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THE EFFECT OF SUCTION AND INJECTION ON UNSTEADY COUETTE FLOW WITH VARIABLE PROPERTIES

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ABSTRACT. The unsteady Couette flow of an incompressible viscous fluid between two parallel porous plates is studied with heat transfer in the presence of a uniform suction and injection considering variable properties. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The fluid is subjected to a constant pressure gradient and a uniform suction and injection through the plates which are kept at different but constant temperatures. The effect of the suction and injection velocity and the variable viscosity and thermal conductivity on both the velocity and temperature fields is studied.

INTRODUCTION

The flow with heat transfer of a viscous incompressible fluid between two parallel plates has important applications in many devices such as aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry. Many researchers have considered this problem under different physical effects [1-5]. Most of these studies are based on constant physical properties, although some physical properties are varying with temperature and assuming constant properties is a good approximation as long as small differences in temperature are involved [6]. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of these physical properties with temperature. The effect of temperature dependent viscosity on the flow in a channel has been studied in the hydrodynamic case [7] and the hydromagnetic case [8,9].

In the present work, the unsteady Couette flow of a viscous incompressible fluid between two parallel porous plates is studied with heat transfer in the presence of uniform suction and injection through the plates with variable physical properties. The upper plate is moving with a constant speed and the lower plate is kept stationary. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature and the two plates are kept at two constant but different temperatures. The fluid is acted upon by a constant pressure gradient. The coupled set of the nonlinear equations of motion and the energy equation including the viscous dissipation term is solved numerically using finite differences to obtain the velocity and temperature distributions at any instant of time.

FORMULATION OF THE PROBLEM

The fluid is assumed to be flowing between two infinite horizontal plates located at the y=±h planes. The fluid between the two plates is subjected to a uniform suction from above and injection from below with velocity $V_o\mathbf{j}$. The motion is produced by a constant pressure gradient dP/dx in the x-direction. The two plates are kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2>T_1$. The viscosity of the fluid is assumed to vary exponentially with temperature while the thermal conductivity is assumed to depend linearly on temperature. The viscous dissipation is taken into consideration. The flow of the fluid is governed by the Navier-Stokes equation which has the form [1,5]

$$\rho(\frac{\partial u}{\partial t}+V_o\frac{\partial u}{\partial y})=-\frac{dP}{dx}+\mu\frac{\partial^2 u}{\partial y^2}+\frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity of the fluid, and u=u(y,t) is the velocity component of the fluid in the x-direction. It is assumed that the pressure gradient is applied at t=0 and the fluid starts its motion from rest and for t>0, the no-slip condition at the plates implies that

$$t=0:\ u=0,\ t>0:\ u=0,\ y=-h\ \&\ u=U_o,\ y=h \tag{2}$$

The energy equation describing the temperature distribution for the fluid is given by [1,10]

$$\rho c_p(\frac{\partial T}{\partial t}+V_o\frac{\partial T}{\partial y})=\frac{\partial}{\partial y}(k\frac{\partial T}{\partial y})+\mu(\frac{\partial u}{\partial y})^2 \tag{3}$$

where T is the temperature of the fluid, c_p is the specific heat capacity of the fluid at constant volume, and k is the thermal conductivity of the fluid. The last term in the left-hand side of Eq. (3) represents the viscous dissipation. The temperature of the fluid must satisfy the boundary conditions,

$$t=0:\ T=T_1 \tag{4a}$$

$$t>0:\ T=T_1,\ y=-h,\ T=T_2,\ y=h \tag{4b}$$

The viscosity of the fluid is assumed to vary with temperature and is defined as, $\mu=\mu_o f_1(T)$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form [7], $f_1(T)=\exp(-a_1(T-T_1))$. In some cases a_1 may be negative, i.e. the coefficient of viscosity increases with temperature [8,9]. Also, the thermal conductivity of the fluid is assumed to vary with temperature as $k=k_of_2(T)$. We assume linear dependence for the thermal conductivity upon temperature in the form $k=k_o[1+b_1(T-T_1)]$ [10], where the parameter b_1 may be positive or negative [10].

