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EFFECT OF POROSITY ON UNSTEADY COUETTE FLOW WITH HEAT TRANSFER IN THE PRESENCE OF UNIFORM SUCTION AND INJECTION

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ABSTRACT. The unsteady Couette flow through a porous medium of a viscous, incompressible fluid bounded by two parallel porous plates is studied with heat transfer. A uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to a constant pressure gradient. The two plates are kept at different but constant temperatures while the viscous dissipation is included in the energy equation. The effect of the porosity of the medium and the uniform suction and injection on both the velocity and temperature distributions is examined.

INTRODUCTION

The flow between two parallel plates is a classical problem that has many applications in accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, fluid droplets and sprays, magnetohydrodynamic (MHD) power generators and MHD pumps. A lot of research work concerning the flow between two parallel plates has been obtained under different physical effects [1-10].

In the present study, the unsteady Couette flow and heat transfer in a porous medium of an incompressible, viscous, fluid between two infinite horizontal porous plates are studied. The fluid is acted upon by a constant pressure gradient, a uniform suction and injection perpendicular to the plates. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The flow in the porous media deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [11-13]. The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The governing equations are solved numerically taking the viscous dissipation into consideration.

The effect of the porosity and the suction and injection on both the velocity and temperature distributions is studied.

DESCRIPTION OF THE PROBLEM

The two non-conducting plates are located at the $y=\pm h$ planes and extend from $x=-\infty$ to ∞ and $z=-\infty$ to ∞. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2>T_1$. The fluid flows between the two plates under the influence of a constant pressure gradient dP/dx in the x-direction, and a uniform suction from above and injection from below which are applied at $t=0$. The upper plate is moving with a constant velocity U_o while the lower plate is kept stationary. The flow is through a porous medium where the Darcy model is assumed [13]. From the geometry of the problem, it is evident that $\partial/\partial x=\partial/\partial z=0$ for all quantities apart from the pressure gradient dP/dx, which is assumed constant. The velocity vector of the fluid is

$$v(y,t) = u(y,t)i + v_o j$$

with the initial and boundary conditions $u=0$ at $t\le 0$, and $u=0$ at $y=-h$, and $u=U_o$ at $y=h$ for $t>0$. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T=T_1$ at $t\le 0$, $T=T_2$ at $y=+h$, and $T=T_1$ at $y=-h$ for $t>0$. The fluid flow is governed by the momentum equation

$$\rho\frac{\partial u}{\partial t} + \rho v_o\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K}u \qquad (1)$$

where ρ and μ are, respectively, the density and the coefficient of viscosity and K is the Darcy permeability [11-13]. To find the temperature distribution inside the fluid we use the energy equation [14]

$$\rho c\frac{\partial T}{\partial t} + \rho c v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2)$$

where c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second term on the right-hand side represents the viscous dissipation.

The problem is simplified by writing the equations in the non-dimensional form. We define the following non-dimensional quantities

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{z} = \frac{z}{h}, \hat{u} = \frac{u}{U_o}, \hat{P} = \frac{P}{\rho U_o^2}, t = \frac{tU_o}{h},$$

$\mathrm{Re} = \rho U_o h/\mu$, is the Reynolds number,
$S = v_o/U_o$, is the suction parameter,
$\Pr = \mu c/k$ is the Prandtl number,
$Ec = U_o^2/c(T_2 - T_1)$ is the Eckert number,
$M = h^2/K$ is the porosity parameter.

In terms of the above non-dimensional variables and parameters, the basic Eqs. (1)-(2) are written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}-\frac{M}{\mathrm{Re}}u, \tag{3}$$

$$\frac{\partial T}{\partial t}+S\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+\frac{Ec}{\mathrm{Re}}\left(\frac{\partial u}{\partial y}\right)^2, \tag{4}$$

The initial and boundary conditions for the velocity become

$$t\le 0: u=0, t>0: u=0, y=-1, u=1, y=1 \tag{5}$$

and the initial and boundary conditions for the temperature are given by

$$t\le 0: T=0, t>0: T=1, y=+1, T=0, y=-1. \tag{6}$$

NUMERICAL SOLUTION OF THE GOVERNING EQUATIONS

Equations (3) and (4) are solved numerically using finite differences [15] under the initial and boundary conditions (5) and (6) to determine the velocity and temperature distributions for different values of the parameters *M* and *S*. The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the *y*-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. Finally, the block tri-diagonal system is solved using Thomas' algorithm. All calculations have been carried out for dP/dx=-5, *Pr*=1, *Pr*=1 and *Ec*=0.2.

RESULTS AND DISCUSSION

Figure 1 presents the velocity and temperature distributions as functions of *y* for different values of the time starting from *t*=0 to the steady state. Figures 1a and 1b are evaluated for *M*=1 and *S*=1. It is observed that the velocity component *u* and temperature *T* reach the steady state monotonically and that *u* reaches the steady state faster than *T*. This is expected, since *u* acts as the source of temperature.

Figure 2 shows the effect of the porosity parameter *M* on the time development of the velocity *u* and temperature *T* at the centre of the channel (*y*=0). In this figure, *S*=0 (suction suppressed). It is clear from Fig. 2a that increasing the parameter *M* decreases *u* and its steady state time. This is due to increasing the porosity damping force on *u*. Figure 2b indicates that increasing *M* decreases *T* and its steady state time. This can be attributed to the fact that increasing *M* decreases *u* and, in turn, decreases the viscous dissipation which decreases *T*.

Figure 3 shows the effect of the suction parameter on the time development of the velocity *u* and temperature *T* at the centre of the channel (*y*=0). In this figure, *M*=0. In Fig. 3a, it is observed that increasing the suction decreases the velocity *u* at the center and its steady state time due to the convection of fluid from regions in the lower half to the center, which has higher fluid speed. In Fig. 3b, the temperature at the center is affected more by the convection term, which pumps the fluid from the cold lower half towards the centre.

CONCLUSION

The unsteady Couette flow through a porous medium of a viscous incompressible fluid has been studied in the presence of uniform suction and injection. The effect of the porosity and the suction and injection velocity on the velocity and temperature distributions has been investigated. It is found that both the porosity and suction or injection velocity has a marked effect on both the velocity and temperature distributions.

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