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ION SLIP EFFECT ON THE UNSTEADY FLOW OF A DUSTY CASSON FLUID THROUGH A CIRCULAR PIPE

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ABSTRACT: In this paper, the transient flow of a dusty viscous incompressible electrically conducting non-Newtonian Casson fluid through a circular pipe is studied taking the ion slip into consideration. A constant pressure gradient in the axial direction and a uniform magnetic field directed perpendicular to the flow direction are applied. The particle-phase is assumed to behave as a viscous fluid. A numerical solution for the governing equations is obtained using finite differences.

1. INTRODUCTION

The flow of a dusty and electrically conducting fluid through a circular pipe in the presence of a transverse magnetic field has important applications such as magnetohydrodynamic generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle-phase viscosity. There have been many articles dealing with theoretical modelling and experimental measurements of the particle-phase viscosity in a dusty fluid (Soo, 1969; Gidaspow et al., 1989; Grace, 1982; Sinclair and Jackson, 1989).

The flow of a conducting fluid in a circular pipe has been investigated by many authors (Gadiraju et al., 1992; Dube and Sharma, 1975; Ritter and Peddieson, 1977; Chamkha, 1994). (Gadiraju et al., 1992) investigated steady two-phase vertical flow in a pipe. (Dube and Sharma, 1975) and (Ritter and Peddieson, 1977) reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses. (Chamkha, 1994) obtained exact solutions which generalize the results reported in (Dube and Sharma, 1975; Ritter and Peddieson, 1977) by the inclusion of the magnetic and particle-phase viscous effects. The heat transfer characteristics of circular pipe flow was studied by many researchers (Yoon et al., 2002; Kim, 2002; Kim, 2003). It should be noted that in the above studies the Hall current as well as the ion slip effects are ignored. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency, is high. This happens when the magnetic field is high or when the collision frequency is low (Crammer and Pai, 1973; Sutton and Sherman, 1965). Furthermore, the masses of the ions and electrons are different and, in turn, their motions will be different. Usually, the diffusion velocity of electrons is larger than that of ions and, as a first approximation, the electric current density is determined mainly by the diffusion velocity of the electrons. However, when the electromagnetic force is very large (such as in the case of strong magnetic field), the diffusion velocity of the ions may not be negligible (Crammer and Pai, 1973; Sutton and Sherman, 1965). If we include the diffusion velocity of ions as well as that of electrons, we have the phenomena of ion slip. In the above mentioned work, the Hall and ion slip terms were ignored in applying Ohm's law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of the electromagnetic force is noticeable under these conditions, and the Hall current as well as the ion slip are important; they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic-force term (Crammer and Pai, 1973; Sutton and Sherman, 1965).

A number of industrially important fluids such as molten plastics, polymers, pulps and foods exhibit non-Newtonian fluid behavior (Nakayama et al. 1988). Due to the growing use of these non-Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow characteristics. Many of the inelastic non-Newtonian fluids, encountered in chemical engineering processes, are known to follow the so-called "power-law model" in which the shear stress varies according to a power function of the strain rate (Metzner et al. 1965). It is of interest in this paper to study the influence of the magnetic field as well as the non-Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle-phase is considered dense enough to include the particulate viscous stresses.

In the present study, the unsteady flow of a dusty electrically conducting non-Newtonian Casson fluid through a circular pipe is investigated considering the ion slip. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing momentum equations for both the fluid and particle-phases are solved numerically using the finite difference approximations. The effect of the magnetic field, the Hall current, the ion slip and the particle-phase viscosity on the velocity distributions of the fluid and particle-phases is reported.

2. GOVERNING EQUATIONS

Consider the unsteady, laminar, axisymmetric horizontal flow of a dusty conducting fluid through an infinitely long pipe of radius d driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The Hall current and the ion slip are taken into consideration and the magnetic Reynolds number is assumed to be very small and consequently the induced magnetic field is neglected (Crammer and Pai, 1973; Sutton and Sherman, 1965). We assume that both phases behave as viscous fluids and that the volume fraction of suspended particles is finite and constant (Chamkha, 1994). Taking into account these and the previously mentioned assumptions, the governing momentum equations can be written as

$$\rho\frac{\partial V}{\partial t}=-\frac{\partial P}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V}{\partial r}\right)+\frac{\rho_p\phi}{1-\phi}N(V_p-V)-\frac{\sigma B_o^2(1+\beta_i\beta_e)V}{(1+\beta_i\beta_e)^2+\beta_e{}^2} \quad (1)$$

$$\rho_p\frac{\partial V_p}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu_p r\frac{\partial V_p}{\partial r}\right)+\rho_p N(V-V_p) \quad (2)$$

where t is the time, r is the distance in the radial direction, V is the fluid-phase velocity, V_p is the particle-phase velocity, ρ is the fluid-phase density, ρ_p is the particle-phase density, $\partial P/\partial z$ is the fluid pressure gradient, φ is the particle-phase volume fraction, N is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce e^{-1} of its original value (Chamkha 1994), σ is the fluid electrical conductivity, $Be=\sigma\gamma B_o$ is the Hall parameter, γ is the Hall factor (Crammer and Pai, 1973; Sutton and Sherman, 1965), B_o is the magnetic induction, Bi is the ion slip parameter, μ_p is the particle-phase viscosity which is assumed constant, and μ is the apparent viscosity of the fluid which is given by,