The problem is simplified by writing the equations in the non-dimensional form. To achieve this, we define the following non-dimensional quantities,

$$(\hat{x},\hat{y},\hat{z})=\frac{(x,y,z)}{h},\hat{t}=\frac{t\mu_o}{h^2\rho},\hat{P}=\frac{\rho h^2 P}{\mu_o^2},\hat{u}=\frac{\rho h u}{\mu_o},\theta=\frac{T-T_1}{T_2-T_1},G=-\frac{d\hat{P}}{d\hat{x}},$$

$\hat{f}_1(\theta)=\exp(-a_1(T_2-T_1))=\exp(-a\theta)$, “a” is the viscosity exponent,

$\hat{f}_2(\theta)=1+b_1(T_2-T_1)\,\theta=1+b\theta$, “b” is the thermal conductivity parameter,

$R=\rho U_o h/\mu_o$, is the Reynolds number,

$Pr=\mu_o c_p/k_o$ is the Prandtl number,

$Ec = \mu_o^2/\rho^2 h^2 c_p(T_2-T_1)$ is the Eckert number,
$S=V_o \rho h/\mu_o$, is the suction parameter,
$\tau_L = (\partial \hat{u}/\partial \hat{y})_{\hat{y}=-1}$ is the axial skin friction coefficient at the lower plate,
$\tau_U = (\partial \hat{u}/\partial \hat{y})_{\hat{y}=1}$ is the axial skin friction coefficient at the upper plate,
$Nu_L = (\partial\theta/\partial \hat{y})_{\hat{y}=-1}$ is the Nusselt number at the lower plate,
$Nu_U = (\partial\theta/\partial \hat{y})_{\hat{y}=1}$ is the Nusselt number at the upper plate,

In terms of the above non-dimensional quantities Eqs. (1) to (4) read (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = G + f_1(\theta)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f_1(\theta)}{\partial y}\frac{\partial u}{\partial y} \qquad (5)$$

t=0: u=0, t>0: u=0, y=-1 & u=1, y=1 (6)

$$\frac{\partial \theta}{\partial t} + S\frac{\partial \theta}{\partial y} = \frac{1}{R\,\Pr} f_2(\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{R\,\Pr}\frac{\partial f_2(\theta)}{\partial y}\frac{\partial \theta}{\partial y} + \frac{Ec}{R} f_1(\theta)(\frac{\partial u}{\partial y})^2 \qquad (7)$$

t=0: θ=0 (8a)

t>0: θ=0, y=-1, θ=1, y=1 (8b)

Equations (5) and (7) represent a system of coupled non-linear partial differential equations which can be solved numerically under the initial and boundary conditions (6) and (8) using the finite difference approximations. The Crank-Nicolson implicit method is used [11]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time levels. The non-linear terms are first linearized and then an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations have been carried out for G=5, R=1, Pr=1, and Ec=0.2.

RESULTS AND DISCUSSION

Figures 1a and b present the time development of the velocity component u at the center of the channel (y=0), for various values of the parameters "a" and S and for b=0. The figures show that increasing the parameter "a" increases u for all values of S as a result of decreasing the viscosity. It is also shown that the steady state time of u increases with increasing "a" for all S. Comparing Figs. 1a and b indicates that increasing S decreases u for all values of "a". Figures 2a and b present the time development of the temperature θ at the center of the channel (y=0), for various values of the parameters "a" and S and for b=0. The figures show that increasing "a" increases θ for all values of S as a result of increasing the viscous dissipation. It is also shown that the steady state time of θ increases with increasing "a" for all values of S. The comparison between Figs. 2a and b shows that increasing S decreases θ for all values of "a". Also, it can be seen from Fig. 2a that θ may exceed the value 1 which is the temperature of the hot plate and this is due to the viscous dissipation.

Figures 3a and 3b present the time development of the temperature θ at the center of the channel (y=0), for various values of the parameters "b" and S and for a=0. Figure 3a shows that, in the case of zero suction, the variation of the temperature θ with the parameter "b" depends on t where a crossover in θ-t charts occurs. For small t, θ increases with increasing "b", however, for large t, increasing "b" decreases θ. This occurs because, at low times, the center of the channel acquires heat by conduction from the hot plate, but after large time, when u is large, the viscous dissipation is large at the center and center looses heat by conduction. It is noticed that the parameter "b" has no significant effect on u in spite of the coupling between the momentum and energy equations. It is also shown in the figures that increasing the parameter "b" decreases the steady state time of θ. Figure 3b indicates that, in the presence of suction, increasing "b" increases θ for all time and leads to the suppression of the crossover in θ-t charts. Comparing Figs. 3a and b shows that increasing S decreases θ for all values of "b".

Tables 1a and b present the variation of the steady state axial and transverse skin friction coefficients at both walls for various values of "a" and for S=0 and 2, respectively. It is clear that increasing "a" increases the magnitude of τ_L and τ_U for the case S=0, as depicted in Table 1a. Table 1b shows that, in the presence of suction, increasing "a" increases the magnitude of τ_U for all values of "a". Increasing "a" increases τ_L for small and moderate values of "a", however, increasing "a" more decreases τ_L. Increasing S decreases τ_L but increases the magnitude of τ_U. Tables 2 and 3 present the variation of the steady state temperature θ at y=0, the Nusselt number at the lower and upper plates for various values of the parameters "a" and "b" and, respectively, for S=0 and 2. It is clear from Table 2 that increasing "a" increases θ, Nu_L and the magnitude of Nu_U for all values of "b". In the suction case, as shown in Table 3, increasing "a" increases θ, but decreases Nu_L and Nu_U for all values of "b". Table 2 indicates that increasing "b" decreases θ and the magnitude of N_{uU} for all values of "a". On the other hand, increasing "b" increases Nu_L for all values of "a" except for negative values of "a" and "b" where increasing the magnitude of "b" decreases Nu_L. Table 3 shows that increasing "b" decreases Nu_L, and the magnitude of Nu_U but increases θ for all values of "a".

