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THE INTRIGUING HUMAN PREFERENCE FOR A TERNARY PATTERNED REALITY

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Three paths through the high spring grass / One is the quicker / and I take it
(Haikku by Yosa Buson, Spring wind on the river bank Kema)

How did number theory degrade to numerology? How did numerology influence different aspects of human creation? How could some numbers assume such an extraordinary meaning? Why some numbers appear so often throughout the fabric of reality? Has reality really a preference for a ternary pattern? Present excursion through the 'life' and 'deeds' of this pattern and of its parent number in different fields of human culture try to offer an answer to some of these questions giving at the same time a glance of the strange preference of the human mind for a patterned reality based on number three, which, like the 'unary' and 'dualistic' patterned reality, but in a more emphatic way, ended up assuming an archetypical character in the intellectual sphere of humanity.

Introduction

Three is not only the sole number that is the sum of all the natural numbers less than itself, but it lies between arguably the two most important irrational and transcendental numbers of all mathematics, i.e., $e < 3 < \pi$. Furthermore, it is the first natural number for which its reciprocal product is bigger than its reciprocal sum, i.e., $3+3 < 3\times3$. It is considered a lucky number. Number three is also rooted in many words, such as triangle, triumvirate, trierarch, triune, trefoil, trilobites, trilogies, trinity, triptych, tricolor (flags), triclinic (crystals), trilingual (persons or countries), ternary, thrice, trivium, tribulations, triumph, trivial, trifling, trial, tree. But let us start from the beginning. A number is an abstract entity used to describe quantity. The most familiar numbers are the natural numbers (0, 1, 2, 3, ….) used for counting and denoted by **N**. Numbers should be distinguished from *numerals*, which are combinations of symbols used to represent numbers, and which allow people to discuss and write about numbers. Some numbers have also a history either as numbers and/or as numerals. In a series of published papers the history and importance of zero, one and two in human culture has been thoroughly analyzed [1-4]. The history of these three numbers offer a kind of fresco of the development not only of mathematics, but also of human culture, which from its origins on endowed some numbers with an esoteric meaning, which can be detected even today in their importance in many religions and sects, in everyday life, and even in science. But, how and when did numerology started? The numerical esoterism started officially with the first scientific school, the Pythagorean School at Crotona, and whose importance goes well beyond numerology, which was a kind of by-product. The famous words of a student of Pythagoras, Philolaus of Crotone [5]: *All things which can be known have number; for it is not possible that without number anything can be either conceived or known*, can be considered the critical moment for a series of mystical and esoteric inferences, which influenced the subsequent development of much pseudoscience and of many religions also. These words were to be echoed through all subsequent ages by different thinkers. Saint Isidore of Sevilla (560-636 A. D.) said something very similar and quite intriguing for a man of faith: *Tolle numerum omnibus rebus et omnia pereunt* (take from all things their number, and all shall perish). Another churchman, the German monk, Hrabanus Maurus, (776-869) preferred to state that *in many holy writings numbers cover secrets that should remain mysterious to everybody who ignores numbers, and it is for this reason that everybody who wants to reach the highest form of knowledge has to master arithmetics*; St. Augustine, instead, held God as the great numerologist [4]. It is said that Gauss held number theory as the Queen of Mathematics, in this way he practically acknowledged the fascination that numbers had on him also. A modern interpretation of the words of Philolaus, which emphasize the strict interplay between science and numbers, was put forward by Isaac Newton (1642-1727) [6] and with more emphasis by Lord Kelvin (1824-1907), one of the fathers of Thermodynamics: *When you can measure what you are speaking about, and express it in numbers, you know something about it. But when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: It may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science*. The strange fascination that numbers had on some churchmen was interpreted in an original way by E.T. Bell, who had to say: *It seems never to have occurred to Saint Augustine of Hippo and his able followers to inquire whether it was their religion or their science that they really believed in* [7].

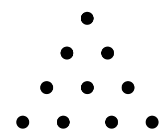
Now, number three and the corresponding ternary pattern derived from it seems central in most of the western civilization, and it has been suggested that it may be a characteristic of the whole Indo-European culture. There are other numerical patterns in human culture and each culture has developed its own preference for a particular numerical pattern, among which, two, four, seven, twelve, nineteen pattern. If, nevertheless a pattern number should deserve the quality of being universal then our preference would fall on number two and on dualism and not three [8], even if 'trialism', in some fields, comes very close to it. This is not only suggested by the widespread occurrence of triads of deities, and other occurrences of this pattern in many religions, but it is confirmed by the important role this pattern plays in literary, philosophic, scientific and everyday domains. Nevertheless, as we shall see in the mathematic section, notwithstanding the apparent 'ideological' importance of numbers two and three, citation of these two numbers in many human data, stands well after number one, and number three after number two.

On a geometrical level, triangles are ubiquitous in Euclidean geometry, and it can surely be stated that they were the first geometrical objects to awaken human curiosity, as it is certified from the fact that the Pythagorean rule for right triangles was already heuristically used by Babylonian and Egyptian architects in land delineations. Everybody knows that there are three laws of thermodynamics, even if some people consider that they are two, others four, and others even five laws of thermodynamics. College students know that Earth is the third planet orbiting the Sun (Mercury, Venus and Earth) that there are three laws of Kepler and three laws of Newton, while three main migrations by our common ancestors out of Africa have been proposed. Real numbers are also divided into three ‘kingdoms’: natural, rational, and irrational. Subatomic particles can be neutral, positively and negatively charged, while the development of Quantum mechanics can be divided into three periods, as we shall see throughout the Physics section. Africa, Asia, and Europe, due to their proximity give rise to a highly differentiated supracontinent. While do the names of the continents normally start with *A*, e.g., Africa, Asia, America, Antarctic, and only the name of Europe starts, in nearly all languages, with the second vocal of the Latin alphabet, *E* ?

Concerning the special place, which is reserved to the occurrence of three in different religions, where it often assumes a mystical meaning, one could imagine that the esoteric meaning that three ended up to assume is deeply related to extensions beyond the intense emotional ties to one and two. The emotionally intense significance of “1” is manifested with the “self” (or self-consciousness), while the emotional significance of “2” is manifested in the equal intense feelings towards the not self, the other, the different, and more generally the entire world outside the self, and this even in connection with one’s mate or with one’s (perhaps violent) competitor. But beyond these two intensely self-centered emotion-based realities, the conscience discovers a third entity that exists beyond me and you, the other, which sometimes can facilitate things, acting as a kind of ‘medium’ between my world and your world, and in some other times can worsen the situation by doubling the foreign world which I have to withstand. The ordinal three is used to designate the third person he/she, and the third person is used in some languages as an official colloquial form either to express deference, or to talk with unknown people or with people with whom no friendship is shared. This way the third person is practically the first unknown person, underlining once again the old meaning of three as the first number of the number system.

Ancient History of Number Three

Old Greeks and Romans considered the number three to be the very first number of the natural number system. In fact, the Greek mathematician Nichomachus regarded number 3 as the first number in the strict sense of this word, for he considered 1 and 2 only the generators of the number system, i.e., a kind of parents (1 = man and 2 = woman) of the whole number system. Nichomachus, who was clearly influenced by Pythagorean number mysticism, preached a kind of “anthropomorphic number religion”, where numbers were endowed with human qualities and could even be character forming. Thus, while 1 and 2 were considered a kind of generators (male and female) of the entire number system, three became to be known as the first number, while ancient civilizations regarded 3 as a symbol for plurality. It has been remarked that human capacity for immediately recognizing concrete quantities very seldom goes beyond four, a capacity which humans share with many other animal species [5, 9-11]. Science and mathematics, practically, starts with Pythagoras and his famous school, the Pythagoreans. The Pythagoreans made the following identifications: 1 was the mind (the One), 2 was the opinion (the first moving away from unity); 3 represented the wholeness (beginning, middle, and end); 4 was the justice (a square deal), and 5 stood for the marriage, since $5 = 2 + 3$ (even numbers were regarded as female, and odd numbers as male). During the Middle Ages the first nine numbers were collected into three triads, which had a special meaning: 1,2,3 ; 4,5,6 ; 7,8,9. These triads had been connected with the old alchemical idea of the planets (the nine spheres) [9] as we shall see in the Chemistry section. In the Middle Ages numerology, three closed the first triad (1, 2, and 3), therefore number 3 was seen as containing the beginning and the middle, and as encoding the overall multiplicity. Lunar animals were often depicted as three-legged as it was considered that the phases of the moon were only three (actually they are eight). Under a later and more precise system, the numbers one through four were identified with the point, line, plane, and solid, respectively. In fact, in Euclidean geometry through two points there is only a line, 3 points are needed to define a plane, and a plane plus a point outside the plane define a solid. Number ten symbolized perfection, as people have ten fingers, and as $10 = 1 + 2 + 3 + 4$. This “triangular numbers” were represented by the Pythagorean tetractys (more in the mathematical section):



Due to the special place of number ten, Pythagoreans believed that the spheres turning around the Earth should also be ten, and to get things straight on this subject (actually they identified only nine), they imagined a counter-Earth always hidden by the sun. They further believed that the whole planetary system was revolving around a mythical and mystical central eternal fire, *Hestia*, which was not the sun, and which was revolving around the Earth. Pythagoreans believed that there is one cosmic mind or soul, and that a small piece of this soul is imprisoned in our body before going through a series of reincarnations into other bodies (metempsychosis). A special life-style could help to avoid the cycle of reincarnations, and allow the soul to ascend directly to the cosmic soul. Hindu influence is here evident; in fact, it seems that Pythagoras sojourned for a while in the Far East [5, 9, 10]. The Pythagorean line of thought and even way of life had an enormous influence on the following generations of philosophers, scientists, thinkers, and religion founders, first of all Plato. Actually, many modern scientists are prone to believe in an objective reality of mathematical objects [12-15, to cite only some, who include in the number Hilbert, Gödel, Hawking, etc.]. Should this be considered a modern version of the ancient Pythagorean number mysticism updated with Plato's philosophical system? Modern research on the Christ has established that Jesus himself knew the rudiments of Greek language, and some rudiments of Greek philosophy, thus, very probably, he knew something about the Pythagorean number mysticism, and so did the evangelists, as some passages of the New Testament seem to disclose [16, 17].

In ancient times (the Roman number IV was a late introduction), numbers 2, 3, and 4 were written as an ensemble of ones, that is, II, III, and IIII, both vertically and horizontally [5, 9-11], a fact which not only underlines the importance of one, but which has also to do with our perception of numbers. The ancient Greek mathematics for a long time tried to cope with three problems, which were never solved: the trisection of an angle, i.e., given an arbitrary angle, construct by means of compass and straightedge alone an angle 1/3 as large as the given angle, the squaring of the circle, and the duplication of a cube. These three problems were born around 450-400 BC and only 2200 years later it was to be proved that all three of them were unsolvable by means of straight-edge and compasses alone. Ancient mathematicians could have been fascinated from the strange correlation of 1 with 3 by the fact that through three points it is possible to trace only a plane. On the other hand, the diagonal of a cube with unit side involves an irrational number, i.e., $\sqrt{3}$. As it is well known, irrational numbers, since their discovery by a student of Pythagoras, plagued the entire history of ancient mathematics, till the advent of modern number theory. On a more religious side, the importance of trinity has surely something to do with the perception of a three-dimensional infinite Euclidean space. Well before becoming a central number in the Christian religion, the Hindu religion highly worshipped number 3, even if in the catholic religion, especially since the dogma of the virgin Mary (established 150 years ago by Pius IX), there is a strange interplay between 3 and 4. Here the cult of the holy trinity is sided in importance by the additional cult of the virgin Mary. Catholics seem, thus, to reintroduce from the back door, after having officially abolished female Gods, the concept of a female God, while the reformed Christian Churches preferred to stick with a male God.

Notation, Symbol, Meaning and Personification of Number Three

Our modern notation for three derives from the Hindu notation, which for this number is practically the same, while Greeks, Romans, and Jews initially used alphabet letters to represent numbers. The number sequence as such begins with three, as number three starts the infinite sequence of numbers. In fact, from the cardinal numbers 3, 4, 5, ... are formed the ordinals, third, fourth, fifth, … , while with the couples two-second and one-first the scenario is completely different. It seems that early man's thinking was one, two, and many, or in some other civilizations: man, woman, and many. It has been suggested that Latin *tres* is related to Latin trans (across, beyond). French language bears memory of *three* as a synonym of *many*, in fact, *trois* (3) is very similar to *très* (very), while the similarity of English *three* with *through* bears also sign of this use. The Egyptian and the Chinese to express many, i.e., the plural, used to write the character for it three times. Thus, *water* was designated by three waves, *many plants* by three plants, *hair* by three hairs. Nowadays numerals *one* and, seldom, *two* are inflected in some languages to agree with the gender of the counted objects. Thus, in Romanian language for one we have un/una, and in Italian language we have uno/una, for two, instead, in Romanian language we have doi/doua, while in Italian it is not inflected. Note that both languages are derived from Latin. However, in the primitive Indo-European languages and in languages derived from them, the first four number words not only had three genders but were also inflected. For three, in four ancient languages, we have

Greek	*Latin*	*Gothic*	*Middle High German*
treis, tria	tres, tria	preis, prija	dri, driu
triôn	trium	prije	dri(g)er
trisí	tribus	prium	drin

The modern Anglo-Saxon languages have similar words for number 3, namely: English, *three*; Dutch, *drie*; Icelandic, *prir*; Danish: *tre*, Swedish, *tre*; High German, *drei*. Celtic languages have a unique common form for the number 3, that is Irish, Welsh, Cornish and Breton: *tri*. Slavic languages have, Czech, *tři* and Russian, *tri*. For the Romance languages we have: French, *trois*; Italian, *tre*; Romanian, *trei*; Spanish, *tres*, Portuguese, *três*. Finally we remember that in the Sumerian language number sequence three was spelled *eš*.

Counting by fingers assigns (as in many tribes of Native Americans) the middle finger to represent three. Further it has been suggested, but not proved, that the personal pronouns, *I*, *Thou*, and *He* were incorporated into the first number words. An interesting feature is that Sanskrit and Celtic languages have preserved an ancient feminine form for three (and four also): Indo-European: *t(r)i-sores*; Sanskrit: *tis-rah* and *čatasrah*; Celtic: *teoir* and *cetheoira*, which can be related to '*sor*', 'wife, woman'. This may have taken its name from the 'third wife', who followed the two principal wives. Number word three is hidden in some other names. Thus, *Drillich*, *Drilch* or *Drell* a triple stranded material used in army jackets, seems to be a Germanization of the Latin '*tri-licium*', i.e., three-threaded. The word *Trense*, 'braid, twist' originates from the Latin *trini-care* (each three), and literally means 'to three', and here the 'braid from three strands', as with hair. In Spanish it gave rise to the verb *trenzar*, whence trenza, 'plaid, braid, cable'. This word was brought to Holland by the Spanish army and it entered Germany as 'trensse', a light bridle with a curb-rein. The French *tresse*, Italian *treccia* have the same root. A *Dreidraht* (literally a three-cord) in Germany is a bore. German *drehen* (to twist, turn), and *Draht* (wire, cord) as well as English *drill* also hide a *three*. The word *trivial* goes back to the Latin noun *trivium* (three roads), and both in Roman and Middle Age times referred to the three basic disciplines of the curriculum (grammar, rhetoric, and dialectics), after which the student could move on to the *quadrivium* (arithmetic, geometry, astronomy, and music). Together they made up the *seven liberal arts*. Trivial has, thus, the meaning of something unimportant, easy, self-evident. The famous fountain *Trevi* in Rome, which has three streams of water, used to be called *Fontana Trivia*. Another word which hides a three is *tribute*, which comes from the Latin *tribus*, and the Indo-European *tri-bhu-s*, where *-bhus* means 'to be'. Originally it meant a 'third or third part' and then a district or 'community' as Rome arose from the three Italic tribes, the Romans, the Sabines, and the Albans. Thus, *tribus* came to mean a tribe, and *tributum* a payment or tax due by the tribe, while *tribune* was in charge of a tribe, and the seat from which he dispensed justice was the *tribunal*. Also *testament* hides a *three*, even if it is quite hard to discover. Everything starts with the old Latin word *tristaamentud* > testamentum, being the third (Latin *tristo* = to be a third party) as the last wills should be read in the presence of a third person as witness. There is also *terzerol* for an old small pocket pistol, which comes from hunting with the aid of a male falcon, which is about a *third* smaller than the female. Can you see the three hidden in the German word *Kümmel* ? This is a misunderstood assimilation of *gimel*, the third letter of the Hebrew alphabet [9].

