proportional to Ha^2. For large times, increasing *Ha* decreases *u* greatly and, in turn, decreases the Joule and viscous dissipations. This accounts for crossing the curves of *T* with time for all values of *K* and higher *Ha*. Figures 4b and 4c show that such a crossover becomes more apparent for higher *K*. The parameter *K* has a negligible effect on the steady state time of *T* for all values of *Ha*.

Figures 5-6 show the time evolution of *u* and *T* at the centre of the channel (*y=0*), respectively for various values of the suction parameter *S* and for *K=* 0, 0.5, and 1. In these figures *Ha=1*. Figure 5 shows that increasing the suction decreases *u* due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. The parameter *K* has a marked effect on the steady state time of *u* for all values of *S*. Figure 6 shows that increasing *S* decreases the temperature at the centre of the channel for all values of *K* and *t*. This is due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel. The effect of *S* on *T* has an apparent effect for all *K*.

CONCLUSIONS

The unsteady MHD Couette flow and heat transfer of a conducting non-Newtonian viscoelastic fluid under the influence of an applied uniform magnetic field have been studied in the presence of a uniform magnetic field as well as a uniform suction and injection. The effect of the viscoelastic parameter, the magnetic field, and the suction and injection velocity on the velocity and temperature distributions has been investigated. The viscoelastic parameter has a marked effect on

the velocity and temperature distributions and their steady state times for all values of the magnetic field and the suction velocity. Also, it is of interest to find that the dependence of the temperature on the magnetic field vary with time for all values of the viscoelastic parameter and higher values of the magnetic field. This results in a crossover of *T-t* charts for higher values of the magnetic field.

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