Table 1. - The Steady State Axial and Transverse Skin Friction Coefficients
(a) S=0, (b) S=2

(a) S=0	a=-0.5	a=-0.1	a=0	a=0.1	a=0.5
τ_L	5.2138	5.4494	5.4976	5.5396	5.5831
τ_U	-2.9647	-4.1236	-4.4978	-5.9102	-6.9482

(b) S=2	a=-0.5	a=-0.1	a=0	a=0.1	a=0.5
τ_L	2.2994	2.3484	2.3527	2.3546	2.3428
τ_U	-3.5060	-5.1159	-5.6472	-6.2401	-9.3602

Table 2. - Variation of the Steady State Temperature and the Nusselt Number
at Both Walls of the Channel with the Parameters "a" and "b" and for S=0

θ	a=-0.5	a=-0.1	a=0	a=0.1	a=0.5
b=-0.5	0.8594	1.0308	1.0888	1.1547	1.4789
b=-0.1	0.8178	0.9290	0.9546	1.0042	1.1997
b=0	0.8107	0.9129	0.9453	0.9812	1.1581
b=0.1	0.8044	0.8989	0.9287	0.9615	1.1232
b=0.5	0.7832	0.8564	0.8789	0.9036	1.0244

Nu_L	a=-0.5	a=-0.1	a=0	a=0.1	a=0.5
b=-0.5	2.0637	2.3867	2.4888	2.5995	3.0493
b=-0.1	2.0626	2.3969	2.5013	2.6153	3.1463
b=0	2.0678	2.4031	2.5073	2.6213	3.1583
b=0.1	2.0741	2.4102	2.5145	2.6283	3.1691
b=0.5	2.1049	2.4436	2.5474	2.6607	3.2074

Nu_U	a=-0.5	a=-0.1	a=0	a=0.1	a=0.5
b=-0.5	-0.9711	-1.6109	-1.8097	-2.0236	-2.8695
b=-0.1	-0.5219	-0.9038	-1.0248	-1.1589	-1.8219
b=0	-0.4715	-0.8166	-0.9256	-1.0464	-1.6473
b=0.1	-0.4319	-0.7464	-0.8454	-0.9549	-1.5023
b=0.5	-0.3358	-0.5673	-0.6391	-0.7181	-1.1144

CONCLUSION

The unsteady Couette flow of a viscous incompressible fluid between two parallel plates has been studied with temperature dependent viscosity and thermal conductivity in the presence of uniform suction and injection. It was found that increasing the viscosity exponent "a" increases the velocity u and the temperature θ for all values of the suction parameter S. Increasing S decreases u and θ for all values of the parameter "a". In the case of zero suction, the effect of "b" on θ depends on the time t and leads to the appearance of crossover in θ-t charts. On the other hand, in the presence of suction, increasing "b" increases θ for all time and leads to the suppression of the crossover in θ-t charts. It was observed that the effect of suction on the velocity u depends greatly on the viscosity parameter. The parameter "b" has a marked effect on the temperature field while its effect on the velocity field can be entirely neglected.

Table 3. - Variation of the Steady State Temperature and the Nusselt Number
at Both Walls of the Channel with the Parameters "a" and "b" and for $=2

θ	a=-0.5	a=-0.1	a=0	A=0.1	a=0.5
b=-0.5	0.2824	0.2992	0.3028	0.3062	0.3163
b=-0.1	0.3127	0.3316	0.3355	0.3391	0.3486
b=0	0.3223	0.3415	0.3454	0.3489	0.3582
b=0.1	0.3320	0.3514	0.3553	0.3589	0.3679
b=0.5	0.3700	0.3894	0.3932	0.3966	0.4044

Nu_L	a=-0.5	a=-0.1	a=0	A=0.1	a=0.5
b=-0.5	0.5391	0.5302	0.5274	0.5244	0.5105
b=-0.1	0.4248	0.4211	0.4187	0.4158	0.3996
b=0	0.4067	0.4037	0.4014	0.3985	0.3817
b=0.1	0.3913	0.3889	0.3866	0.3837	0.3662
b=0.5	0.3464	0.3459	0.3436	0.3404	0.3205

