

## **UNSTEADY COUETTE FLOW WITH HEAT TRANSFER OF A VISCOELASTIC FLUID CONSIDERING THE HALL EFFECT**

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**ABSTRACT.** The unsteady Couette flow of an electrically conducting incompressible non-Newtonian viscoelastic fluid between two parallel horizontal non-conducting porous plates is studied with heat transfer considering the Hall effect. A sudden uniform and constant pressure gradient, an external uniform magnetic field that is perpendicular to the plates and uniform suction and injection through the surface of the plates are applied. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are taken into consideration. Numerical solutions for the governing momentum and energy equations are obtained using finite difference approximations. The effect of the Hall term, the parameter describing the non-Newtonian behavior, and the velocity of suction and injection on both the velocity and temperature distributions is examined.

### **INTRODUCTION**

The flow of an electrically conducting viscous fluid between two parallel plates in the presence of a transversely applied magnetic field has applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays. Hartmann flow of a Newtonian fluid with heat transfer, subjected to different physical effects, have been studied by many authors [1-9]. These results are important for the design of the duct wall and the cooling arrangements. The rectangular channel problem has later been extended also to fluids obeying non-Newtonian constitutive equations. The hydrodynamic flow of a viscoelastic fluid has attracted the attention of many authors [10-13] due to its important industrial applications [11]. In most cases the Hall term was ignored in applying Ohm's law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamics is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [4]. Under these conditions, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term.

Attia [8] has studied the influence of the Hall current on the velocity and temperature fields of an unsteady flow of a conducting Newtonian fluid between two infinite non-

conducting horizontal parallel stationary and porous plates. This problem is extended here to the case of a non-Newtonian viscoelastic fluid where the upper plate is moving with a uniform velocity. The flow is subjected to a uniform and constant pressure gradient, a uniform suction from above and a uniform injection from below, and an external uniform magnetic field perpendicular to the plates. The Hall current is taken into consideration while the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The two plates are kept at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers and flow meters. The Joule and viscous dissipations are taken into consideration in the energy equation. The governing momentum and energy equations are solved numerically using the finite difference approximations. The inclusion of the Hall current as well as the non-Newtonian fluid characteristics leads to some interesting effects, on both the velocity and temperature fields.

### FORMULATION OF THE PROBLEM

The geometry of the problem is shown in Fig. 1. The fluid is assumed to be incompressible, viscoelastic and flows between two infinite horizontal parallel non-conducting plates located at the  $y=\pm h$  planes and extend from  $x=-\infty$  to  $\infty$  and from  $z=-\infty$  to  $\infty$ . The upper plate is moving with a uniform velocity  $U_0$ . The lower and upper plates are kept at two constant temperatures  $T_1$  and  $T_2$  respectively, with  $T_2>T_1$ . The flow is driven by a uniform and constant pressure gradient  $dP/dx$  in the  $x$ -direction, and a uniform suction from the above and injection from below which are applied at  $t=0$ . A uniform magnetic field with magnetic flux density vector  $B_0$  is applied in the positive  $y$ -direction which is assumed to be also the total magnetic field, as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4]. The Hall effect is taken into consideration and consequently a  $z$ -component for the velocity is expected to arise. The plates are assumed to be infinite in the  $x$  and  $z$ - directions which makes the physical quantities do not change in these directions. Thus, the velocity vector of the fluid, in general, is given by

$$\vec{v}(y,t) = u(y,t)\vec{i} + v(y,t)\vec{j} + w(y,t)\vec{k}$$

It is because of the conservation of mass, i.e.,  $\nabla \cdot \vec{v} = 0$  and due to the uniform suction the velocity component  $v(y,t)$  is assumed to have a constant value  $v_0$ .

The fluid motion starts from rest at  $t=0$ , that is  $u=w=0$  for  $t \leq 0$ . The no-slip condition at the plates implies that  $u=U_0$ ,  $w=0$  at  $y=h$  and  $u=0$ ,  $w=0$  at  $y=-h$ . It is also assumed that the initial temperature of the fluid is  $T_1$ , thus the initial and boundary conditions of temperature are  $T=T_1$  at  $t=0$ ,  $T=T_1$  at  $y=-h$ ,  $t>0$  and  $T=T_2$  at  $y=h$ ,  $t>0$ . If the Hall term is retained, the current density  $\vec{J}$  is given by

$$\vec{J} = \sigma \{ \vec{v} \wedge \vec{B}_o - \beta (\vec{J} \wedge \vec{B}_o) \} \quad (1)$$