We already told that **i**n many cultures, the notation for the numbers *one*, *two*, and *three* is very similar: in Roman numerals they are I, II, and III; in Chinese and in Brahmin, the same notation is used but with tally marks written horizontally, - , =, and ≡. The tally marks disappeared with number four, and other numerals were introduced in these languages. Psychologists explain that the reason for the shift from a simple tally notation to one involving more convoluted symbols is the difficulty humans have in visually differentiating similar patterns with more than three identical elements, i.e., how to tell at a glance the difference between IIIII and IIIIII ? Instead, it is much easier to differentiate at a glance between V and VI. To designate numbers 0 to 9, Sanskrit texts of Indian scientific astronomy, whence both our modern place-value numeration and the number zero originate, not only use familiar words for them, i.e., tri for three, but also a large collection of other Sanskrit words, each with a meaning that evoked certain numerical value (i.e., things, ideas, persons or animals denoted by the words were taken as number symbols). Thus, three was evoked by the three primordial properties (guna, triguna), the three worlds (loka, bhuvana), the three divisions of time (kāla, trikāla), the three Vedic fires (agni, vaiśvānara, vahni, dahana), the three eyes of Shiva (trinetra, Haranetra) [10]. The original Sanskrit words for the numerals 0 to 9 are: *śûnya (0)*, *eka*, *dvi*, *tri*, *catur*, *pañca (5)*, *şaț*, *sapta*, *asta*, *nava (9)*. Here, it is interesting to note the extraordinary similarity between some Sanskrit numeral words and our number words. The Hindu-Arabic numeral system, from which our numeral system has been derived, uses slightly modified tally marks for *1*, *2*, and *3*. Thus, while *1* has undergone nearly no modification, *2* and *3* are evidently based on the horizontal Chinese lines without lifting the pen. The notation for three underwent nearly no changes from its Indian (Gvalior) notation, through the Sanskrit-Devangari (Indian), West Arabic (Gubar), till our modern notation, which was settled upon during the 15th century.

Biology

Three sits at the core of molecular biology within the *encoding-decoding* machinery and in the concept of *codon-anticodon*. All living organisms share certain similarities: they can replicate and the replicator is a DNA molecule, and they can convert the information stored in DNA into products essentials for the function of the organism thanks to the 'encoder' messenger m-RNA, which transcribes the information stored in the DNA and gives it over to the 'decoder', the transfer t-RNA. Codons are triplets of adjacent nucleotides on the m-RNA, while the anticodons are the complementary triples on the t-RNA, which is bonded to an amino acid. Thus the information stored in the DNA is translated into the protein machinery, which, at its time, is essential for the transmission of the genetic machinery. Actually with a triplet of four nucleotides, $4^3 = 64$ combinations are possible, i.e., it is possible to encode all the twenty amino acids, plus many redundancies, while with a doublet codon it would only be possible to encode only $4^2 = 16$ amino acids with no redundancies. Redundancies play, normally, a positive role in any transmission code system as they allow minimize errors. [18]

An unhappy result (trisomy) connected with number three occurs when a human embryo has a triplet of chromosomes instead of the normal pair. Living cells divide by mitosis (normal replication) except for sex cells (gonads) whose nuclei contain only one from each pair of chromosomes. Humans have 23 pairs of chromosomes, 22 of them (autosomes) are exactly matched, and are inherited from the father and mother, respectively. The 23rd pair is the sex chromosome containing normally an X chromosome inherited from the mother, and from the father either a Y chromosome for male offspring, or another X chromosome for a female offspring. It so happens that sometimes the embryo develops with a triplet instead of a pair of chromosomes, and in most cases such a genetic defect does not result in a live birth. There are some exceptions which do result in live births, and the individuals have a so-called "trisomy". The most common is the trisomy-21 or the *Down syndrome*, also known as mongolism because of the slanted eyes; this is an autosomal disorder involving a third chromosome 21 so that the individual has 47 instead of the normal 46 chromosomes. In the USA about 1 in 700 live births is associated with the Down syndrome. The probability of such a genetic defect increases sharply with the age of the mother, reaching 1 in 16 live births for 45-year-old mothers. The life expectancy and the mental aptitudes of people with trisomy-21 are lower than for normal individuals, but people with mongolism are more affectionate. Less frequent is the autosomal-22 disorder which also results in mentally retarded people. More severe autosomal disorders with survival prognostics less than one year are: trisomy-13 (Patau syndrome) occurring with a probability of 1/5,000 and resulting in harelip, cleft palate, and eye/brain/circulatory system defects; and trisomy-18 (Edwards syndrome) – 1 for every 10,000 live births affecting practically every organ system.

Genetic defects involving the sex chromosomes (non-autosomal disorders) are also widespread. An XXY zygote (Klinefelter syndrome; 1 for 2,000 live births) is a sterile male with many female characteristics. An XXX zygote ("metafemale", 1 for 1,000 live births) is a female with limited fertility. An XYY zygote (1 for 1,000 newborn males) is a fertile and usually normal male; however, it seems that in penal and mental institutions there are 20 XYY trisomic for every 1,000 males, indicating that subtle effects on the mind may be present in such individuals.

The human eye, which allows vision, works thanks to two kinds of light-sensitive cells situated in the retina of the eye: the rods and the cones. Rod cells are responsible for vision at very low intensity (a good human eye fires with two photons, a normal eye fires with 5 photons) but cannot resolve sharp images or colour. Cones, instead, can resolve sharp images and colour but require a higher light intensity (with very poor light and at night, the vision is just black-and-white). There are *three* types of cones: cones sensitive to the red light (called red cones), cones sensitive to green light (green cones), and cones sensitive to blue light (blue cones). The normal coloured day vision is based on an imbalance between the stimulation levels of the different cell cone types. Thus, the cones of a normal person cover long (red), medium (green), and short wavelengths (blue). A small percentage of males, and an even smaller percentage of females, and many animals are Daltonic, i.e., colour blind as they do not have cones. An even smaller percentage of humans are *protanopic* (1% males and 0.02% females), i.e., they are red-colour blind: instead of red they see a kind of psychedelic colour, and they are unable to distinguish between red and green. Green blindness is called *deuteranopia*, and people who suffer from this defect are unable to see the green part of the spectrum. Blue blindness is called *tritaopia*, these last people are unable to distinguish between blue and yellow. The ability to distinguish colors exist only in some vertebrates, including, among others, man and the other primates, fish, amphibians, some reptiles, some birds, bees, and butterflies. Color blindness affects 20 times as many males as females, it is a sex-linked recessive defect. A woman must inherit the trait from both parents to be color-blind. A color-blind man and a woman of normal color vision have daughters, who have normal color vision but are carriers of the trait, that is, the daughters may have color-blind sons and daughters, who are carriers. The sons of a color-blind man and a woman with normal vision themselves have normal vision and are unable to pass the color-blind trait on to offspring. The son of a normal man and a carrier woman may be color-blind, and the daughter of such a union may be a carrier. Thus, color blindness tends to skip generations.

There are several anatomical formations in humans, whose names are derived from 3, because evolution resulted in three favorable formations. We can flex out arms by the concerted action of two muscles, the *biceps brachii*, and the *triceps brachii*. There are actually a few other triceps mucles also having three attachment points, less known to the general public: *soleus* and *gastrocnemius* muscles, in the leg. Then there is the *tricuspid* valve in the heart (the right atrioventricular valve, with three flaps) – the left atrioventricular valve has two flaps and is called bicuspid or mitral valve. The *trigeminal* nerve is the largest of the cranial nerves, and is the fifth one from the dozen pairs of cranial nerves that we have. It contains both sensory and motor fibers, and serves for moving and feeling organs on the face, forehead, jaws, and teeth. Trigeminal neuralgia ('tic douloureux') can produce quite sharp painful sensations in the nervous path to the jaws and eyes.

The three kingdoms of matter are mineral, vegetable, and animal. An old way to divide organisms was to divide them into two kingdoms, animal (animalia) and vegetal (plantae), while a less older one was to divide them into three kingdoms: animalia, plantae, and unicellular protista. A better scheme, which dates from the 19th century, divides all living organisms into five kingdoms: Monera (bacteria), Protista, Fungi, Plantae, and Animalia. These five kingdoms have been subdivided into three groups: (i) Monera, i.e., very simple unicellular organisms with no nucleus (mitochondria, bacteria, and other organisms), (ii) Protista, i.e., unicellular complex organisms, and (iii) Fungi, Plantae, and Animalia, i.e., multicellular organisms. This scheme coexisted with a scheme based on two divisions of the domain of life: Prokaryotae (bacteria, etc.), and Eukaryotae (animals, plants, fungi, and protists). Recent work on this scheme has shown, however, that the domains of life are, actually, three: Bacteria, Archea (recently discovered), and Eukaryota. No one of these groups is ancestor to the other, and each shares certain features with the others as well as having unique characteristics of its own. Note that the viruses, which may be crystallized, are not endowed with life in the same sense that eukaryotes, archeans, and bacteria are (even if viruses are of considerable chemical, biological and medical interest), because they need the machinery of living cells. This means that viruses must have appeared on the evolutionary scale after Archea. The three eras of the multicellular life, which started around $570 \cdot 10^6$ years ago, are: Palezoic, Mesozoic, and Cenozoic. Biological evolution gave rise to the dinosaurs, during the Mesozoic, which lasted from $225 \cdot 10^6$ till $60 \cdot 10^6$ tears ago. Nobody knows how life on earth would have evolved if the asteroid that fell close to the Yucatan peninsula about 60 million years ago had had another trajectory. The disappearance of the dinosaurs made way for mammals, which could develop undisturbed from small mammals into big mammals, thus, we are the result of post-dinosaur biological evolution. Among the most spectacular dinosaurs, triceratops had three long pointed horns, one on the nose and two one-meter-long ones above the eyes. Other life forms, which found that ternary symmetry was favorable are *trigonia*, which are mollusks that appeared about 200 million years ago and are still extant, and *trilobites*, which are extinct fossil arthropods that dominated the seas about half a billion years ago. Some trilobites were half a meter long and weighed 5 kg, but others were very small. It seems that they did not have "ternary symmetry" but rather the standard "bilateral symmetry", and perhaps the same is true of "trigonia". The *Triassic* geological period (from about 200 till 250 million years ago) was so named by Friedrich von Alberti in 1834 because rocks of this age in central Germany were of three kinds: Bunsandstein (the oldest), Muschelkalk, and Keuper (the youngst). During the triassic all landmasses formed one single supracontinent called Pangaea.

Today there is little doubt that humanity (genus Homo) was born in Africa and that from there humanity spread out over the world, and three types of Homo have been recognized: Homo habilis (appeared $2 \cdot 10^6$ years ago), Homo erectus ($1.6 \cdot 10^6$ years ago), and homo sapiens ($5 \cdot 10^5$-$2.5 \cdot 10^5$ years ago), i.e., ourselves. The Neanderthal man, who is, normally, considered a subspecies (with bigger brain), for a while coexisted with the homo sapiens till he underwent a complete extinction. Some suspect that he was 'eaten' by the homo sapiens. Furthermore, it seems that there were three great human Diasporas of the homo sapiens out of Africa [19, 20].

The three-field agricultural system was introduced in Europe in the Middle Ages (before Justus von Liebig's discovery of mineral fertilizers of which, more below) and replaced the old two-field system in which half of the land was left fallow in each season. By leaving fallow only a third and rotating crops (cereals and soil-enriching legumes) on the other two thirds, it was possible to reduce the risk of famine. For a long time it was known that organic fertilizers enhanced crop yields, and all the guano from the coast of South America, particularly the Chinchas and the Lobos Islands near Peru's coast, found its way to Europe. Liebig introduced in 1840 the idea of using synthetic nitrogen fertilizers, but it was only after Wilhelm Ostwald discovered how to oxidize ammonia to nitric acid and after Fritz Haber discovered how to synthesize ammonia from elements that industrial production of ammonium nitrate, and more recently of urea, that industrial production of nitrogen fertilizers became widespread. Nowadays the *three* main mineral fertilizers, nitrogen, phosphorus, and potassium (NPK) are routinely administered to the soil.

Chemistry

During Middle Ages the three triads, 1,2,3; 4,5,6, and 7,8,9 had been connected by alchemists with the old alchemical idea of the planets and metals, i.e.,

$$1 \equiv \text{gold} \equiv \text{sun},\ 2 \equiv \text{silver} \equiv \text{moon},\ 3 \equiv \text{Tin} \equiv \text{Jupiter}$$
$$4 \equiv \text{Gold} \equiv \text{Sun},\ 5 \equiv \text{Quicksilver} \equiv \text{Mercury},\ 6 \equiv \text{Copper} \equiv \text{Venus}$$
$$7 \equiv \text{Silver} \equiv \text{Moon},\ 8 \equiv \text{Lead} \equiv \text{Saturn},\ 9 \equiv \text{Iron} \equiv \text{Mars}$$

Thus, 1 was similar to 4 and 2 to 7. This strange association between numbers, planets, and metals, and between odd and even numbers, is probably due to the Babylonians. Alchemy, which was superseded by modern chemistry during the 18th century, was an interesting set of practices where the scientific and the pseudo-scientific aspect were not always easy to differentiate. In early times, alchemy was centred on two elements, *Sulphur* & *Mercurius*, which built the primeval dualistic 'chemical' system. These two elements were supposed to be the essence from which matter was composed, and it was only later in the development of alchemy (by Paracelsus, 1493-1541) that a third "element" was added, table salt, and, thus, alchemy ended up with a triad. The Greek God Hermes (the Roman Mercurius) had a stab where two snakes were symmetrically wounded around it, and it seems that they symbolized mercury and sulphur [21, 22].

An important ecological problem nowadays is the ozone hole centered on the ozone molecule, which is a triatomic molecule made up of three oxygen atoms, O_3. The greenhouse effect, instead, which is not always a negative effect, is due, mainly, to two other triatomic molecules, CO_2 and H_2O, furthermore this last molecule, water, is essential for life. But let us start from the beginnings of chemistry. Johann Wolfgang Döbereiner in 1817 was the first to note that *triads* of elements allowed some properties (density, atomic weight) of the second member of the triad to be deduced as the mean of the first and third elements: Cl-Br-I, Li-Na-K, Ca-Sr-Ba. At that time only 48 elements were known, and their atomic weights were uncertain because there was no clarity about the concept of molecule. The modern concept of molecule was re-introduced by Cannizzaro, who described, imposed, and showed how to use Avogadro's ideas on the subject at the first international chemical congress organized by Kekulé in 1860 in Karlsruhe [22]. Around that time the number of known elements had reached 63. Although Mendeleev is justly credited with the discovery of the Periodic System of Elements, other scientists might be considered to have almost arrived at the same results around the same time, among which, A.-E.-Béguyer de Chancourtois, Lothar Meyer, and William Odling. However, it was only D. I. Mendeleev who in February 17, 1869, ordering the 63 then-known elements according to their atomic weights, dared to invert the order for pairs of elements when chemical properties required this (Te-I and Co-Ni, to which Ar-K would be added later), to predict the existence of new, missing, elements and to interpolate quantitatively their properties in 1871. Thus he obtained the ordinal number, which we now call the atomic number Z. Mendeleev's greatest achievement was to leave blank spaces where these elements were predicted to be situated. Unlike other chemists who had produced Periodic *Tables*, Mendeleev became sure that he had discovered a *System* of elements. He had based his classification mainly on atomic weights, but had taken into consideration also valences of elements evidenced by formulas of their oxides and hydrides, isomorphism, and the Dulong and Petit's law. Perhaps De Chancourtois, who declared, echoing the Pythagorean "everything is number" that "the properties of elements are properties of numbers", could have reached Mendeleev's achievement had he not been trained as a geologist. The third element of the modern periodic table based on atomic numbers is *Li*, a highly reactive element, but chemistry has another and very famous three concealed in the very first element, Hydrogen. This element with its three stable isotopes is the only element whose isotopes have separate names. The three stable isotopes are called protium, ^{1}H, deuterium, ^{2}H (or *D*), and the radioactive isotope (half-life approximately 13 years, a weak beta-emitter) is called tritium, ^{3}H (or *T*), because it has an atomic weight of three atomic mass units (Daltons), having in the nucleus (triton) one proton and two neutrons, and it is the less stable of the triple. This isotope is of paramount importance in nuclear fusion reactions, which are responsible for the solar energy, and in the synthesis of labeled compounds for studying reaction mechanisms in chemistry.