Nu_U	a=-0.5	a=-0.1	a=0	A=0.1	a=0.5
b=-0.5	1.7446	1.3378	1.2287	1.1162	0.6346
b=-0.1	0.9422	0.6985	0.6345	0.5692	0.2951
b=0	0.8314	0.6092	0.5511	0.4919	0.2449
b=0.1	0.7392	0.5349	0.4818	0.4278	0.2029
b=0.5	0.4878	0.3335	0.2938	0.2536	0.0886

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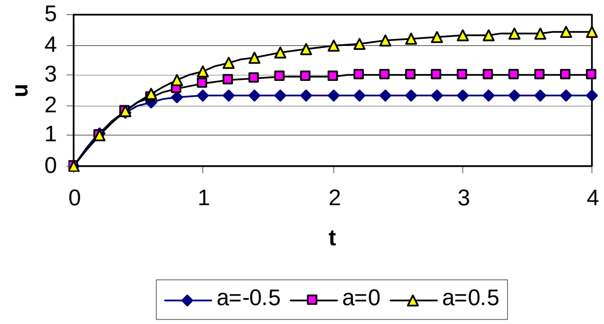
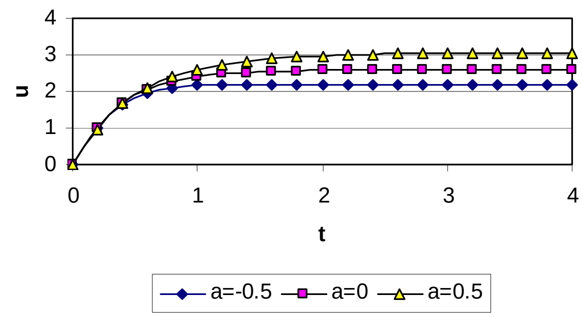
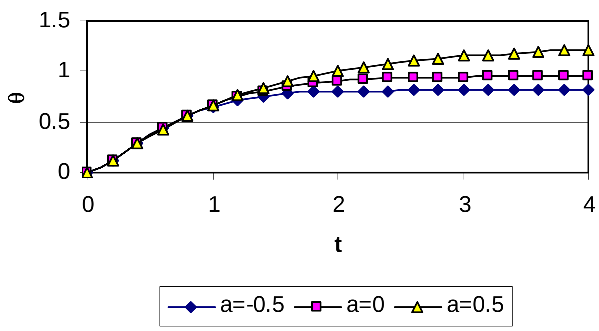


(a)

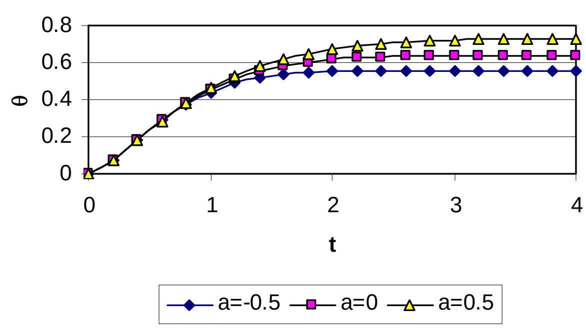


(b)

Fig. 1 Time development of u at y=0 for various values of a and S
(a) S=0; (b) S=1. (b=0)

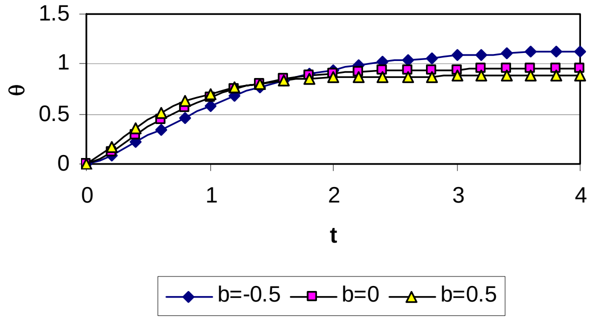


(a)

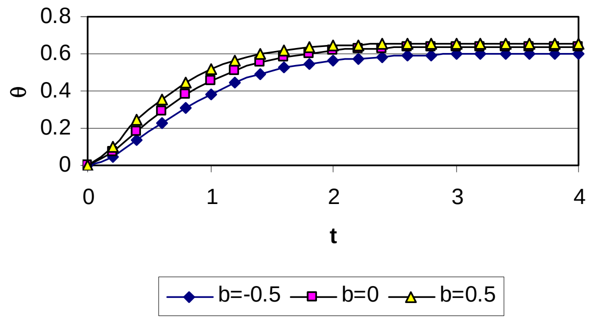


(b)

Fig. 2 Time development of θ at y=0 for various values of a and S
(a) S=0; (b) S=1. (b=0)



(a)



(b)

Fig. 3 Time development of θ at y=0 for various values of b and S
(a) S=0; (b) S=1. (a=0)