where  $\sigma$  is the electric conductivity of the fluid and  $\beta$  is the Hall factor [4]. Equation (1) may be solved in  $\vec{J}$  to yield Lorentz force vector in the form

$$\vec{J} \wedge \vec{B}_o = \frac{-\sigma B_o^2}{1+m^2} \{ (u+mw)\vec{i} + (w-mu)\vec{k} \}$$

where  $m=\sigma\beta B_o$  is the Hall parameter. The fluid motion is governed by the momentum equations [14]

$$\rho \left( \frac{\partial u}{\partial t} + v_o \frac{\partial u}{\partial y} \right) = -\frac{dP}{dx} - \frac{\sigma B_o^2}{1+m^2} \{ u+mw \} + \frac{\partial \tau_{xy}}{\partial y} \quad (3)$$

$$\rho \left( \frac{\partial w}{\partial t} + v_o \frac{\partial w}{\partial y} \right) = -\frac{\sigma B_o^2}{1+m^2} \{ w-mu \} + \frac{\partial \tau_{zy}}{\partial y} \quad (4)$$

where  $\rho$  is the density of the fluid,  $\tau_{xy}$  and  $\tau_{zy}$  are the components of the shear stress of the viscoelastic fluid given respectively as [10]

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} - \frac{\mu}{\alpha} \frac{\partial \tau_{xy}}{\partial t} \quad (5a)$$

$$\tau_{zy} = \mu \frac{\partial w}{\partial y} - \frac{\mu}{\alpha} \frac{\partial \tau_{zy}}{\partial t} \quad (5b)$$

where  $\mu$  is the coefficient of viscosity and  $\alpha$  is the modulus of rigidity. In the limit  $\alpha$  tends to infinity or at steady state, the fluid behaves like a viscous fluid without elasticity. Solving Eqs. (5a) and (5b) for  $\tau_{xy}$  and  $\tau_{zy}$  in terms of the velocity components  $u$  and  $w$  we obtain

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \frac{1}{\alpha} \frac{\partial}{\partial y} \left( \mu \frac{\partial}{\partial t} \left( \mu \frac{\partial u}{\partial y} \right) \right), \quad (6a)$$

$$\frac{\partial \tau_{zy}}{\partial y} = \frac{\partial}{\partial y} \left( \mu \frac{\partial w}{\partial y} \right) - \frac{1}{\alpha} \frac{\partial}{\partial y} \left( \mu \frac{\partial}{\partial t} \left( \mu \frac{\partial w}{\partial y} \right) \right), \quad (6b)$$

where the two terms  $(1/\alpha^2) \frac{\partial}{\partial y} (\mu \frac{\partial}{\partial t} (\mu \frac{\partial \tau_{xy}}{\partial t}))$  and  $(1/\alpha^2) \frac{\partial}{\partial y} (\mu \frac{\partial}{\partial t} (\mu \frac{\partial \tau_{zy}}{\partial t}))$ , which are proportional to  $(1/\alpha^2)$  have been neglected. Substituting Eqs. (6a) and (6b) in the momentum Eqs. (3) and (4) yields

$$\rho \left( \frac{\partial u}{\partial t} + v_o \frac{\partial u}{\partial y} \right) = -\frac{dP}{dx} - \frac{\sigma B_o^2}{1+m^2} \{u + mw\} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\mu^2}{\alpha} \frac{\partial^3 u}{\partial t \partial y^2} \quad (7)$$

$$\rho \left( \frac{\partial w}{\partial t} + v_o \frac{\partial w}{\partial y} \right) = -\frac{\sigma B_o^2}{1+m^2} \{w - mu\} + \mu \frac{\partial^2 w}{\partial y^2} - \frac{\mu^2}{\alpha} \frac{\partial^3 w}{\partial t \partial y^2} \quad (8)$$

The temperature distribution is governed by the energy equation [16]

$$\rho c_p \left( \frac{\partial T}{\partial t} + v_o \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_o^2}{1+m^2} \{u^2 + w^2\} + \mu \left( \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right) \quad (9)$$

where  $c_p$  and  $k$  are, respectively, the specific heat capacity at constant pressure and the thermal conductivity of the fluid. The second and third terms on the right-hand side represent the Joule and viscous dissipations respectively. Equations (7), (8) and (9) can be made dimensionless by introducing the following dimensionless variables

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{t} = \frac{tU_o}{h}, \hat{u} = \frac{u}{U_o}, \hat{w} = \frac{w}{U_o}, \hat{p} = \frac{P}{\rho U_o^2}, \hat{T} = \frac{T - T_1}{T_2 - T_1}$$