In organic chemistry many molecules which have three functional groups have a special importance, the ATP with three phosphate groups is essential for life, it is 'the immediate energy provider' of life, storing the energy which comes from the degradation of carbohydrates and plays a central role in the energetic needs of all living organisms. The conversion of glucose into carbon dioxide and water plus chemical energy stored as adenosine triphosphate (ATP) is the reverse of the photochemical assimilation of carbon dioxide performed by plants and microscopic algae. This "oxidation" of glucose which allows animals (including humans, feeding on carbohydrates synthesized by plants with the input of solar energy) to exist and is nowadays shared by plants and animals has a complicated mechanism. Hans Krebs, who flew from the Nazi regime to England where he was knighted, formulated in 1937 a cyclic mechanism where citric acid plays a key role. This is the so-called *Tricarboxylic Cycle* or the *Krebs cycle*. It involves several enzymes and is a keystone in biochemistry. The synthesis of ATP in living cells proceeds by a "molecular motor" working in three strokes, i.e., in waltz time, as demonstrated by P. D. Boyer, J.E. Walker and J.C. Skou, who were awarded the 1997 Nobel Prize in chemistry for their discovery of how the enzyme ATP-synthase (or F_1F_0 ATPase) works. Protons emerging from the intramembrane space in the F_1F_0 complex drive the mobile F_1 stalk that rotates about 50 –

100 times per second and releases for each rotation by 120º an ATP molecule formed from an ADP molecule (adenosine diphosphate) and a phosphate anion; this endoergonic process is coupled energetically with the previous oxidative process, and in plants with photoassimilation.

The most famous explosives, trinitrotoluene (TNT) and Alfred Nobel's invention (dynamite, based on glycerol trinitrate), have three nitro groups. Triglycerides (fats) are compounds with three chains of fatty acids esterified with a glycerol molecule, and they serve for long-term storage of energy that could be used in moments of shortage of food, but with no shortage of food this can results in fat people. Triarylmethane (particularly triphenylmethane) derivatives play an important role in organic chemistry and in the dyestuff industry. The first stable free radical, triphenylmethyl, was discovered in 1900 by Moses Gomberg. Triphenylmethyl cations are so easily formed that they result by spontaneous dissociation of triphenylmethyl chloride in sulfur dioxide. By attaching electron-donor groups to the *para*-positions of triphenylmethyl cations the resonance stabilization is enhanced, and colored substances result (triphenylmethane dyes).

Graphs can be *planar* if they can be written on a plane with no crossing edges or *non-planar* when they cannot. Kuratowski proved that all non-planar graphs have as subgraphs either a complete graph K_5 (the regular graph of degree 4 with five vertices that are all interconnected) or a graph $K_{3,3}$ (the cubic graph with six vertices represented in Fig. 1A by the third graph). This figure has one non-planar graph on 6 vertices ($K_{3,3}$) and two non-planar graphs on 8 vertices, which evidently have $K_{3,3}$ as a subgraph. When two vertices may be connected by multiple bonds we have a multigraph. Valence isomers of annulenes are cubic multigraphs; Fig. 1B represents the carbon skeleton (scaffold) of valence isomers of benzene $(CH)_6$, where each vertex symbolizes a CH group. Only planar connected multigraphs are included in Fig. 1B; the names of the five compounds are: benzene, bicyclopropenyl, Dewar-benzene, benzvalene, and benzprismane; the last graph is the unique planar graph on 6 vertices shown in Fig. 1A. [23]

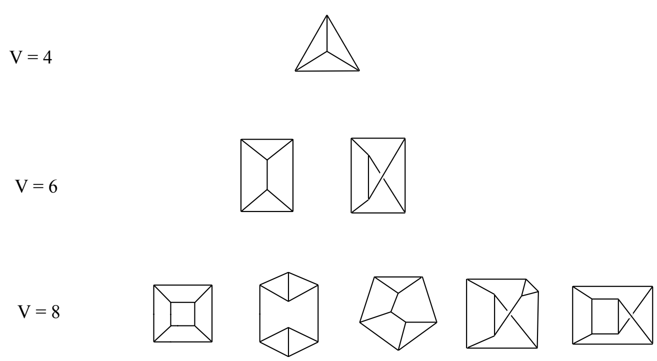


Figure 1A. Connected cubic graphs with V vertices; non-planar graphs have crossing edges.

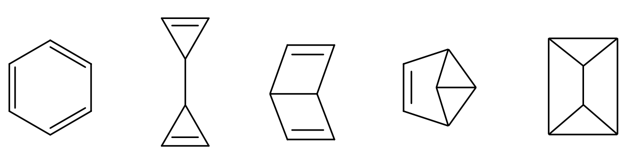


Figure 1B. Connected planar cubic multigraphs on 6 vertices (valence isomers of benzene)

In graph theory and chemistry, graphs with all vertices having degree 3 (cubic graphs), are relevant to enumerating valence isomers of annulenes $(CH)_{2k}$. Thus, tetrahedrane and cyclobutadiene are the only valence isomers of [4]annulene. All planar cubic graphs with six vertices (valence isomers of benzene or [6]annulene) have been synthesized. Most of the valence isomers of [8]annulene and [10]annulene are also known, and some of them have very interesting properties. Another chemical application of cubic graphs is in finding all possible cycles in hydrogen-depleted molecular graphs of hydrocarbons, by ignoring vertices of degree 1 or 2, and by replacing vertices of degree four by a pair of connected vertices of degree three. The resulting homeomorphic cubic graph allows (via a simple algorithm) to find the smallest set of smallest rings.

During the last period of the 20^{th} century chemists became interested in knot theory, as they started to create knotted molecules, which are thought to have special properties different from the corresponding unknotted molecules [24]. This type of molecule would enlarge the meaning of 'isomer', and compel to talk about a corresponding 'knotted isomer' of a molecule. Since there are many topologically distinct types (even millions) of *knotted* molecules, a single sequence of atoms would generate a huge number of '*knotted isomers*'. Now, how many atoms are needed to construct the simplest knotted (nontrivial) molecule? Well, it seems that this number is six, which might be able to build a *trefoil* knot molecule, related to number three. So far the simplest and first knotted molecule constructed during the year 2000 is a trefoil knot with more than 100 atoms on the string (and more as side-chains), which is shown in Figure 2.

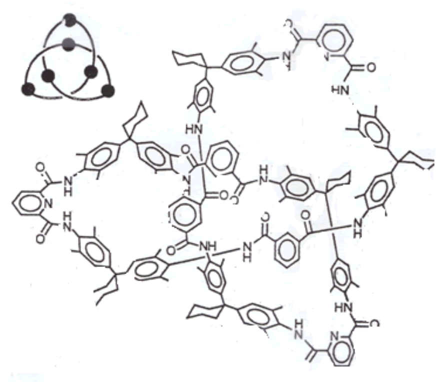
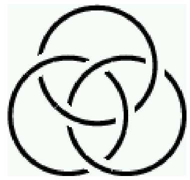


Figure 3. The simplest knotted molecule constructed so far (from ref. 21).

Biochemists also became interested in knot theory as the negatively-charged double-helix DNA molecule is highly packed (wrapped around positively-charged histones), with a multitude of protein molecules around it (the chromatin complex), and the whole aggregate resides in the cell nucleus, with a diameter of about one micrometer. Nonetheless, even if it is highly packed, DNA performs the three tasks, *recombination*, *transcription*, and *replication*. Now, as all these operations are under the control of specific enzymes, they must know how to perform knot theoretic operations on the huge DNA-protein complex. To unravel this phenomenon has become a main focus of research of biochemists and mathematicians. DNA are polymers and, like all polymers, have three features that makes them different from small molecules: *chain entanglement*, *summation of intermolecular forces*, and *time scale of motion*, as polymers move intramolecularly more slowly than small molecules do [25]. In addition to knotting there is also the related possibility of topologically linking of molecules without chemical bonds. In fact simple catenanes have been known for some time and they involve pair linking. However, there is also the possibility of triplet linking, exemplified by the Borromean rings, i.e., the design of three interlinked circles,



The three rings taken together are inseparable, but removing any one ring causes the other two to fall apart. Because of this property, they have been used in many fields as a symbol of strength in unity. Attempts to synthesize a molecule mimicking the Borromean rings are on the way.

Chemical kinetics tells us that tri-molecular reactions, i.e., a reaction due to a simultaneous encounter of three chemical bodies (molecules, radicals, atoms, ions, ..), which might explain some of the third-order reactions, are highly improbable. Third-order reactions are normally explained by a sequence of bimolecular and monomolecular elementary steps. Note that the order of a reaction is an experimental parameter characterizing the kinetic rate law.

Security windows for cars can be made from two panes of glass held together by a transparent sheet of plastic (triplex glass). It was discovered serendipitously in 1903 when Edouard Benedictus dropped on a hard floor a glass bottle of collodion (a solution of cellulose nitrate) that had been left without a stopper. To his surprise the shards of the broken bottle did not fly apart, but were held in place by the thin film of cellulose nitrate. His invention became widespread and mandatory after 1920 for windshields. Nowadays for other car windows (but not windshields in USA) triplex glass competes with tempered glass, which on breaking shatters into many small pieces.

Literature and Music

Literature is grounded on propositions, and the simplest proposition requires three 'parameters': the *subject*, the *predicate*, and the *copula*, while three propositions are required to complete the simplest argument: the *major premiss*, the *minor*, and the *conclusion*. The most famous ancient literary opus, Homer's *Iliad* tells of the destruction of Troy by the Greek army, and the cause of the war was a triple-choice problem, centered on a golden apple on which it was written '*to the fairest*', and which was thrown by the self-invited Goddess of discord and strife, Eris, during the marriage party of Thetis and Peleus, Achilles' future parents. Invited to this party were three Goddesses, Aphrodite (Goddess of love), Athena and Hera (Jupiter's wife). Paris, son of the King of Troy, had been chosen by Jupiter, who wished to avoid getting into trouble by having to choose himself, to give the golden apple to the fairest, and Paris awarded it to Aphrodite, who secured for Paris the love of the beautiful Helen, wife of Menelaus, king of Sparta. What happened afterwards is well-known. One of the greatest poems ever written is surely *The Divine Comedy* by Dante Alighieri (1265-1321) from Florence, and whose structure turns around the number three. Actually the sobriquet *Divine* was added after the death of Dante, as Dante named it just *Comedy.* It consists of three parts, *Hell*, *Purgatory*, and *Paradise*, each of which is made up of 33 cantos, and the basic rhythmic structure of the cantos is the '*terzina*' a three-verse unit. Furthermore, at the entrance of the Hell appears the Greek-Roman figure, *Cerberus*, the three headed dog. This last figure reminds the triple-headed Slavic Moon-God, which, again, reminds the Teutonic lunar goddess which forms a trio with her two daughters. The famous 20th century English poet T.S. Eliot had to say about Dante: "Dante and Shakespeare divide the modern world between them, there is no third". Many people, following their nationality, would surely add a third, but why not a fourth or a fifth or even more?

Carlo Gozzi from Venice (1720-1806) was a poetic fable writer, many of whose fables were transformed into theatre works or even operas, for example, Puccini's *Turandot*, in which three characters appear, *Ping*, the grand chancellor, *Pang*, the general purveyor, and *Pong* the chief cook. Gozzi's most famous fable is, *l'Amore delle tre melarance* (The Love of the three oranges), which was put into music in 1921 by S. Prokofiev. The short story *Three Sisters* by Jane Austen (1775-1817) was written in epistolary form in 1792, and is her juvenile work. Much better known is Anton Chekhov's play with the same title. Reading Alexandre Dumas' (1808-1870) *The Three Musketeers* (1844) everybody is certainly amazed by the fact that the 3 musketeers were practically 4. Gertrude Stein's (1874-1946) *Three Lives*, it is the first of Stein's works, which established her position as a master of the English language and expositor of the 20th century woman. Jerome K. Jerome (1859-1927) wrote a classic of the humoristic literature, *Three Men in a Boat (to Say Nothing of the Dog)*. The collection of three plays by Eugene O'Neill (1888-1953): *The Airy Ape*, *Anna Christie*, *The First Man*, won the best drama of 1922 and the Pulitzer Prize. Agatha Christie (1880-1976), whose books underwent more editions than the Bible, also paid a tribute to number three with the story the *Three Blind Mice*. The Persian tale *The Three Princes of Serendip*, where the heroes make fortunate discoveries by chance, inspired Horace Walpole in the 18th century to invent the word serendipity, which means discovery by chance. Talking about movies, *The Third Man* is one of the most unforgettable movies ever done. It was made soon after the second world war, in 1950, by Carol Reed, and masterly played by three actors, Joseph Cotten, Alida Valli, and Orson Welles. To the fame of this movie contributed its famous leitmotiv, *the third man theme*, by Anton Karas, played on an old-fashioned Greek guitar, but to which the same Welles seems to have contributed (to both music and film). In the field of cartoon movies we have Walt Disney's three small ducks, and the three small pigs, and there is a Tri-Star Film Company and a Three Pines Press, which edits East-West studies, especially Chinese culture and Daoism. The reader should note that in Chinese *three* sounds like *life*, while Japanese *Haikkus* had always three verses.

A triad in music is a three-note chord in which there exist, a *root*, a *3rd*, and a *5th* musical note. The following example is a 'C' triad made up of three notes: 'C', 'E', and 'G' (do, mi, sol). In this triad the bottom note is the root, the middle note is the third, and the top note is the fifth. Furthermore, in music the point of reference is the *middle C*, which serves as a midpoint between the base and the treble clefs in addition to functioning as a point of reference from which to describe voice ranges (one, two octaves above middle C). Practically, the history of Opera starts with Claudio Monteverdi's (1567-1643) famous trilogy: *L'Orfeo* (1607), *Il Ritorno di Ulisse in Patria* (1641, the Comeback of Ulysses to his Homeland), and the *L'Incoronazione di Poppea* (1642, The crowning of Popea). Actually, he never conceived of composing a trilogy, but only these three survived. Much of Baroque instrumental music, such as the seminal *Concerti Grossi* by Arcangelo Corelli (1653-1723) and the *Concerti* by Antonio Vivaldi (1678-1741), were normally divided into three movements, while the operas, with some exceptions, were divided into three acts. The *fast-slow-fast* division of the concert, which would become the norm for succeeding generations of concert composers, seems to have been introduced by Tomaso Albinoni (1671-1751) with his Opus 5 composed in 1707. Albinoni introduced also the principle of the fugue into the solo concerto. His instrumental music attracted the attention of J. Sebastian Bach (1685-1750), who wrote at least two fugues on Albinoni's themes and used his basses for harmony exercises for his pupils. The famous Albinoni Adagio is a 1945 reconstruction by Remo Giazzotto of a fragment from a slow movement of a trio sonata. The three-movement division of many instrumental music lasted through the classic and romantic ages till nowadays, like the normal three acts of a theater piece, and opera. To cite only some 20th century examples, the *violin Concertos* by Sergei Prokofiev (1891-1953, he died the same day Stalin died) have three movements. The work *Contrastes* by Bela Bartók composed in

1938 (1881-1945) is made up of three movements. The *Kammerkonzert* (Chamber Concerto) by Alban Berg (1885-1935) composed in 1924 is also made up of three movements. The opera *Pierrot Lunaire* composed in 1912 by Arnold Schönberg (1874-1951) consists of three acts, normally called parts. Luciano Berio's (1925-2003) *Concertino* for clarinet, violin, celeste, harp, and strings has also three movements. A well-known form of music and ensemble is the *Trio*, composed for and played by three players. Coming back to operas, another famous opera trilogy is Mozart's (1756-1791) enchanting Italian opera trilogy, whose librettos were written by Lorenzo da Ponte (1749-1838): *Le Nozze di Figaro* (1786), *Don Giovanni*.(1787), *Così fan Tutte* (1790). During the 19th century we can find other famous trilogies: Verdi's (1813-1901) mid-fifties Trilogy: *Trovatore*, *Rigoletto*, and *Traviata*. Actually, Verdi's operatic curriculum has been divided into celebrated trilogies. Wagner's (1813-1883) *Ring* Tetralogy, is actually (like the three-four Musketeers) a Trilogy, as, even if it consists of four operas: *Das Rheingold*, *Die Walkürie*, *Siegfried*, and *Gotterdämmerung*, Wagner lowered the first opera into the status of a prologue. Franz Schubert (1797-1828) composed during his short lifetime several operas, which are seldom performed, nevertheless his music may be heard in the operetta *Dreimädlerhaus* (the House of the three girls). Giacomo Puccini (1858-1924) around 1912 composed another trilogy, known as the *Trittico (Triptych)* made up of three one-act operas, completely independent from each other, *Il Tabarro*, *Suor Angelica*, and *Gianni Schicchi*. Very few people know the 1874 Spanish novelette by Pedro de Alarcón, *El Corregidor y la Molinera* (the Governor and the miller's wife). Well, this novelette became in the hands of the Spanish composer Manuel de Falla (1876-1946) the famous music for the ballet *El sombrero de tres picos* (the three-cornered hat) composed in 1919, in which number three makes its way into the title. *The Threepenny Opera* (Dreigroschenoper) was written and composed by the couple Brecht-Weill in 1928 in Berlin, and it was a huge success, and has since then become one of the most often played 20th century operas. Brecht's play originated from the 1728 play by the English poet and dramatist John Gay (1635-1732), the *Beggar's Opera*, which was translated into German by one of his three mistresses, Elizabeth Hauptmann (Brecht's wife Helene Wiegel, and Ruth Berlau were the other two, but there were other some minor cases). Gay's play was turned into music by the English composer J. Christoph Pepusch of German origin, and had a huge success in its days, giving Gay the possibility, to continue his life of drinking and sporting. Gay, the following year wrote another play, *Polly*, which was prohibited, as immoral, by the then prime-minister Walpole. The story of Beggar's Oper does not end here as Opus 43 by the 20th century English composer Benjamin Britten is, Beggar's Opera. Thus, at the very end, there were three Beggar's Opera.