$$\mu=\left(K_c+\sqrt{\frac{\tau_o}{\left|\frac{\partial V}{\partial r}\right|}}\right)^2$$

where K_c is the coefficient of viscosity of a Casson fluid, τ_o is the yield stress, and $\left|\partial V/\partial r\right|$ is the magnitude of the velocity gradient which is always positive regardless of the sign of $\partial V/\partial r$. In this work, ρ, ρ_p, μ_p, φ, and B_o are all constant. It should be pointed out that the particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid-phase.

The initial and boundary conditions of the problem are given as

$$V(r,0)=0, V_p(r,0)=0, \quad (3a)$$

$$\frac{\partial V(0,t)}{\partial r}=0, \frac{\partial V_p(0,t)}{\partial r}=0, V(d,t)=0, V_p(d,t)=0 \quad (3b)$$

where d is the pipe radius.

Equations (1)-(3) constitute an initial-value problem which can be made dimensionless by introducing the following dimensionless variables and parameters

$$\bar{r}=\frac{r}{d}, \bar{t}=\frac{tK_c}{\rho d^2}, G_o=-\frac{\partial P}{\partial z}, k=\frac{\rho_p\phi}{\rho(1-\phi)}, \bar{\mu}=\frac{\mu}{K_c}$$

$$\overline{V}(r,t)=\frac{K_c V(r,t)}{G_o d^2}, \overline{V}_p(r,t)=\frac{K_c V_p(r,t)}{G_o d^2},$$

$\alpha = Nd^2 \rho / K_c$ is the inverse Stokes' number,
$\beta = \mu_p / K_c$ is the viscosity ratio,
$\tau_D = \tau_o / G_o d$ is the Casson number (dimensionless yield stress),
$H_a = B_o d\sqrt{\sigma / K_c}$ is the Hartmann number (Sutton et al. 1965).

By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs. (1)-(3) can be written as (the bars are dropped),

$$\frac{\partial V}{\partial t}=1+\sqrt{\mu}\frac{\partial^2 V}{\partial r^2}+\frac{\mu}{r}\frac{\partial V}{\partial r}+k\alpha(V_p - V)-\frac{H_a^2(1+\beta_i\beta_e)V}{(1+\beta_i\beta_e)^2+m^2} \tag{4}$$

$$\frac{\partial V_p}{\partial t}=\beta\left(\frac{\partial^2 V_p}{\partial r^2}+\frac{1}{r}\frac{\partial V_p}{\partial r}\right)+\alpha(V-V_p) \tag{5}$$

$$\mu=\left(1+\sqrt{\frac{\tau_D}{\left|\frac{\partial V}{\partial r}\right|}}\right)^2$$

$$V(r,0)=0, V_p(r,0)=0, \tag{6a}$$

$$\frac{\partial V(0,t)}{\partial r}=0, \frac{\partial V_p(0,t)}{\partial r}=0, V(1,t)=0, V_p(1,t)=0 \tag{6b}$$

The volumetric flow rates and skin-friction coefficients for both the fluid and particle phases are defined, respectively, as (Chamkha, 1994)

$$Q=2\pi\int_0^1 rV(r,t)dr, Q_p=2\pi\int_0^1 rV_p(r,t)dr, C=-\frac{\partial V(1,t)}{\partial r}, C_p=-\beta k\frac{\partial V_p(1,t)}{\partial r} \tag{7}$$

3. RESULTS AND DISCUSSION

Equations (4) and (5) represent a coupled system of nonlinear partial differential equations which are solved numerically under the initial and boundary conditions (6), using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method (Mitchell et al. 1980 and Evans et al. 2000) is used at two successive time levels. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas algorithm (Mitchell et al. 1980 and Evans et al. 2000). Computations have been made for α=1, γ=1, and k=10. Grid-independence studies show that the computational domain $0<t<\infty$ and $0<r<1$ can be divided into intervals with step sizes Δt=0.0001 and Δr=0.005 for time and space respectively. It should be mentioned that the results obtained herein reduce to those reported by Dube et al. (1975) and Chamkha (1994) for the cases of non-magnetic, inviscid particle-phase (B=0), and Newtonian fluid. These comparisons lend confidence in the accuracy and correctness of the solutions.

Figures 1a and b present the time evolution of the velocity of the fluid V and dust particles V_p at the center of the pipe, respectively, for various values of the ion slip parameter Bi and the Hall parameter Be and for Ha=3, τ_D=0 and B=0.5. Both V and V_p increase with time and V reaches the steady state faster than V_p for all values of Be and Bi. It is clear from the figures that increasing Be or Bi increases both V and V_p while its effect on their steady state times can be neglected. This is due to the decrease in the effective conductivity which reduces the damping magnetic force on V. Figures 2a and b present the time evolution of the velocity of the fluid V and dust particles V_p at the center of the pipe, respectively, for various values of the ion slip parameter Bi and the Hall parameter Be and for Ha=3, τ_D=0.05 and B=0.5. It is clear from Figs. 1 and 2 that increasing τ_D decreases V and V_p for all Be and Bi. Also, the effect of Be and Bi on both V and V_p becomes more pronounced for higher values of τ_D.