We also define the following dimensionless parameters,

$$S = \frac{v_o}{U_o}, \text{ the suction parameter,}$$

$$Re = \frac{U_o \rho h}{\mu}, \text{ is the Reynolds number,}$$

$$Ha = B_o h \sqrt{\sigma / \mu}, \text{ the Hartmann number,}$$

$$Pr = \frac{\mu c_p}{k}, \text{ the Prandtl number,}$$

$$Ec = \frac{U_o^2}{c_p (T_2 - T_1)}, \text{ the Eckert number,}$$

$$K = \frac{\mu^2}{\rho \alpha h^2}, \text{ the viscoelastic parameter,}$$

In terms of these dimensionless quantities, Eqs. (7), (8) and (9) may be written, after dropping all hats for convenience, as

$$\frac{\partial u}{\partial t} + \$ \frac{\partial u}{\partial y} = -\frac{dp}{dx} - \frac{Ha^2}{\text{Re}(1+m^2)} \{u + mw\} + \frac{1}{\text{Re}} \frac{\partial^2 u}{\partial y^2} - K \frac{\partial^3 u}{\partial t \partial y^2} \quad (10)$$

$$\frac{\partial w}{\partial t} + \$ \frac{\partial w}{\partial y} = -\frac{Ha^2}{\text{Re}(1+m^2)} \{w - mu\} + \frac{1}{\text{Re}} \frac{\partial^2 w}{\partial y^2} - K \frac{\partial^3 w}{\partial t \partial y^2} \quad (11)$$

$$\frac{\partial T}{\partial t} + \$ \frac{\partial T}{\partial y} = \frac{1}{\text{Re Pr}} \frac{\partial^2 T}{\partial y^2} + \frac{Ha^2 E_c}{\text{Re}(1+m^2)} \{u^2 + w^2\} + \frac{E_c}{\text{Re}} \left( \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right) \quad (12)$$

The initial and boundary conditions for the velocity and temperature in the dimensionless form are written as

$$u(y,0) = w(y,0) = u(-1,t) = w(-1,t) = w(1,t) = 0, u(1,t) = 1, \quad (13)$$

$$T(y,0) = T(-1,t) = 0, T(1,t) = 1 \quad (14)$$

Equations (10)-(12) represent a system of partial differential equations which is solved numerically under the initial and boundary conditions (13) and (14), using the finite difference approximation. The Crank-Nicolson implicit method [15] is used at two successive time levels. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximation in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm [15]. Grid-independence studies show that the computational domain  $0 < t < \infty$  and  $-1 < y < 1$  can be divided into intervals with step sizes  $\Delta t = 0.0001$  and  $\Delta y = 0.005$  for time and space, respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when any one of u, w, T, and their gradients for the last two approximations differs from unity by less than  $10^{-6}$  for all values of y in  $-1 < y < 1$  at every time step. Computations have been made for  $dp/dx=5$ ,  $\text{Re}=1$ ,  $\text{Pr}=1$ , and  $E_c=0.2$ . In order to examine the accuracy and correctness of the solutions, the results for the non-magnetic and Newtonian cases are compared and shown to have complete agreement with those reported by Attia [8].

## RESULTS AND DISCUSSION

Figures 2-4 show the time development of the profiles of the velocity components  $u$  and  $w$  and the temperature  $T$  respectively for various values of time  $t$  and for  $K=0, 0.5, \text{ and } 1$ . The figures are evaluated for  $Ha=3$ ,  $m=3$ , and  $\$=1$ . Figures 2 and 3 and tables 1 and 2 indicate that the parameter  $K$  has an apparent effect on the growth of  $u$  and  $w$  with time and that, for all  $K$ ,  $u$  and  $w$  increase for a period of time and then decrease as time develops for all  $y$ . The time at which  $u$  and  $w$  start decreasing increases for all values of  $K$ . Also, the time at which  $u$  starts decreasing is larger than that of  $w$ . Figures 2 and 3 and tables 1 and 2 show that, for small  $t$ , increasing  $K$  decreases  $u$  and  $w$  for all  $y$ . For large  $t$ , increasing  $K$  increases  $u$  and  $w$  for all  $y$ . Figure 4 and table 3 show that the temperature profile reaches its steady state monotonically. In general, increasing  $K$  decreases  $T$  for all  $y$  and  $t$ . However, for large values of  $t$ , increasing  $K$  decreases  $T$  for all  $y$ . Also, for the upper half region at small time, increasing  $K$  increases  $T$  slightly. It is observed also that the velocity component  $u$  reaches the steady state faster than  $w$  which, in turn, reaches the steady state faster than  $T$ . This is expected as  $u$  is the source of  $w$  as a result of the inclusion of the Hall current, while both  $u$  and  $w$  act as sources for  $T$ . It is noticed that the effect of  $K$  on  $u$  and  $w$  and their steady state times is more pronounced than its effect on the temperature  $T$ .