Nowadays nearly everybody a bit versed in opera knows that *The Three tenors* are, José Carreras (Spanish), Placido Domingo (Mexican), and Luciano Pavarotti (Italian). It is interesting to note in this context that the *Tibor Rudas* organization who 'marketed' the three tenors, is now advertising '*The three sopranos*': Kathleen Cassello, Kallen Esperian, and Cynthia Lawrence, all three Americans. It is surely a strange and innocent coincidence that the names of the three most famous German language composers start with B: J.S. Bach, L. van Beethoven (1770-1827), and J. Brahms (1833-1897).

Concerning triptychs let us do a digression into other forms of art. The triptych paintings, a central piece surrounded by two lateral pieces, was a widely used form of painting during the late middle ages, and it was normally used as the main piece in many altars of Romanic and Gothic churches, and in Italy it was called 'Pala'. Triptych paintings slowly disappeared during the Renaissance, even if Sandro Botticelli (1445-1510) used to draw triptychs, which were later dismembered to end in three different museums over the world. This form of art re-appeared in the 20th century. Many famous paintings of the painter Francis Bacon (1909-1992) not only *are* triptychs, but they are also *called* triptychs.

Mathematics and Logic

George Gamow's *One, Two, Three, ..., Infinity* [26] emphasizes that there is a significant generalization from, e.g., 1-dimensionality to 2-dimensionality, and to 3-dimensionality, with the next significant generalization being to infinite dimensionality. Euclidean geometry covers objects belonging to the three dimensions, line, plane, three-dimensional objects. Euclid (ca. 330 - ca. 275 BC), in fact, considered also the problem of defining a point: a point is that object which has position and no magnitude. Here all Greek mathematics failed to infer number zero. The degrees of comparison, which go well beyond mathematics, are also three: $<$, $=$, and $>$. In the introductory section it was mentioned that the number three was a lucky number. This has nothing to do with numerology. A *lucky number* is, instead, an accurate mathematical category. Write down all the odd numbers, 1, 3, 5, 7, ... The first odd number after 1 is 3, so strike out every third number from the list and you get: 1, 3, 7, 9, 13, 15, 19, … Now the first odd number after 3 is 7, so strike out every 7th number from the second list and you obtain: 1, 3, 7, 9, 13, 15, 21, ... The numbers that remain after this procedure has been repeated for each remaining odd number in order are called lucky numbers. Hence the lucky numbers begin with: 1, 3, 7, 9, 13, 15, 21, 25, 31, 33, … Many asymptotic properties of the prime numbers are shared by the lucky numbers. The asymptotic density is just the same, i.e., $1/lnN$. The frequencies of the twin lucky numbers (here: 7, 9 and 13, 15) and twin prime numbers are similar. The Goldbach conjecture can also be adapted to lucky numbers (every whole number greater than nine can be written as the sum of three odd primes). Never heard about a *three-sum number* ? Well, a three-sum number is such that its second digit is the sum of its first digit and third digit, i.e., if the three digit number is abc, then $b = a + c$. This kind of number has the interesting property that abc must be divisible by 11, which is easy to prove:

$$abc = 100a + 10b + c = 100a + 10(a + c) + c = 110a + 11c = 11(10a + c)$$

Three integers, a, b, and c that satisfy the relationship $a^2 + b^2 = c^2$ are called Pythagorean triples. There are infinitely many such numbers and there also exists a way to generate all the triples. Let m and n be integers with $m < n$, then be

$$a = n^2 – m^2, \quad b = 2mn, \quad c = m^2 + n^2$$

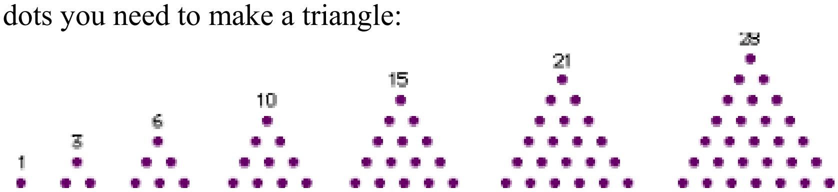
$n^2 – m^2$, $2mn$, and $m^2 + n^2$ always form a Pythagorean triple. The proof is simple: squaring a, and b we arrive at $c^2 = a^2 + b^2 = (m^2 + n^2)^2$. The geometric form of this formula was known to Euclid, and used by Diophantus to obtain Pythagorean triples with special properties, even if he did not care to check if it could be possible, in this way, obtain all triples. Here are the first triples T(m, n) for m between 1 and 5 and n between 2 and 6.

T(1, 2) = (3, 4, 5)
T(1, 3) = (8, 6, 10); T(2, 3) = (5, 12, 13)
T(1, 4) = (15, 8, 17); T(2, 4) = (12, 16, 20); T(3, 4) = (7, 24, 25)
T(1, 5) = (24, 10, 26); T(2, 5) = (21, 20, 29); T(3, 5) = (16, 30, 34); T(4, 5) = 9, 40, 41)
T(1,6)=(35,12,37); T(2,6)=(32,24,40); T(3,6)=(27,36,45); T(4,6)=(20,48,52); T(5,6)=(11,60,61)

Many patterns can here be noted, and with their help it is possible to construct all the subsequent triples. Compare, for example, T(1, 2) with T(2, 4), and with T(3, 6). The fact is that for m and n coprime of different parities eq. (2) yields coprime numbers a, b, and c.

Conversely all Pythagorean triples based on coprime numbers can be obtained in this way. All other Pythagorean triples are multiples of coprime triples: *ka*, *kb*, *kc*. Three integers are said to be coprimes or relatively prime if and only if they have no common factor other than 1 and – 1, or equivalently, if their greatest common divisor is 1. For example, 8 and 35 are coprimes, but 12 and 33 are not, as they are both divisible by 3. An interesting theorem about coprimes states that the probability that two randomly chosen integers are coprimes is $6/\pi^2$. From $a^2 + b^2 = c^2$, we can obtain, $(a/c)^2 + (b/c)^2 = x^2 + y^2 = 1$. This means that finding Pythagorean triples is equivalent to locating points, P(x, y), on the unit circle, centred on the origin, for which both x and y are rational. Now, let us pull the unit circle a little aside, and let us consider the unit circle centred at $C(\sqrt{2}/k, \sqrt{3}/k)$, as *k* grows P(x, y) approaches the origin, but for no *k* does such a circle contain a rational point. It is here interesting to recall that the recent proof of Fermat's last theorem tells us that the curves $x^n + y^n = 1$with $n > 2$ contain no rational points [excluding the trivial ones, P(0, ±1) and P(±1, 0)]. Actually, a simple curve like the one described by $y = e^x$, has as a single rational point, P(0, 1).

Triangular numbers (*t-nums*), mentioned in the introduction, take their name by association with square numbers and cubic numbers (putting all numbers from 1 to *n* in a square or cubic lattice yields n^2 and n^3, respectively). They form the series 1, 3, 6, 10, 15, 21, 28....Flocks of birds often fly in this triangular formation. Their properties were first studied by ancient Greek mathematicians, particularly the Pythagoreans, and are of the form 0+1+2+3...+n, or n(n+1)/2. They can be arranged in a triangle, and can be thought of as the numbers of dots you need to make a triangle:



The triangular numbers are also found in the third diagonal of Pascal's triangle:

1
1 1
1 2 **1**
1 **3** **3** 1
1 4 **6** 4 1
1 5 **10** **10** 5 1
1 6 **15** 20 **15** 6 1
1 7 **21** 35 35 **21** 7 1
1 8 **28** 56 70 56 **28** 8 1

There are many interesting features of *t-nums*. [27, 28] Adding any two consecutive *t-nums* is a square number (6+10, 10+15). A notable t-num is the 36th t-num, 666, which in *Revelations* is considered the number of the beast. Of course 36 (6*6) is itself a t-num.

Trigonometry, i. e, the triangle-based measurement, is a branch of mathematics that from its undefined beginnings, seemingly in Babylonian times, was associated with architecture and, particularly, with astronomy. Hypparchus of Rhodes (2nd century BC), who lived in Alexandria is credited for its official foundation, even if other Alexandrians like Eudoxos of Cnidus (408-365 BC), and mainly Euclid (3rd century BC), the author of the *Elements*, had already introduced spherical geometry before him. But the real father of trigonometry, who is also the greatest astronomer from antiquity, and who also operated in Alexandria, was

Ptolemy Claudius (2nd century BC), with his work, *Mathematical Composition*, know later as the *Great Composition*, and even later with the Arab title, *Almagest* (the greatest). Actually Alexandrian trigonometry used chords instead of sines, which were defined in India, like zero and the decimal system for number notation. The first work containing the first table of sines, which dates from the 4th or 5th century AD is the *Surya Siddhanta*; there we find the calculus of sines of the angles which are multiples of 3°45' to a full circle. Indian astronomers defined also the cosine and a function today no longer in use, i.e., the *versine* = 1 – cosα. They are also credited to have defined the tangent and the cotangent. Before closing this section let us come back to Ptolemy, who is also responsible for much *pseudoscience*. [2, 5] In fact, his book the *Tetrabiblos* represents a kind of sidereal religion to which much of the ancient world succumbed. In it Ptolemy adopted with no hesitation the superstitions and prejudices of his time. The resulting pseudoscience, astrology, continues to be attended nowadays by a vast audience, inclusive by 'men and women of letters'.

There is an interesting conjecture in number theory, known as the Collatz problem, Syracuse problem, Kakutani's problem, Hasse algorithm, and Ulam's problem, but is better known as the "hailstone sequence". This concerns the behavior of the function which converts any positive odd integer *x* into *3x +1* (and any positive even integer *x* into *x/2*) forming then a sequence, which will always produce the value 1, after repeated iterations. Starting, for example with *x* = 5, the following sequence is obtained: 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, …… ; with *x* = 7 we have, instead, the following sequence: 3, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, and from here on you have the previous case. Actually any starting value seems to end with the repeating cycle: 4, 2, 1, 4, 2, 1, 4, 2, 1. The name *hailstone sequences* originated from the fact that they go up and then they fall down like a hailstone before crashing to earth. Up to now no sequence has been found that does not end in the given cycle, nevertheless it continues to be an unproved conjecture.

There is an interesting 3-fold structure to Melzak's general "Bypass Principle" which he advocates as a powerful yet simple approach to complexity [29]. He identifies a great number of problems all soluble via a general 3-step procedure: *first*, transform to an equivalent problem; *second*, solve the equivalent problem; and *third*, transform this equivalent solution back to the original problem. For example, in mathematics (with applications in physics, chemistry, biology, and engineering) a scheme to compute a function of a matrix, say the function $f(\mathbb{X})= e^{\mathbb{X}}$ of a matrix $\mathbb{X}$, one proceeds: *first*, transform $\mathbb{X}$ to a diagonal form $\mathbb{D}$ *via* a unitary transformation *i.e.*, $\mathbb{U}\mathbb{X}\mathbb{U}^\dagger = \mathbb{D}$; *second*, make the easy computation of the function $f(\mathbb{D})$; and *third*, transform $f(\mathbb{D})$ back to the original representation to obtain the desired $f(\mathbb{X})$, as $\mathbb{U}f(D)\mathbb{U}^\dagger$. Moreover, Melzak illustrates many other examples in mathematics, and the sciences, and argues that the idea applies also to politics, philosophy, and even theology.

Continued fractions are an interesting mathematical tool apt to discover regularities even where there seems to exist no regularities at all, for example, the irrational numbers. Had they be known by Pythagoreans they could probably have accepted the irrational numbers. The following expression: $[a_0; a_1, a_2, a_3, …., a_n, …]$ is a compact notation for a simple continued fraction (having all numerators equal to 1), where a_0 is the integer part, and the remainder is the fractional part, while the single a_i values are the partial quotients (positive integers) i.e., the continued fraction written in full form is:

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cfrac{1}{a_5 + \ldots\ldots}}}}}$$

If the number of terms a_i is finite, the continued fraction represents a rational number. Continued fractions allow us to represent also approximate values of irrational numbers as partial ratios. In fact, if we truncate an infinitely continued fraction after a finite number of steps, then a rational approximation to the infinite expanded irrational is achieved, and these are called the initial convergents, or rational convergents, and they are denoted by p_n/q_n. Continued fractions of irrational numbers are infinite in length, and allow in many cases to detect a hidden order within the irrational realm. They were discovered by the Indian mathematician Aryabhata in the 6^{th} century. He used them to solve linear equations. They were passed to Arabic mathematicians and emerged in Europe with Fibonacci, in the 13^{th} century [2]. Their name and a detailed study of some of their properties, are due to the Oxford mathematician, John Wallis, who described them in his *Arithmetica Infinitorum* published in 1653. The digits of an irrational number such as π are totally random, which may be taken to mean that every sequence of digits must appear in its infinite expansion – every phone number on the earth, all of our names suitably encoded, all the works of humanity (also suitably encoded), so that in principle we do not have to build the strange machine conceived by Gamow [26] to have all the story of the world with all the achievements of any kind in it, the less than infinite (but anyway monstrously huge) random sequence of an irrational number is enough for it. The irrational $\sqrt{3}$ may be expressed as the following regular continued fraction,

$$\sqrt{3} = [1; 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, \ldots.]$$

with initial convergents: 2, 5/3, 19/11, and 26/15, where the last convergent, 26/15, can be easily remembered. π instead gives rise to the following random continued fraction,

$$\pi = [3; 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1, 84, \ldots]$$

Its initial convergents are: 22/7, 333/106, 355/113, and 103993/33102, of which the most simple and most used is 22/7. Most of the irrational numbers (i.e., e, and $\sqrt{2}$) have very regular sequences. The mathematician Paul Levy discovered that the growth rate of the rational convergents is determined by a fundamental constant $L = \exp(\pi^2/12\ln 2) = 3.2758229187\ldots\ldots$. The three most famous symbols of mathematics are π, e, and i. They are linked together in the Euler relationship, $e^{i\pi} = -1$, which can be written as $e^{i\pi} + 1 = 0$. This relation was described by Richard Feynman as “the most remarkable formula in mathematics”.

Graham’s number is often cited as the largest number that has ever been put to practical use; it has to do with combinatorics. Let N^* be the smallest dimension n of a hypercube such that if the lines joining all pairs of corners are two-coloured for any $n \geq N$, a planar complete graph K_4 of one colour (tetrahedron-like graph) will be forced. An answer to this combinatorial problem was proved to exist by Graham and Rothschild (1971), who also provided the best known upper bond, known as Graham’s number, given by $N^* \leq 3\uparrow\ldots\ldots$(64 arrows)… $\uparrow 3$. This arrow notation means: $3\uparrow 3 = 3^3$; $3\uparrow\uparrow 3 = 3\uparrow 3\uparrow 3 = (3^3)^3$; $3\uparrow\uparrow\uparrow 3 = 3\uparrow\uparrow 3\uparrow\uparrow 3 = [(3^3)^3]^3$; thus, the previous number is hardly imaginable. Graham and Rothschild also showed that N^* must be at least 6. More recently Exoo (2003) has shown that N^* must be at least 11 and has provided experimental evidence suggesting that it is actually even larger [30].

Trivalent g-cages are fascinating regular graphs of degree three which have the smallest number g of vertices for a circuit (girth g). Table 1 presents their numbers of vertices and the names associated with them. These are the only cages that have been proved so far, and in some cases they have extraordinary symmetries. The first 10-cage published in 1972 is mentioned as ‘Balaban’s 10-cage’ in “Pearls in Graph Theory” and it is shown on its cover [31]. The search for trivalent cages of higher girth continues (Exoo published promising approaches).