Figures 3a and b present the time evolution of the velocity of the fluid V and dust particles V_p at the center of the pipe, respectively, for various values of the ion slip parameter Bi and the Hartmann number Ha and for Be=1, τ_D=0 and B=0.5. It is clear that increasing Ha decreases V and V_p and their steady state times for all values of Bi due to the increase in the damping magnetic force. The figures indicate also that the effect of Bi on V and V_p becomes more pronounced for higher values of Ha. Figures 4a and b present the time evolution of the velocity of the fluid V and dust particles V_p at the center of the pipe, respectively, for various values of the ion slip parameter Bi and the Hartmann number Ha and for Be=1, τ_D=0.05 and B=0.5. It is shown in these figures that increasing τ_D decreases both V and V_p for all values of Bi and Ha.

Figures 5a and b present the time evolution of the velocity of the fluid V and dust particles V_p at the center of the pipe, respectively, for various values of the ion slip parameter Bi and the viscosity ratio B and for Ha=1, Be=1 and τ_D=0. The figures indicate that increasing B decreases both V and V_p and their steady state times for all values of Bi. The effect of the parameter Bi on V and V_p is more apparent for higher values of the parameter B. Figures 6a and b present the time evolution of the velocity of the fluid V and dust particles V_p at the center of the pipe, respectively, for various values of the ion slip parameter Bi and the viscosity ratio B and for Ha=1, Be=1 and τ_D=0.05. It is clear that increasing τ_D decreases V and V_p and their steady state time for all values of B and Bi. Table 1 presents the steady state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters τ_D and Bi and for Ha=3, Be=1 and B=0.5. It is clear that increasing the parameter Bi increases Q, Q_p, C, and C_p for all values of τ_D. It is also shown that increasing τ_D decreases Q, Q_p, C, and C_p for all values of Bi.

Table 2 presents the steady state values of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of the parameters Bi and B and for Ha=3, Be=1 and τ_D=1.5. It is clear that, increasing Bi increases Q, Q_p, C, and C_p for all values of B and its effect becomes more pronounced for smaller values of B. Increasing the parameter B decreases the quantities Q, Q_p, and C, but increases C_p for all values of Bi.

4. CONCLUSIONS

The transient MHD flow of a particulate suspension in an electrically conducting non-Newtonian Casson fluid through a circular pipe is studied considering the ion slip. The governing partial differential equations are solved numerically using finite differences. The effect of the magnetic field parameter, the Hall parameter, the ion slip parameter, the Casson number, and the particle-phase viscosity on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases is studied. It is shown that increasing the magnetic field or the viscosity ratio decreases the fluid and particle velocities, while increasing the Hall parameter or the ion slip parameter increases both velocities. It is found that, the effect of the ion slip on the fluid and particle velocities is more apparent for higher values of the magnetic field, the Casson number or the viscosity ratio.

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Table 1

The steady state values of Q, Q_p, C, C_p for various values of Bi and n and for Ha=3, Be=1, B=0.5

τ_D=0	Bi=0	Bi=1	Bi=3
Q	0.1361	0.1429	0.1559
Q_p	0.0325	0.0342	0.0374
C	0.2395	0.2468	0.2606
C_p	0.1634	0.1715	0.1867

τ_D=0.025	Bi=0	Bi=1	Bi=3
Q	0.1297	0.1361	0.1482
Q_p	0.0309	0.0325	0.0356
C	0.2275	0.2344	0.2474
C_p	0.1557	0.1632	0.1774

τ_D=0.05	Bi=0	Bi=1	Bi=3
Q	0.1232	0.1291	0.1402
Q_p	0.0294	0.0309	0.0336
C	0.2154	0.2219	0.2339
C_p	0.1479	0.1549	0.1679

Table 2

The steady state values of Q, Q_p, C, C_p for various values of Bi and B and for Ha=3, Be=1, τ_D=0.05

B=0	Bi=0	Bi=1	Bi=3
Q	0.1672	0.1778	0.1986
Q_p	0.1299	0.1373	0.1517
C	0.2658	0.2770	0.2986
C_p	0	0	0

B=0.5	Bi=0	Bi=1	Bi=3
Q	0.1232	0.1291	0.1403
Q_p	0.0294	0.0309	0.0336
C	0.2154	0.2219	0.2339
C_p	0.1479	0.1549	0.1679

B=1	Bi=0	Bi=1	Bi=3
Q	0.1169	0.1222	0.1321
Q_p	0.0159	0.0164	0.0181
C	0.2093	0.2152	0.2260
C_p	0.1594	0.1666	0.1799