Table of trivalent g-cages

girth g	vertices	graph name or notation	multiplicity
3	3	tetrahedron K_4	1
4	6	Thomsen graph $K_{3,3}$	1
5	10	Petersen graph	1
6	14	Heawood graph	1
7	24	McGee graph	1
8	30	Tutte graph	1
9	58	Biggs; Evans; McKay graphs	18
10	70	Balaban; O’Keefe-Wong graphs	3
11	112	Balaban graph	1
12	126	Benson graph	1

During the 19th century the mathematician and astronomer S. Newcomb noticed that the percentage of numbers that starts with digit one was nearly three times larger than the percentage of numbers that started with 4, and he even found a formula for the percentage of experimentally derived numbers that start with digit *d*, i.e., $log[(d + 1)/d]$, which means that the first digits of such sets of numbers are highly skewed towards the first ones [32] (see Figure 3).

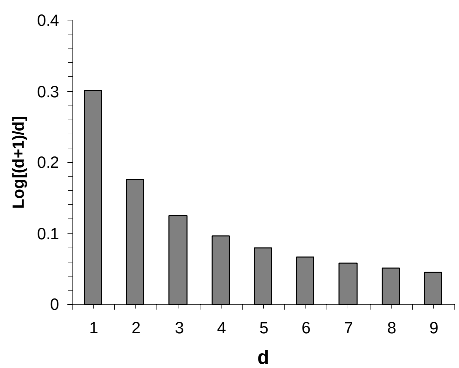


Figure 3. Benford’s law for the first digits.

This problem was further studied by Frank Benford in 1938, who gave the name to this law, and by Roger Pinkham in 1961, who showed that the law is scale invariant. But for which mathematical kind of numbers is this law valid? The numbers should neither be totally random nor overly constrained. Thus, it is not valid for lottery numbers which are totally random, nor for numbers from Olympic Games or from the ages of scientists (in years) as these latter numbers are highly constrained. As Pinkham discovered, the distribution of digit frequencies should be ‘scale invariant’ to obey Benford’s law. Thus the law is valid for processes with many influences, like the fundamental physical constants, vapour pressures, street addresses, populations of towns and cities, tax returns, and in fact it has been used to detect fraud in this last domain. In natural sciences, experimental (unbiased) sets of numbers which have units may be anticipated to obey Benford’s law, as the choice of units provides the scale (and consequent scale invariance). This finding shows that number three is not that important as it seems.

Frank Ramsey, the founder the Ramsey theory (1928), never saw his fundamental paper, with which he founded the theory that bears his name, as he died at the age of 26 shortly before its publication. His theory involves coloured graphs, and the basic problem it tries to solve is the amount of order which can be found in a most disordered system. For example, let us take a complete graph whose edges can be either red or blue. A graph is called complete if every vertex pair is connected. A complete graph of order p (i. e. with p vertices) is denoted by K_p and is r-regular, where r is its regularity, and $r = p - 1$. A graph is r-regular if it has all vertices with the same degree r, so that all complete graphs are regular but not all regular graphs are complete. Let us examine the coloured complete graph with six vertices, the K_6 graph (with 15 edges). Clearly, the most orderly graph would be a K_6 with all edges monochromatic (red or blue), and the most disorderly case would either have seven red edges and eight blue edges, or vice-versa. Now, can we join the six vertices in a way that no triangle is monochromatic, or is this an impossible task? In fact, it has been proved that this is impossible. A K_6 bicoloured graph has always a monochromatic triangle (red or blue). Note that there are about 30,000 possible colourings (2^{15}), but a bit of mathematics can prove this without having to draw all the 30,000 colourings. Thus, we can ask the question: how big must the order p of a bicoloured K_i graph be in order to have three vertices joined by monochromatic (red or blue) edges? This number is called a *Ramsey number*, and in this case it is $R(3,3) = 6$. Note that you could also have asked the question: "in a room with six people, chosen at random, what is the minimum number of people who are either all friends or all strangers (precluding non-strangers, who are enemies)?" Note that in this case the two numbers of the Ramsey number must add up to six. It has been shown that $R(2,5) = R(5,2) = 5$, $R(3,4) = R(4,3) = 9$, $R(4,4) = 18$. It has also been proved that $R(5,3) = 14$, $R(6,3) = 18$, and $R(7,3) = 23$. *Ramsey's* theorem tells us that however much orderliness we want, we can find it as long as the graph we are given is big enough. This theorem tells us that $R(5,5)$ surely exists even if up to now it has been impossible to find its value, i.e., $R(5,5)=$? Note that with growing p in a bicoloured K_p complete graph the colourings can become so huge to be intractable even by the fastest computer. For example with $p = 23$, we have $(23 \times 22)/2 = 253$ edges, and $2^{253} = 1.45 \cdot 10^{76}$ possible colourings, well, the whole universe has presumably $\sim 10^{80}$ atoms.

In the chemistry section we discussed about *trefoil knots* and other types of (chemical) knots, a subject that originated in mathematics. If a knot is tied on a string and then the two loose ends are glued together a trefoil knot is obtained, with three crossings. A special branch of mathematics studies this kind of object. There are many kinds of distinct knots: the trivial knot (or not-knot) is just a string with the two ends glued, the figure-eight knot with four crossings, and for more complicated knots there are many non-equivalent figures with the same number of knots. The trefoil knot is the simplest non-trivial knot. Over 1.7×10^6 non-equivalent topologically distinct knots with 16 or fewer crossings have been identified [24].

Another subject cited in the chemistry section concerned graphs with all vertices having degree 3, i.e., cubic graphs, which are 3-regular graphs (Figure 4). The last one is an example of a disconnected cubic graph. Cubic graphs on n nodes exist only for even n. The number of cubic graphs on n = 2, 4, 6, 8, ... nodes are 0, 1, 2, 6, 94, 540, 4207,....[33].

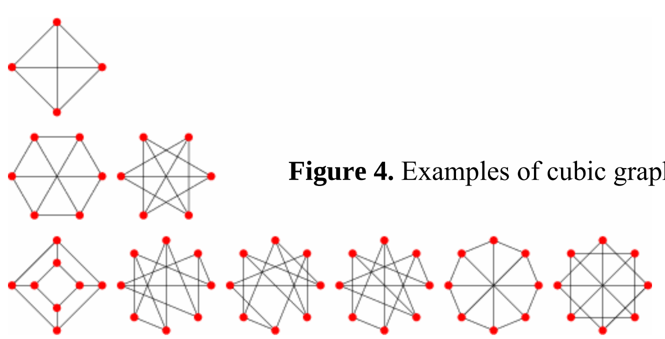


Figure 4. Examples of cubic graphs with n = 4, 6, and 8.

Logic from the time of Aristotle onwards is based on the law that *"tertium non datur"*. Aristotle had been the first to codify the laws of logic in the fourth century BC, and thus he is generally credited as the founder of Logic. He assumed that a proof must consist of a sequence of statements. Following him it was possible to start with some premises and definitions and end with the desired conclusion. Each statement, excluding the premises, in a sequence of statements, had to follow from previous ones according to one of a fixed set of relationship called *syllogisms*, such as,

All men are mortal
Socrates is a man
____
(therefore) Socrates is mortal

That is, if it is true that 'all As have property B', and that 'the individual X is in A', then it is also true that 'X has property B'. The syllogisms express what we could call the *rules of inference,* such that the validities (i. e. the truth values) of the premises are transmitted to the conclusions, and there are rules that can be applied to determine whether a purported proof is valid or not. It is interesting to note that when Penrose [14] wants to explain to the reader the idea of defining concepts in terms of sets, he uses an example centred on the number three.

Encrypting your credit card number: a three-number procedure

Every time you send your credit card number through the Internet you surely ask yourself if some 'clever guy' fishing in the Internet will be able to steal it. Maybe he can really fish it, but what he will find is a strange message impossible to decrypt. The actual encryption of a message, i.e., of a credit card number, N, on the Internet follows a *three-number procedure*, the encryption key, e, the modulus, M, and the decryption key, d. First of all, your credit card number, N, is replaced by its binary counterpart, $И$ (using a method such as ASCII), then it is transformed into a number called *ciphertext*, C, using the following algorithm,

$$C = И^{e} \bmod M$$

where C, is the remainder when $И^e$ is divided by M. C can then be decrypted with the decryption key, d, in the following way,

$$C^{d} \bmod M = И$$

where $И$ is the remainder when C^d is divided by M. The whole process can be written in the following way,

$$C^{d} \bmod M = И = (И^{e})^{d} \bmod M = И^{ed} \bmod M$$

Thus, an encrypting-decrypting system needs to find numbers e, $И$, and d, such that raising the (binary counterpart) credit card number (or any message), $И$ to the power ed modulo M, is equivalent to raising a number to the power of 1 in normal arithmetic. The most used system to date allows finding numbers e and d only if you know the factors of M. And here lies the security of the system. M is chosen to be the product of two giant prime numbers. Current technology is not able to factorize numbers with more than 230 digits, and the product of the two giant prime numbers is chosen so as to be more than 230 digits long. Thus, even if somebody were to steal your encrypted credit card number (note that the encryption follows at once, as soon as you write down your credit card number), he would not be able to decrypt it (with current algorithms) in a time smaller than the age of our universe, unless the two giant prime numbers were known [34]. Actually, it is much easier to steal the number and the name of credit card holder peeping over her or his shoulders while she or he is using it and then use number and name to purchase via Internet at somebody else's expenses, a commonly used procedure by 'card-number-stealers'.

Philosophy, Politics, Geography and History

While Kant's philosophy may be considered dualistic in character, Hegel's (1770-1831) philosophy is based on the concept of dialectics, which, reminiscent in its formal structure of Aristotelian logic, is made up of three moments: thesis, anti-thesis, and synthesis. Here thesis and antithesis are opposite contrasting views, out of whose conflict there arises the third view – the synthesis. With his dialectics Hegel argued to justify even geographical facts as the existence of the three continents: Eurasia, America, and Africa. Actually, this kind of dialectics originated, as many other ideas, in ancient Greece with Heraclites of Ephesus (c. 535-475 BC), the philosopher of the change, the theoretician of the birth of the entire reality from the movement, i.e., the *becoming* as a result of the conflict between *being* and *not-being*. It is told that he said that *one can never step in the same river twice*. Marx (1818-1883) was, instead, fascinated by the concept of change as the result of the struggle between two classes, and, again, a central position in this struggle is occupied by three classes: nobility, bourgeoisie, and proletarians. Marx viewed governmental-societal rule to subsequently reside through historical evolution with three classes, in correspondence with thesis, anti-thesis, and synthesis. Furthermore, the three sources of Marxism were German philosophy, English political economy, and French socialism. All three together gave birth to the huge mess of the real communism.

Also politics seem to share a strange preference for number three. In the former Soviet Union a form of government was the *Troika*, whose well-known ancestor is the Roman *Triumvirate*. Normally, all of them ended badly, as a single person achieved to shift or to eliminate the other two. The first Roman triumvirate was composed by Caesar, Pompey, and Crassus, while the second at the death of Caesar, by Octavius, Marcus-Antonius, and Marcus Lepidus, and everybody knows how did they end. Even Napoleon started his lightning career with a triumvirate, which nearly nobody remembers. *Triremes* were ancient warships with three rowers on each side operating a separate oar. In the 6^{th} – 4^{th} century BC, a Greek trireme carried about 200 men. It had a *trierarch* as commander, who sometimes financed and owned the ship. Three caravels were, instead, the number and kind of ships with which Columbus made his first journey across the Atlantic to the Caribbean and 90 were the men carried by the three ships. Columbus made three journeys across the Atlantic to the Caribbean Islands.

During the cold war, USA developed a three-fold nuclear defense of (*i*) nuclear bombers, (*ii*) land-based missiles, and (*iii*) submarine-based missiles, while the C^3 military philosophy (of a decade ago) is based on (*i*) communication, (*ii*) command, and (*iii*) control. The three separate branches of a democratic Government are the Executive, the Legislative, and the Judicial. Everybody knows about the *three world wars*, where the most catastrophic, the third one, never took place, but for a long while, during the cold war period, terrorized humanity. The first two were much less publicized before their start but this did not avoid their 'successful' birth, while the much more publicized "third world war" disappeared from the scene with the disappearance of the "evil empire", as Reagan called Soviet Union. Preceding the first-world war was the *Triple Alliance* between Germany, Austria-Hungary, and Italy, which was confronted by the *Triple Entente* among England, France, and Russia. However, in 1915 Italy, after reclaiming hopelessly two "Tri-" regions (Trient, and Triest) in the hands of Austria-Hungary, shifted over to the triple entente. The Second World War started in 1939 after the Molotov-Ribbentropp pact between the two greatest dictators of the 20th century, Hitler and Stalin, was concluded. They, clearly, did not mind to share the responsibility for the atrocious war they started, even if Stalin had time to clear up his responsibility ending, in time, in the right field, thanks to a dictator, who was even crazier than him, and who grounded the 'Dritte Reich' (the Third 'Reich'). This pact had a secret clause dealing with dividing Poland between Nazy Germany and Soviet Union, and the annexation of Bessarabia, of the three Baltic states and of Southeastern Finland by the Soviet Union. The *Three-country Axis*, Berlin-Roma-Tokyo, dissolved in 1943, when fascists in Italy finally noticed that their head, Mussolini, was quite 'crazy' and jailed him. An attempt done in 1944 (July 24) to kill Hitler by some German officers, headed by the officer and aristocrat Claus von Stauffenberg and backed up by a resistance group made up of intellectuals (Hans von Dohnányi, Freya von Moltke, who launched a secret resistance group with her husband in 1940) ended quite badly. If the three world wars were two the three Punic wars between the Roma and Carthago were really three, and they ended with the destruction of Carthago.

Communism all over the world worshipped in a nearly 'divine' way the triad, Marx, Engels, and Lenin. The attempt to add to this triad a fourth 'entity', Stalin, and even a fifth one, Mao, ended soon quite badly, but even the previous triad has today lost any meaning. Coats of arms, instead, survive, even if it is an ancient practice, which was used to characterize noble families and nations, and which continues to be used even today. Many coats of arms enclose three symbols as can be seen in Figure 5.

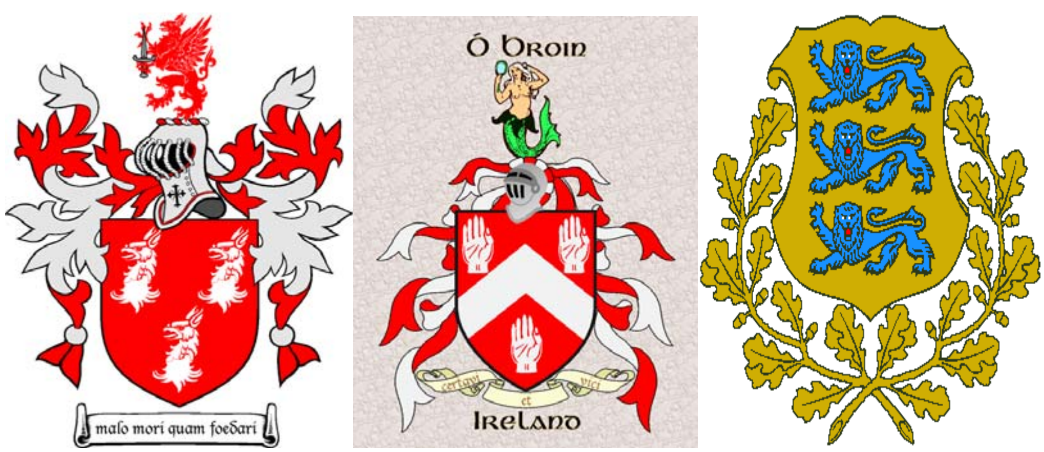


Figure 5. Three different coat of arms, which use a triad in their set of symbols.

There are three cities called Tripoli ('three cities' in Greek): the second city of Lebanon after Beirut, Libya's capital, and a small city in Central Peloponese, Department of Arcadia (Greece). There are two "Three Rivers" districts, one in England (Hartfordshire), and another in Québec, Canada (Trois Rivières), while Pittsburg, PA, USA, is the "three rivers" city. The highest peak (2,864 m) in the Julian Alps, about 60 km North-West of Slovenia's capital Ljubljana, is called Triglav in Slovenian, and Tricorno in Italian, it has indeed three peaks. The delta of Bangla Desh is made up of three rives: Gange, Nrahmaputra and Maghna.

Physics

Three appears in a number of lopsided (yet quite fundamental) triads, with two of the members forming some sort of duality, and a third member mediating between the other two, for example in Galileo's renowned writings we have Salviati, Simplicio, and Sagredo. Time flows from past to future through the present moment. This time flow from a strict relativistic point of view is only apparent as in a four-dimensional block universe time is already there as the other three dimensions, just as in a mathematical equation where the two sides of the equation is mediated by an equal (=) sign, where after and before are not time-related (or where left and right are not direction-related). This finds its chemical equivalent in the interpretation of the two sides of a chemical equation mediated by two arrows, in a representation of a chemical equilibrium, where also there is no time flow. James Jeans had to say that the tapestry of spacetime is already woven throughout its full extent, both in space and time, so that the whole picture exists, although we only become conscious of it bit by bit. A human life is reduced to mere thread in the tapestry. It was with Minkowski (and not with Einstein as it is generally credited) that space and time became particular aspects of a single 4-D continuum, where all motional phenomena become static phenomena in a 4-D spacetime. The whole history of a physical system is laid out as a changeless whole. The three time directions, past, present, and future, which are so important for humans, do not have any deep reality in both classical, relativistic, and quantum mechanics, a fact that intrigues many physicists and intrigued even Einstein, even if recent findings seem to throw some new light on the subject. Thermodynamics, instead, with the concept of entropy allows for a T-symmetry (time-symmetry) breaking, especially for bulk materials, i.e., it allows for what we constantly experience, the flow of time. There is one observed process in particle physics in which appears a T-violation, the decay of the neutral Kaon. This violation resulted first from an observed violation in CP-symmetry detected in 1964 in the Brookhaven National Laboratory (BNL). Now, as the three symmetries, the CPT-symmetry, must hold, then there must be a balancing, once the CP-symmetry is broken a T-symmetry violation should be assumed in order to preserve CPT-symmetry. In 1998 the T-violation in decays of neutral kaons was directly observed at the Fermilab. CPT-symmetry is a fundamental symmetry of physical laws under transformations that involve the inversions of charge, parity, and time simultaneously. There is a theorem that derives the preservation of CPT-symmetry for all of physical phenomena assuming the correctness of quantum laws. If you consider a mirror-image of our universe where all objects have momenta and positions reflected by an imaginary plane (corresponding to a parity inversion), and where matter is replaced by anti-matter (corresponding to a charge inversion), and the time is reversed ($-t$ instead of t), then the preservation of CPT-symmetry would mean that the present and the mirror-image universes are identical. The CPT transformation would simply take one into another. This CPT-symmetry is recognized to be a very fundamental property of physical laws. CP-violation and T-violation seems to work also in some processes involving strong nuclear forces, a fact that would explain, if confirmed, why in our universe there is much more matter than anti-matter, once it is excluded that our universe started out already with this asymmetry. Another T-violation happens during the act of measurement. This act collapses the wavefunction in the Copenhagen interpretation of quantum mechanics, and still await for a solution in this interpretation. Other interpretations, instead, like the de Broglie-Bohm-Bell and the Gherardi-Rimi-Weber interpretations of quantum mechanics avoid this pitfall.

The history of quantum mechanics may be divided into three main periods. The first began with Planck's theory of black-body radiation in 1900; it may be described as the period in which the validity of Planck's constant was demonstrated but its real meaning was not fully understood. The second period began with the quantum theory of atomic structure and spectra proposed by Niels Bohr in 1913. Bohr's ideas gave an accurate formula for the frequency of spectral lines in many cases and were an enormous help in the codification and understanding of spectra. Nonetheless, they did not represent a consistent, unified theory, constituting as they did a sort of patchwork affair in which classical mechanics was subjected to a somewhat extraneous set of so-called quantum conditions that restrict the constants of integration to particular values. True quantum mechanics appeared in 1926, reaching fruition nearly simultaneously in three forms, namely, the matrix theory of Max Born and Werner Heisenberg, the wave mechanics of Louis V. de Broglie and Erwin Schrödinger, and the transformation theory of P.A.M. Dirac and Pascual Jordan. These different formulations were in no sense alternative theories; rather, they were different aspects of a consistent body of physical law.

Relativity, considers the three dimensions of physical reality just as an approximation of a more complicated reality where the three spatial dimensions are intermixed with the time dimension, while some speculative string theories mix in several additional "compactified" dimensions. The three time directions and the three spatial dimensions have an interesting parallel in the three types of forces, which physicists are trying to unify: the strong, the electroweak, and the gravitational forces. The electroweak force is a unification of the weak nuclear force and the electromagnetic force, achieved not so long ago by three scientists: Glashow (1932-), Weinberg (1933-), and Abdus Salam (1926-1996). The standard model of particle physics teaches that Fermions are divided into three doublets of leptons and three doublet of quarks, while three different quarks (up, down and strange) make up a baryon (neutron, proton, and other hadrons). Furthermore, each quark carries one of the three types of "strong charge", also called "color charge", which has nothing to do with the colors of visible light, but it is just a property of quarks. The great unification (GUT = grand unified theory) of the three forces, actually only occurs in string and especially in superstring theory, which to date has no experimentally verified new predictions. This last theory came out from the consequence that quantum field theory and general relativity are incompatible. The attempt to combine these two theories in quantum gravity gives rise to unsolvable problems, like the breakdown of the renormalization procedure, usually used for eliminating infinities from calculations of physical quantities in quantum field theory. By replacing point-like particles with one-dimensional extended strings, superstring theory overcomes this problem. There are other problems due to this unification, such as the black hole problem (their origin, their thermodynamic properties, which seem incompatible with quantum mechanics), which superstring theory overcomes by modifying general relativity theory rather than quantum mechanics. Now, superstring theory requires a property called supersymmetry (also known as SUSY), which can be described in terms of six extra space dimensions, which curl up into a tiny geometrical space (in some string theories, the number of extra dimensions is even higher), and whose size would be comparable to the string length $\mathbf{L_{st}} \sim 10^{-33} - 10^{-31}$ cm (5×10^{-33} cm being the Planck length). Existing accelerators cannot resolve distances shorter than about 10^{-16} cm. Supersymmetry implies that every known elementary particle must have a 'superpartner', no pair of the known particles being supersymmetry partners of one another. Thus, supersymmetry requires the existence of a new elementary particle for every known one. The European accelerator called the *Large Hadron Collider* (LHG), which will begin to operate in Geneva around 2005 is credited by some to have the possibility to check SUSY. During the years 1984-1985 superstring theory became one of the most active areas of theoretical physics, which it has remained ever since. In those times there were five different types of superstring theories, each requiring ten dimensions (nine space and one time): type I, type IIA, type IIB, E8×E8 heterotic (HE, for short), and SO(32) heterotic (HO, for short). It was on this occasion that the term 'theory of everything' (TOE) was popularized. In 1994 there was a second revolution in superstring theory, which achieved to unify all five superstring theories, thus now there is a general overarching superstring theory, which allows many different perturbative expansions (even more than five, some of them allowing even for 11-dimensional space-time). It is interesting to note that Aristotle in his *Physics* considered that space in general had six dimensions: up, down, left, right, front, and behind, but at the same time he took that single bodies are sufficiently determined by three dimensions only: height, amplitude, and length. For him the general space is the last limit of an enormous body, i.e., the last celestial sphere, and it is the specific place of all the single bodies, which for him means the frontier which divides a body from the other, i.e., space is the container of bodies, and are limited by him [35, and also Aristotle, Phys. Ausc. IV]. Concerning the dimensionality problem of objects, and especially of such objects known as fractal sets which (following the mathematical theory of dimensions) do not have integer dimensions, mathematicians have found that if the *Milky Way* is seen as a fractal object then it should have dimensions d, between 1.8 and 2.6, i.e., $1.8 < d < 2.6$ [36].

The *tripod* is an object which provides a stable support for a 3-dimensional object, against a 2-dimensional surface using the fewest possible legs, while *triangulation* is used for locational purposes on the Earth's surface, i.e., one can always know where one is if one has three directions from three general (non-collinear) points.

In classical Hamiltonian mechanics the number of dimensions that a dynamical system of particles can assume is huge for N particles: $3N$ space dimensions and $3N$ momenta, all of which are represented in a $6N$ dimensional-phase space by a point. The relevance of the $6N$ dimensions is that, granted the manner of interaction, the dynamical evolution to later times is entirely determined by the position in this 6N-dimensional space (while any less information is generally insufficient).

The two dual spaces of a *Gelfand triple* and the intermediate L_2-normed Hilbert space H are relevant in quantum mechanics. Gelfand triplets of spaces H are of key importance in the formulation of quantum mechanics with H an L_2-normed Hilbert space, or Schwartz space with stronger convergence criteria, containing many of Dirac's generalized functions. There are three imaginary basis elements {i,j,k} of the quaternions, while the number of regular polytopes in any sufficiently high dimension (≥5) is three, corresponding to the hypercube, hypertetrahedron, and hyperoctahedron. "*period three implies chaos*" is the Li and Yorke's sufficient condition that a one-dimensional map leads to deterministic "chaos". Such a one-dimensional mapping (or 1-variable real function f(x)) may be iterated, i.e., $f^{(m+1)}(x) = f(f^{(m)}(\mathbf{x}))$, where $f^{0}(x) \equiv x$. A prototypical being the "discrete logistic" $f(x) = \alpha(1 - x)$, where α is a parameter characterizing the mapping. Interest focuses on the behavior of $f^{(m)}(x)$ after many iterations; it is often independent of x. The simplest such behavior is that $f^{(m)}(x)$ approached a fixed point $x_1 = f(x_1)$, but a more general (asymptotic) behavior is that $f^{(m)}(x)$ approaches an *m-cycle* of distinct values $x_1 = f(x_m)$, $x_i+1 = f(x_i)$, i= 1, 2, …., m-1. The *chaotic* circumstance is marked by the absence of any finite stable m-cycle, i.e., the circumstance of an ∞-cycle. Generally, 1-dimensional mappings may be linear re-scaled by a linear parameter α and lead to different such behaviors, with the consequence that if α is increased, a 3-cycle (i.e., a period of 3) is realized, then also (at lower values of α) chaotic behavior is also realized.

Matter normally exists in three fundamental phases: solid (with definite volume and shape), liquid (with definite volume but without definite shape), and gas (without definite shape and volume). Crystalline solids have ordered lattices, whereas there is no order in gases and liquids, therefore the liquid and gas phase (both are "fluid phases") may interconvert above the critical point without a phase transition. However, some new materials recently discovered belong to new domains, like the liquid crystals, the plasmas, the Bose-Einstein condensate, and even the old glasses have properties that are not easy to ascribe to one of the three fundamental phases. The thermodynamic triple point can be defined as the lowest integer dimension in which the three fundamental phases occur simultaneously, which for water is located at 0.01°C and 0.06 atm (4.56 mm Hg). The thermodynamic description of a pure substance at equilibrium is tri-dimensional, in fact, it depends on just three parameters: p, V, T or p, n, T or U, V, n or U, S, T, where p is pressure, V the volume, T the Kelvin temperature, n the number of moles, U the internal energy, and S the entropy, but other descriptions are possible, each with three parameters. *Tribology*, instead, has nothing to do with three as it derives from the Greek word *tribos* meaning rubbing, but, look, it includes three subjects: friction, wear and lubrication.

As already introduced in the Biology section, human color vision is based on three colors, which were defined by Newton as primary colors, and whose different percentage combinations give rise to all possible colors: blue, green, and red. To ascertain this just take a magnifying glass and look from very near at the texture of your color TV screen when it is on. Combining light of two primaries gives the secondary colors: blue and green yield cyan; blue and red, magenta; and red and green, yellow. Probably many of us heard at school that the three primary colors are: red, yellow, and blue; however, these are *subtractive colors*, in contrast to the former ones which are called *additive fundamental colors*. In subtractive colors the colors are produced by subtracting light, as in printing, or when mixing colors from a paint box. The subtractive mixing of colors leads to the removal of certain wavelengths of light by absorption by an object, and the color we observe comes from the light reflected from the object: the components of the light source minus the color adsorbed. The paint producing the blue of a Beato Angelico fresco absorbs (or subtracts) red and green but reflects blue light to our eyes. A better set of subtractive colors are: magenta, yellow, and cyan. Adding mixing of the three *additive fundamental colors* produce dramatic effects in movies, theatres, discos, night clubs, TV, and computer monitors. It should be emphasized here that some animals are bi- or mono-chromatic, and some animals are even almost blind to visible light, i.e., their visual field uses a different range of frequencies than most mammals, e.g., honeybees which perceive ultraviolet, while bats 'see' soundwaves. It should also be emphasized here that color perception is sensitively dependent on processes in the brain, and that there is a rather complicated wiring that converts the signals from our retina to what we actually perceive. Several giants of physics (Newton, Maxwell, and Schrödinger) have described in a fundamental fashion the mathematical 3-foldedness of (human) color perception. John Dalton (1766-1844), the father of the modern atomic theory of matter, along with his other researches also became interested in color blindness, a condition that he and his brother shared. The (wrong) results of this work were published in an essay, "*Extraordinary Facts Relating to the Vision of Colors*" (1794), in which he postulated that deficiency in color perception was caused by discoloration of the liquid medium of the eyeball.

Again, in reference to the Biology section, the German engineer Felix Wankel unknowingly imitated the molecular motion producing ATP when he invented his rotating "*Wankel engine*". Instead of a piston moving in a cylinder and requiring a mechanism for converting linear motion into circular motion as we have in the 4-stroke or 2-stroke engine, Wankel's motor has a triangular-shaped rotor, which functions similarly to the internal combustion engine.

Equilibrium thermodynamics has normally three laws (or postulates, or axioms in axiomatic thermodynamics) regarding energy, entropy and the zero temperature limit. Some specialized textbooks on the subject talk only about two laws, the energy and the entropy law, while other textbooks talk about four laws, including a zeroth-law, which is needed for the definition of equilibrium (and of temperature). Beyond the equilibrium circumstance, other textbooks talk even of five laws, i.e., the Onsager-Prigogine law of irreversible thermodynamics for the rate of entropy production. Recently there has also been talk about a thermodynamic law defining the highest attainable temperature, the temperature of the '*big bang*.'

There are three Newton's laws of motion. The *first law* says (at least in modern form) that there exist so-called inertial frames of reference such that every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it. This is essentially Galileo's concept of inertia and for this reason it is usually termed the law of inertia. The *second law* tells that the relationship between an object's mass *m*, its acceleration

a, and the applied force is $\boldsymbol{F} = m\boldsymbol{a}$. Acceleration and forces are vectors (here in boldface), and in this law the direction of the force is the same as the direction of the acceleration. Aristotle's dynamics was in direct contradiction to both the 1st and 2nd law, instead, making the seemingly "common sense" postulate that force was needed to maintain velocity, and the higher the velocity maintained, the higher this requisite force. The *third law* tells that for every action there is an equal and opposite reaction. The cause-effect dualism of Aristotle came somewhat close to this law (if one discounts the lack of recognition of relevant forces). Actually, Aristotle's common sense failed to recognize the role played by frictional forces. Kepler's three laws of planetary motion are of a different (descriptive) nature: (*1*) planets move on ellipses, with the sun at one focus; (*2*) the planet-sun line sweeps out equal areas in equal time; and (*3*) the ratio of the squares of the revolutionary periods for two planets is equal to the ratio of the cubes of their semi-major axes: $(P_1/P_2)^2 = (R_1/R_2)^3$. From Newton's point of view Kepler's first law implies that the planets are continually accelerating (when they approach the sun, and decelerating in the other case), and that the magnitude of the force must vary as $1/d^2$, where d = distance between the planet and the sun. If one reads Kepler's second law with Newton's insight, this implies that the force must be directed towards the sun from the planet. Newton tells us that Kepler's third law implies that the force must be proportional to the product of the masses for the planet and the sun, i.e., behind all three Kepler's laws hides just one law, Newton's gravitational law, $F = GmM/d^2$, where G is a universal gravitational constant. This law may be considered as the fourth of Newton's laws (but is in fact properly of a different kind, describing a special type of force, of which in Newton's time it could be imagined that there could many such forces). The tidal effect of gravity is the effect that causes distortion of the falling bodies in a gravitational field. Here on Earth it is responsible also for the raising tides of the oceans due to the presence of the gravitational field of the moon (and to a lesser but not insignificant amount also due to the field of the sun). The magnitude of the tidal distortion actually falls off as the inverse cube of the distance, d^{-3}, from the centre of attraction, being described in terms of differences of two (or more) centres of gravitational force. Nevertheless, one should be reminded here that the volume of the ellipsoid into which the sphere initially distorts is equal to that of the original sphere (with no loss of volume). Interactions that involve an inverse cube law include dipole-dipole interactions. The potential energy between two polar molecules, with dipole moments μ_1 and μ_2 interact as $V \propto f\mu_1\mu_2/r^3$, where f is a directional factor.

Knot theory made its first appearance in physics with W. Thomson, Lord Kelvin (1824-1907), who suggested a knot model for the atoms, in a period when mathematicians had not yet fully developed knot theory, and physicists, but not chemists, did not yet like the concept of discrete atomic entities. Lord Kelvin suggested that the different atoms were actually different knots tied in the "ether" that was believed to fill space. With this concept physicists and mathematicians developed a 'periodic table' of distinct knots.

The Three Body Problem continues to cause many "nightmares" to physicists and mathematicians. If we define an optimal theory a theory that can master a n-body problem and then derive from it through a series of successive simplifying steps all the (n-1)-, (n-2)-,……., 4-, 3-, 2-body problems, then our present physical theories either, classical or quantum mechanical, are far from being optimal theories. Up to now the three-body problem continues to be unsolved, in a traditional analytic sense, so that for numerical results approximations are needed. It should be noted, nevertheless, that with a two-body model, physics can not only often solve the equations of motion analytically, but also can explain the ever-growing mass of data. Unfortunately, as soon as we add a third body, the equations of motion become unsolvable, so that the hope to find a closed-form analytical solution are dispensed with, and "only" approximate, normally numerical, methods are at hand. Giuseppe Lodovico Lagrangio (1736-1813, better known with his French name, J.-L. Lagrange), a famous mathematician and physicist, tried to master this problem. He was born in Turin where he was the co-founder of the local Academy of Sciences; in 1766 he succeeded Euler in Berlin, and in 1787 he was called to Paris. He became a count under Napoleon, and he died in Paris. As told by Fourier, he always spoke with a very strong Italian accent, a fact which caused troubles with his students. He was the first to show that with the following three conditions one could approximately but accurately solve the three-body problem: (*i*) the three bodies should move in the same plane; (*ii*) the mass of one of them has to be so small as to be negligible; and, possibly, (*iii*) the three bodies should move in unison (maintaining the same relative position with respect to one another). A system made of the Earth, the Moon, and a spacecraft all three in the same plane is a good example of a solvable three-body problem. Considering a two-body system, then the points at which the third small coplanar body may be found are known as the *Lagrange points*, and they are five. Taking a coplanar Earth – Moon – space-station L system (E, M, L_i) they are, qualitatively (see Figure 5): $E—L_1\text{-}M$; $E—M\text{-}L_2$; $L_3\text{-}E—M$. The L_4 and L_5 points of the spacecraft build the symmetric vertices of two triangles with E and M as common basis. At these two points there may accumulate debris from space probes. Thanks to this methodology the NASA achieved recently to pin-point a launched artificial satellite that had become lost

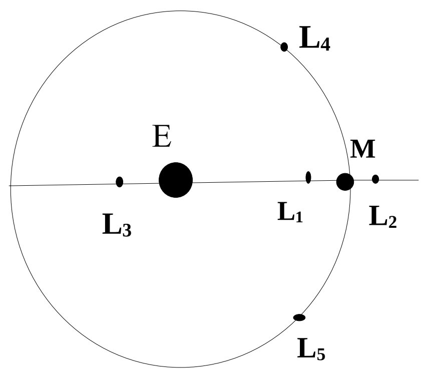


Figure 6. The *three-body* problem with its five characteristic Lagrange points, $\boldsymbol{L_i}$ (i = 1 to 5) where the small third body can be found. Here, ***E*** is the Earth and ***M*** the Moon.

Under the name of the three-body problem generations of physicists searched for a solution to the equations of the Earth, Jupiter, and the Sun. It is now clear that no closed-form algebraic solution can be found for this problem. Thus, how can we answer the question of the stability of Earth orbit? The answer came only during the middle of the 20th century (building on earlier foundational ideas of Poincaré), when it was solved by three mathematicians: the Russian A.N. Kolmogorov (1903-1987) showed a way how to prove the stability of Earth orbit without solving for it, while the Russian, V. Arnold (1937-) and the German-American J. Moser (1928-1999) completed the proof, which is known as the *KAM* theorem. Actually, if one were to multiply Jupiter's mass by 100 the problem would be treatable with Lagrange's method, whence the Earth's orbit would become unstable and erratic, and after not so long the Earth would leave the solar system [37].

The Earth-Moon system is sometimes considered a double planet, and it is the center of this system, rather than the center of the Earth itself, that describes an elliptical orbit around the Sun in accordance with Kepler's laws. It is also more accurate to say that the Earth and Moon together revolve about their common center of mass, rather than saying that the Moon revolves about the Earth. This common center of mass lies beneath the Earth's surface, about 4800 km from the Earth's center. A similar system is built by planet Pluto and his moon Charon. It is well-known that the Moon appears to go through phases, it is, instead, less known that these phases are not restricted to the Moon. Phases can be observed with the two inner planets as well, i.e., Mercury and Venus. It is, nevertheless, impossible to observe phases of the outer planets, which having their orbits lying outside the orbit of the Earth, can never be between the Sun and the Earth. Actually, due to their position not all the phases of Mercury and Venus can be seen, i.e., the full phases. In fact, these two inner planets will never be on the opposite side of the Earth from the Sun. Clearly, they can be on the other side of the Sun from the Earth, but, who is able to see through the Sun?

Quantum theory in some respect is quite analogous to classical mechanics. The quantum two-body system (of the hydrogen atom) is exactly solvable. As soon as quantum mechanics is applied to the case of helium, attempts to solve the problem exactly were soon set aside and approximate quantum methods were hastily developed and successfully applied, in contrast with the classical mechanics case, where approximate methods took about a century after Newton. There is another interesting analogy between quantum and classical mechanics, in this domain. Planetary mechanics is qualitatively analyzed in the 'non-interacting' planet approximation: planetary orbits are ellipses plus small perturbations from other planets. Quantum chemistry is qualitatively typically presented in the 'non-interacting' (or at least self-consistently interacting) electron approximation. The filling of the electrons in the shells (*1s*, *2s*, *2p*, …) make sense if one ignores electron–electron repulsion (at least in so far as correlation is concerned). Actually, the shell model for atoms is our best explanation for the periodic table of the elements, and it is a powerful experimental tool. The key for the three-body problem consists in finding a sufficient number of conserved quantities, like energy, momentum, angular momentum, etc., but, as there are not enough conserved quantities, the three-body problem is said to be 'non-integrable' (in the sense that the number of conserved quantities is insufficient to reduce the Schrödinger equation to ordinary differential equations via separation of variables), while the two-body system is 'integrable'.

In quantum mechanics the *triplet state* is a very important state, which accommodates three independent orientations of overall electron spin. Furthermore this state is the stable (ground) state of many molecules and atoms, i. e., those with two unpaired electron spins. For example the singularly important stable state of molecular oxygen (O_2) is a triplet state. The same holds for nuclear spins, and thus there are two isolable forms of molecular hydrogen, *ortho*-hydrogen (triplet state, with parallel nuclear spins) and *para*-hydrogen (singlet state); their interconversion has applications in quantifying the activity of heterogeneous catalysts.

Because nowadays laser techniques have made distance measurements highly accurate, *trilateration* tends to replace triangulation (measurement of two angles plus one side's length for determining the two other sides of a triangle). Electronics began with diodes and triodes, which were glass bulbs with electrodes. The *triode* (with three electrodes) was the actual workhorse because a small variation of the voltage between the cathode and the anode could control instantaneously the electrical current flowing though the triode. Nowadays transistors have replaced the fragile electronic tubes, and miniaturization has reached almost its limit.

Religion

Here the importance of the number three assumes aspects that are found in no other domain. It appears in a number of fundamental triads, again with two of the members forming some sort of duality, and a third member either mediating between the other two or giving rise to them: God, Adam and Eve, and Adam, Eve and the Serpent. The three religions having in common the bible are: Judaism, Christianity, and Islam, Judaism being the common source. The trinitarian God creed in Christianity is based on number three: God, the father, God the son, and God, the Holy Ghost. The third, the Holy Ghost, is a kind of mediator between the other two, and its sex was never exactly defined, while the other two are 'male' Gods (at least till today). The representation of the Holy Ghost as a dove may surely be due to Greek-Roman religion influences. The holy Christian family is represented by Joseph, Mary, and Jesus, i.e., father, mother, and son. Strange enough, in a period where families had a lot of children the holy family had only one, even if interpretations seem to diverge on this subject, as many Christians and some religion specialists, citing some passages from the New Testament, believe that Jesus had brothers. The Christian God triad reflects probably the influence of Hindu religion with its three male Gods, Vishnu, Brahma and Shiva, but it should here be said that the concept of "Trinity" plagued many other religions as well. Before entering into the Hindu religion let us remember that the Babylonian religion worshipped three gods: Ea, Any, and Enil. The Hindu religion is the world's oldest, has no founder and no single holy book, but has many texts including the four Vedas along with the Upanishads. India's main religion turns around the *Trimurti*, which is surely the oldest form of a 'Trinitarian' God. The term *Trimurti* means '*having three forms*', and is applied to the three Hindu Gods: Vishnu, the God of preservation, Brahma, the God of creation, and Shiva, the God of destruction who had 3 eyes. In the famous epics, the Mahabharata, the poet counts 33,333 prophets [3]. Besides the all-important Trimurti, Hindus worship a rich set of Gods and Goddesses. Buddhism also became fascinated with the number three, as is testified by the *three precious jewels*, and by the *Buddha*, *Dharma* and *Sangha*. The African moon Goddess, *Ashanti*, is three people, two black and one white. In the ancient Celtic society a triune mother-Goddess was worshipped, and she was the most important of all the continental Celtic deities. A pre-Islamic Goddess, *Manant*, is a three-fold goddess representing the three holy virgins: Al-Itab, Al-Uzza, and Al-Manat. The ancient Egyptian religion God, Thoth, the Thrice Great, the supreme power, was considered by the Alchemists as their ancestor, and was identified with Hermes in the time of the Ptolomies, from whence came the Thrice Great Hermes '*Trismegistus*', progenitor of the *Emerald Tablets* describing the mysteries of Alchemy. In later alchemical times, when these old Gods were banished by the incoming Christian religion, the divine Hermes lost his divinity and was reduced to a simple but powerful magician.

The Zoroastrian religion, sometimes known also as Mazdaism, is a very old religion (ca., 1400-1200 BC), which influenced Judaism, and through it Christianity (especially Manicheism) and Islam, and whose holy book is the *Avesta*, of which only the hymns are attributed to Zoroaster. Modern Zoroastrians, who consider their religion monotheistic, worship Ormazd, the wise Lord (Ahura Mazda), who has three personifications: Ormazd himself, Spenta Minyu, and Angra Minyu (Ahriman). Ahriman originally was opposed to Ormuzd, who created the Earth as a battlefield to fight in the next 3000 years the evil Ahriman, i.e., till AD ~2400. Human beings have free will to choose between Ohrmazd and Ahriman, but once the choice is made it cannot be changed anymore, but here, even those who go to Hell, after a period of time go to Heaven. Purification rituals and confessions are important in Zoroastrianism. Small Zoroastrian groups survive in Iran and in India around Mumbay (Bombay), and even smaller groups exist in the larger American and Canadian towns. All of them together make up ca. 150,000 adepts. The community living now in India came from Persia after persecutions they suffered during Islam in the 8th century, after that in 6th century the Caliph Omar granted them the status of *People of the Book*. Two very famous modern Zoroastrians are the deceased Freddy Mercury of the group Queen and the Symphonic conductor Zubin Mehta.

A kind of "Trinity" existed also in the ancient Roman religion. The centre of Rome and of any Roman town was the Capitolium, where the three gods, Jupiter, Juno and Minerva, were worshipped. It is an interesting fact that two of them were women, an unacceptable concept by the Christian religion. Nevertheless, Christianity in its Catholic branch, with the huge importance of the cult of the Virgin Mary, seems to pay a belated tribute to female deities. Interestingly, this kind of female 'God' is the only 'God' who seems to appear here and there nowadays, in fact, among other apparitions, she appeared in 1917 to three unknown young Portuguese shepherds, of which only the ten-years-old girl could speak with the Virgin. Tradition tells that she wrote three letters to be given over to the pope. Here a strange parallel with UFO abductions is at play; in fact UFOs seem to abduct otherwise little-known people. In the Greek-Roman religion an important place was held by the Triad of Muses. Even if the Old Greek-Roman religion was the most joyous religions ever existed, this did not prevent Greeks and Romans (together with all other civilizations of those times), from finding slavery a quite normal matter, not even worth of discussion. In any event, in this religion a considerable place was reserved to the *Three Graces*. In contrast with the three Furies, who nearly nobody remembers, they were the goddesses of joy, charm, and beauty. They were the daughters of the God Zeus and the nymph Enymone. They were: *Aglaia* (spendor), *Euphrosyne* (Mirth), and *Thalia* (Good cheer). They presided over banquet, dances, and all other pleasurable events, and brought joy and goodwill to both mortals and Gods. They are associated with the arts, and were believed to endow artists and poets with the ability to create beautiful work of arts. In art they are usually represented as naked young maidens dancing in a circle. Their myth was re-enacted during the Renaissance and many famous artists, like Raffaello Sanzio (1483-1530) and even Rembrandt (1606-1669) left interesting works of art about them. With the Reform and Counter-reform their myth slowly died out, as well as the representation of the naked body, which resulted in putting pants even to the naked bodies of the frescos of the Sistine Chapel by Michelangelo. Not even the last restoration of this chapel, done more than ten years ago, allowed to get rid of those pants, even if the technology to get rid of them was at hand. The Renaissance was imbued by the Platonic philosophy and the idea of Platonic love and beauty patronized by the renaissance Florentine philosopher (and priest) Marsilio Ficino (1433-1499), the grounder of the Platonic Academy, and author of a *Platonic Theology*. In this philosophy the representation of a naked body was the quintessence of art and it represented the concept of beauty and love.

Numerology as an orthodox method of research in Mediaeval theology stems from St. Augustinus of Hippo (353-430), who concluded that *number is the unshatterable foundation of the Absolute*, and that the *deity is the great numerologist* (following in this Plato), who knows all numbers because his understanding is infinite. Conversely, the deity is omniscient because he knows all the numbers (or all the “true” names of the numbers). *Number* is therefore necessary and sufficient for the existence of the deity. Christendom ended by strongly emphasizing the concept of trinity, re-developing the esoteric doctrine of the sacred three, which was already highly developed when Augustine took it over amplified it, and passed it on, enriched by his own contributions, to the theologians of the Middle Ages. The main difficulty here was to show how the Pythagorean numerology sanctions the equality 3 = 1. By the time Saint Augustine tackled the problem of three this difficulty had been overcome. The Council of Constantinople (c. 381) had officially recognized the transcendental arithmetic of the holy as the foundation of Christian theology. The Sevillan polymath, Bishop St. Isidore (570-636) [6] confirmed the propagation of the Pythagorean gospel. Thus the emphatic role of number three was taken over by the Christian religion. Already the Old Testament tells of the three sons of Abraham. The Bible teaches that three were the famous kings of Israel: Saul, David and Salomon, while we are the descendents of Shem, Ham, and Japheth. An important moment in the history of Israel was reached with Abraham, Isaac, and Jacob. The three attributes of God are: omniscience, omnipresence, and omnipotence, and this is why three denotes the divine perfection, who created the Earth on the third day. Moses asked three things from God: that the Shekinah might rest on Israel, that it might rest on none but Israel, that God's ways might be known to him (Beracheth, fol. 7, col. 1). Three attributes appear again with the Christian three-fold nature of man: spirit, soul, and body. The life of Jesus is plagued by number three, from its beginnings till its end. At

Christ's birth three wise men (the Magi, or the three Kings) worship the new born Jesus with three gifts. On the mount Thabor during the transfiguration Jesus was together with Elias, and Moses, and this same transfiguration was observed by three Apostles, Peter, John, and James. Jesus was crucified at the third hour when he was 33 years old, and it was for three hours that darkness shrouded the Calvary, where there were three crosses (Jesus plus two 'unworthy' companions). Christ's crucifixion was witnessed by the *Three Maries*. About the ninth hour Christ cried three words, "my God, my God, why hast thou forsaken me" (Matt 27:46). The inscriptions on the cross were in three languages. Jesus was denied by his apostle Peter three times. Jesus resurrected on the 3rd day, and, tradition believes that his tomb was again visited by the *Three Maries*, who came into Christ's sepulcher bringing spices they had prepared, and found it empty. By the way, these three Maries remind strongly the three Greek graces. Jesus, finally, appeared three times to his disciples. The central role of the number three here could strike a modern theologian as a kind of slightly blasphemous numerology. Number three had a quite long career in Christian religion with an interesting history. To worship the mystery of the Trinity in 1193 St. John de Matha founded the monastic order of the *'Trinitarians*'. One of the tasks of this order was the redemption of captives from Muslims. This order was instrumental in freeing Cervantes after he had been for five years a slave of the Bey of Algiers, and after many unsuccessful attempted escapes. Around the year 540 A.D. during Justinian's reign, there arose the dispute among the fathers of the Christian Church known as the *Three Chapters* problem (tria kephálaia). These were propositions anathematizing (*i*) the person and writings of Theodore of Mopsuestia, (*ii*) certain writings of Theodor of Cyrus, and (*iii*) the letter of Ibas to Maris. At a very early stage of the controversy the incriminated writings came to be spoken of as the *Three Chapters.* The history of the controversy may also be divided into three periods: (i) the first one ended with the arrival of Pope Virgilius at Constantinople; (ii) the second ended with his ratification of the 2nd Council of Constantinople, in which the three chapters were condemned; (iii) the third period ended with the final healing of the schisms in the West caused by papal ratification of the aforementioned council. The anathematization started with an edict of the Emperor Justinian in 544 A.D [38].

How many Christians know how many Creeds there are? Well, there are three creeds, known as the *Three Historic Ecumenical Creeds*: the *Apostles' Creed*, the *Nicaene Creed*, and the *Athanasian or Trinitarian Creed*. The *Apostles' Creed*, as we now know it, dates back from the 8th century. It is a revision of the so-called Old Roman Creed, which was used in the West by the 3rd century. The Old Roman creed has its roots in the New Testament itself. It does not come from the Apostles, but its roots seem apostolic. It serves as a Baptismal symbol, i.e., it describes the faith into which we are baptized, and is used in the rites of Baptism, and of affirmation. This is the Creed which is normally used for private devotions. The *Nicaene Creed* appeared in the East, when the Council of Nicaea (A.D. 325) rejected the teaching of Arius, and expressed its position by adopting one of the current Eastern symbols and inserting into it some anti-Arian phrases. At the Council of Constantinople (381) some minor changes were made, and it was reaffirmed at the Council of Chalcedon (451). The Nicaene Creed says "I believe in one holy Catholic and Apostolic Church." Today the term denotes one of three main distinct branches of Christianity, the other two being Orthodox and Reformed churches. All present believers are members of one branch and forbidden to receive communion with members of the other two. The *Athanasian Creed* is of uncertain origin, but it is Western in character. For a certain time it was supposed that it was conceived during the life of the great theologian of the 4th century, Athanasius. Actually, it dates from the 5th or 6th century. With this creed the Church condemned two deviations from the official Bible teaching: the denial that God's Son and the Holy Spirit are of one being with the Father, the other the denial that Jesus Christ is true God and true man in one person. It declares that whoever rejects the doctrine of the trinity and the doctrine of Christ is without the saving faith; it is, in fact, considered the *Trinitarian creed* and read aloud in corporate worship on Trinity Sunday [39]. No wonder that there are three main branches of Christianity: Catholic, Orthodox, and Reformed Christian Churches.

Let us here recall that the trinity problem, i.e., a 'trinitarian' conception of the Church development through history had caused the posthumous condemnation (1215) of the ideas of the catholic Cistercian abbot Joachim da Fiore (1130-1202, from Cosenza, he achieved to be cardinal), which delineated a historic-philosophic interpretation of the concept of the Trinity. He divided history into three successive and fundamental ages: the age of the Father was the age of the old biblical Testament, which was characterized by humanity suffering under the power of an all-encompassing divine law; the age of the Son, which started with Christ and was surmised to end in 1260, was the age of the New Testament that had granted the humans the role of sons of God; the age to come was the age of the Holy Ghost after 1260. In this last stage humanity would be directly in touch with God and enjoy the complete freedom that had been preached by the Christian message. This new humanity was presumed to abolish the organized Church, which was thought to be overly influenced by earthly considerations and which was said to have relinquished the teachings of the New Testament. Only during this last stage would God's ideas be fully and deeply understood and not just literally as it happened in his days. Although da Fiore's original ideas had been well received during his life, after his death his teachings were condemned by the Lateran council in 1215, and in 1263 the Church classified all his ideas and books as pure heresy. Still, his ideas influenced William of Ockham, among others, and the spiritualist movement of the friars, which also ended up excommunicated and persecuted [40]. It should be reminded that nearly in the same period (1209-1249) the Church condemned and persecuted also the Cathari heresy (Albigeses) in southern France, which ended up completely extirpated, while their writings went completely destroyed.

The trinitarian problem made a very famous victim in the person of the Spanish theologian, philosopher and anatomist Michel Servetus (1511-1553), who attacked the creed of the trinity in his famous book *De Trinitate Erroribus* (1531) and ended up burned on the main square of Geneva by the order of Calvinists. Of note here is the fact that Calvin was his former friend, but as it often happened (and happens) theological questions were deemed more important than humanitarian sentiments. Servetus ended up influencing Faustus Socinus, (1539-1604) from Siena, Italy, the founder of the anti-trininitarian movement called Socinianism (together with Bernardino Ochino, Giorgio Blandrata, and the brother of Faustus, Laelius). To avoid persecution for his religious ideas, Faustus and his friends had to flee from Italy to Poland, here they settled down in Krakow, where they organized the Minor Reformed Church of Poland. Socinianism represented the extreme and last attempt to reconcile Christianity with humanism and rationalism. The doctrine of the Holy Trinity was rejected, as well as hell, original sin, and baptism; the scriptures were considered authoritative but not exempt from errors and were interpreted in the light of the new rationalism. The sacraments were viewed as spiritual symbols, and Jesus was considered a man, who held testimony of God and who, with his example, showed how to reach salvation. Socinian ideas were compiled in 1605 in the famous *Krakovian catechism* (1605). The movement became known as the Polish Brethren, and it refused to hold serfs or to participate in war. Persecution started in 1610 when Jesuits arrived in Poland and influenced the King Sigismund August III (1587-1632) on the subject. In 1611 the Socinian Jan Tyskiewicz refused to recant and was burned in the main square of Warsaw. Persecution peaked in 1658 when the movement was given the triple option (*a*) to convert to Catholicism, (*b*) leave the country, or (*c*) be executed. Some of its members settled down in Holland, where they helped in liberalizing the Reformed doctrine. The movement disappeared completely in 1811, to reappear again in Poland, after the fall of the Communism [41]. Freeman Dyson (1923-), a famous living physicist considers himself a Socinianist [15].

There is a site in the web that has collected, and continues to collect any kind of more or less serious proofs of God's existence. Most of them are based on the ternary structure of Aristotle's logic, like the 3rd proof, which uses an Ontological argument: (*i*) I define God to be *X*, (*ii*) Since I can conceive of *X*, *X* must exist, (*iii*) Therefore, God exists, all withstanding that one likely can also conceive of not *X* [42].

For centuries church calendars in the East and the West have agreed that there are twelve days of Christmas, and that they begin with 25th December and end on January 6. The history of Christmas is intertwined with that of Epiphany. The commemoration of Baptism (also called the day of light, i.e., the illumination of Jesus) was also known as the birthday of Jesus because he was believed to have been reborn in baptism. In some records Christmas and Epiphany were referred as the first and second nativity; the second being Christ's manifestation to the world. In the early days of Christianity it was believed that Christ will be reborn again a third time, a belief that in the Catholic church faded away, while in some reformed churches and ins some sects is still extant. The twelve days of Christmas start, in the West, with the Christmas day, i.e., the date of the Feast of Christ's birth, adopted in the 4th century by the Western Christian Church. These twelve days end with the Feast of Epiphany, celebrated on January 6, also called 'The Adoration of the Magi', or the 'Manifestation of God'. This feast is known as the day of the three kings, or wise men, or magi: Gaspar, Melchior, and Balthasar, who presented the Christ child with three gifts: gold, frankincense, and myrrh. Some of the Eastern Christian Churches, instead, recognize January 6 as the celebration of the nativity [43]. These three kings should represent the three old continents, Africa, Asia, and Europe, and they remind us of three other men from the Old Testament, namely the three sons of Noah, whose generations spread over those three continents: Ham, Japheth, and Sem. Note the coincidence of these two last triplets with the three great monotheistic Mideastern religions arising *via* the descendants of Abraham: Judaism, Christianity, and Islam. These last triples have, actually, a strange parallel with the recent evolutionistic finding of the three diasporas of the Homo sapiens out of Africa. Let us close this section with two random triples with no intention to draw any parallelism: Neptune's (Poseidon's) Trident and the Pope's Triple Crown.

Gematria is not a chapter of Geometry but a kind of numerology the cabalists believed in, and still believe today, and that associated each letter of the Torah (the old Testament's first five books) to specific numbers. It was conceived that in the letters of the Torah were concealed thousands of secret meanings. These mysteries were extracted from the text by a *Gematria* process: numbers assigned to letters were added, multiplied, and manipulated in an attempt to disclose the hidden mysteries. Similar techniques were, and still are, widespread among Christian (and Muslim mystics), even if they are no more officially endorsed by the Church. Thus, in Gematria, *444* is the gematria number of the word Damascus, which is thought to be the oldest city in the world, *666* is the gematria number of the beast in Revelations, but it is also the gematria number of man, symbolizing human wisdom but also imperfection; *888* is the gematria number of the name Jesus, ad, finally, *999* is the gematria number connected with judgement, and it is the numerical value of the phrase *my wrath*.

Finale

*Alle gute Dinge sind drei (*all good things are three, a German saying)

The third time it's the charm (an English saying)

These two sayings are just a specimen in the plethora of examples we can find in the ordinary life, which show how number three pervades everyday life, to cite all examples would make of this paper a book. Nevertheless, some well known everyday ‘three’ are worthy to cite. The eternal ‘love triangles’ (he, she and the other), always persecuted, even with the most cruel methods (even today in some Islam countries), but never deracinated. Who has not ever heard about the *three good things I can tell about her* ? The genies of many fables and jokes always grant *three wishes*, which, phonetically, remind one of the three witches (see Shakespeare’s *Macbeth*). Modern supermarkets instead of allowing for three wishes allow to *take three for two*, to induce buying more for less. There are three strikes in baseball, three sets in a tennis match, three minutes in a boxing round, a triple somersault from a trapeze bar to a catcher's hand. The alphabet is referred to as the ABC, and something is as easy as ABC, while many organizations and television networks use three letter abbreviations, which makes them easier to be memorized: USA, AMA, DOE, FBI, SOS, CIA, CB, CNN, SSN (social security number), TGF (Thank God it’s Friday). In many Western European countries and in the USA the convention is to assign two names to an individual, resulting in two initials preceding the surname. Slogans and verbal rituals of any kind consist in many cases of three words, "veni, vidi, vici", "libertè, egalitè, fraternitè", "we shall overcome", "tell the truth, the whole truth, and nothing but the truth", "those who can, do; those who can't, teach; and those who can't teach, teach the teachers !" Similarly to the “three wishes”, it was reported by sales trainers that consumers prefer products with “three choices” such as whole/low-fat/no-fat (milk, yogurt, cheese). It was also reported that three-beat slogans are the more effective: reduce, reuse, recycle; the few, the proud, the Marines; the Domino’s pizza delivery slogan in US is FFF: fact, friendly, free-delivery. Also, commands go with three words: lights, camera, action!; ready, aim, fire !; ready, steady, go!; and the starter for a race will say "one, two, three, go". The internet addresses show the type of organization by three letters following a dot, where the “dot-com” is nowadays commonplace, even if other types of organizations are not uncommon, namely: ‘.gov’, ‘.org’, ‘.mil’, ‘.edu’, and so on. Who does not remember the three monkeys, one hiding his eyes, the second his mouth and the third his ears? This is a very old symbol, which seems to have originated in India, where it was first recognized that the number system is composed of positive, negative and zero numbers.

On the three ages of man "four legs in the morning, two legs at noon, and three legs at night" is based the very old and wide distributed riddle of the Sphynx, which is cited in Sophocle's Oedipus rex. As the figure of Oedipus was central in the development of Freudian psychosexual theory let us tell that the structural model of this theory was centered on three 'entities': the *Id* (we are born with it), the *ego* (it develops in the first three years after birth), and the *superego* (the moral part of us, it starts to develop after five). It should be reminded that the scientific bases of the psychoanalysis are rather wacky [44, 45]. An American neurologist recently proposed a triune primate brain, which is structured like the layers of an archeological site: the oldest and the most internal brain, the *Reptilian* brain, which is responsible for the mechanical behavior, and which encompasses the brain stem and the cerebellum; the midway brain, or the *Paleomammalian* brain, which includes the limbic system, which contains the hippocampus, the thalamus and the amygdala, and which is responsible or the emotional behavior, and the external brain, the neocortex of the primates or the *Neomammalian* brain, which is responsible for the rational behavior. Actually, this subdivision reminds the real division of the brain into three parts: the forebrain, which includes the cerebral cortex (highly developed in humans), the brainsteam, and the hindbrain. The forebrain develops and controls the possibility to feel, learn, and remember, the brainsteam send and receives information, and the hindbrain (made up of medulla oblongata, pons and cerebellum) coordinates the movements. [46]

In all our previous papers [1-4] there has always been a place for the *Fibonacci* series. Has number three or the ternary pattern nothing to do with such a subject? Well, The Fibonacci *Q*-Matrix is a three element matrix defined by,

$$\begin{pmatrix} F_2 & F_1 \\ F_1 & F_0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

where F_n is a Fibonacci number, then

$$Q_n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$$

Its basic properties, which can be found in [47, 48], include among others: $|Q_n| = |Q|^n$, which gives a series of relations with ternary structure,

$$F_{n-1}F_{n+1} - F_n^2 = (-1)^n,$$

and

$$Q^{n+1}Q^n = Q^{2n+1}$$

and

$$\begin{pmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{pmatrix}\begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} = \begin{pmatrix} F_{2n+2} & F_{2n+1} \\ F_{2n+1} & F_{2n} \end{pmatrix}$$

Present excursion about a pattern and its characterizing number that did much more than history was done with the intent to help the reader to catch a glimpse about the way some numbers, and, especially, the number three, have ended occupying a much wider place in the history of human culture than they deserved either as one of the ten digits, all on an equal foot, of our decimal system, or in the order stated by the Benford law. If there are a vast plethora of fundamental dualities in religion, in philosophy, in the sciences including mathematics, and in everyday life, as has recently been reviewed [1], there seems also to be many non-trivial *tri-alities*, much beyond religion, which however seems to be the most affected by the fascination number three exerted on mankind. Who can exclude that the repeated appearance of number three in science is not due to the importance this number has in the religious, literary, and philosophical background of many scientists? But, one may surmise that these triads or trinities in religion may in part gain mystery simply from not being dualities. It should be remembered that many 'trialities' could also be read as dualities, as 'tetralities' or even as sextets, as in biology, thermodynamics, mechanics and particle physics. For example, if in the genus Homo the Neardenthal man is considered not a subspecies but a species, as it is suggested by many experts, than we have four types of Homo, furthermore, the same could be done with the Cro-Magnon man, actually considered a subspecies. Furthermore, the Australopithecus, the first hominid, who appeared around $4 \cdot 10^6$ years ago, also in Africa, has been excluded from this classification. Some of the appeal the triune pattern has, surely, originates from the fact human mind likes to classify using simple patterns. Number three is not trivial like one or two and it keeps the classification (or chances) far from inflation. The triune pattern offers, thus, a very easy way to learn through patterns, which is the easiest way to learn and teach.

Latest News

A group at the department of Chemistry and Biochemistry at the University of California, Los Angeles, achieved to synthesize the first molecular Borromean Ring [49].

Acknowledgements

We would like to draw the attention to the internet mathematical magazine *plus.math.org*, to which we are so much indebted (some figures inclusive). Furthermore, we would like to add here an interesting remark by Ivan Gutman about two unstable hydrogen isotopes recently obtained in laboratory: ^{4}H, with three neutrons and a proton, obtained by bombarding tritium with fast moving deuterium nuclei, and ^{5}H, with four neutrons and a proton, obtained by bombarding tritium with fast moving tritium nuclei [50]. Thus, up to now, there are five hydrogen isotopes: two highly stable, *H*, and *D*, one relatively stable, *T*, and two unstable, ^{4}H, and ^{5}H. We decided to close the reference section with the Trinajstić paper on number five, [51] as it was this paper, which first awoke our curiosity about numbers.

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