Figures 5-7 show the effect of the Hall parameter  $m$  on the time development of  $u$ ,  $w$  and  $T$  at the centre of the channel ( $y=0$ ) respectively for various values of the Hall parameter  $m$  and for  $K=0, 0.5, \text{ and } 1$ . In these figures  $Ha=3$  and  $\$=1$ . Figure 5 shows that  $u$  increases with increasing  $m$  for all values of  $K$  as the effective conductivity ( $\sigma/(1+m^2)$ ) decreases with increasing  $m$  which reduces the magnetic damping force on  $u$ . In Fig. 6, at large times, the velocity component  $w$  increases with increasing  $m$  as  $w$  is a result of the Hall effect. On the other hand, at small times,  $w$  decreases when  $m$  increases. This happens due to the fact that, at small times  $w$  is very small and then the source term of  $w$  is proportional to  $(\mu/(1+m^2))$  which decreases with increasing  $m$  ( $m>1$ ). This accounts for the crossing of the curves of  $w$  with  $t$  for all values of  $K$ . An interesting phenomenon is observed in Figs. 5 and 6, which is that, when  $m$  has a nonzero value the component  $u$  and, sometimes,  $w$  overshoot. For some times they exceed their steady state values and then go down towards steady state. The time at which overshooting occurs increases greatly with increasing  $K$ . It is also shown in Fig. 5 and 6 that the time at which  $u$  and  $w$  each reaches the steady state increases as  $m$  or  $K$  increases. As shown in Fig. 7, increasing  $m$  decreases  $T$  at small times and increases it at large times. This is due to the fact that, for small times,  $u$  and  $w$  are small and an increase in  $m$  increases  $u$  but decreases  $w$ . Then, the Joule dissipation which is also proportional to  $(1/(1+m^2))$  decreases. For large times, increasing  $m$  increases both  $u$  and  $w$  and, in turn, increases the Joule and viscous dissipations. This accounts for the crossing of the curves of  $T$  with time for all values of  $K$ . The time at which the crossing occurs increases with the increase in  $K$ . It is also observed that increasing  $K$  decreases  $T$  for all values of  $m$  and this effect becomes more pronounced for small  $t$  and large  $m$ . This is because, when  $t$  is small, increasing  $K$  decreases both  $u$  and  $w$  and their gradients which decreases the Joule and viscous dissipations. The figure shows also that the time at which  $T$  reaches its steady state value increases with increasing  $m$  or  $K$ .

Figures 8-10 show the effect of the Hartmann number  $Ha$  on the time development of  $u$ ,  $w$  and  $T$  at  $y=0$  with time respectively for various values of Hartmann number  $Ha$  and for  $K=0, 0.5, \text{ and } 1$ . In these figures  $m=3$  and  $\$=1$ . Figure 8 shows that increasing  $Ha$  decreases  $u$  as it increases the damping force on  $u$ . On the other hand, Fig. 9 indicates that, unless  $Ha$  is large ( $Ha=5$ ), increasing  $Ha$  increases  $w$  as it increases the source term of  $w$  which is

proportional to  $(Ha^2u)$  for various values of  $K$ . Figure 9 presents an interesting phenomenon which is the appearance of the crossing of  $w$  curves with time for large values of  $Ha$  and all values of  $K$ . Increasing  $Ha$  increases  $w$  for small  $t$  and decreases  $w$  for large  $t$ . This is because for small  $t$ ,  $w$  is small and increasing  $Ha$  increases the source term of  $w$  and then increases  $w$  for all values of  $K$ . Small values of  $K$  increases  $w$  more, then with the progress of time, the resulting large increase in  $w$  decreases  $u$  more as it increases the damping force on  $u$ . Hence, the large decrease in  $u$  as well as the large increase in  $w$  decrease greatly the source term of  $w$  which reduces  $w$  more and results in the crossing appears in the figure. It is also clear from Figs. 8 and 9 that  $u$  and  $w$  overshoot for large values of  $Ha$  while the Hall effect is considered ( $m=3$ ). The overshooting in  $u$  and  $w$  decreases with increasing  $K$  due to the decrease in both  $u$  and  $w$  for small  $t$ . It should be pointed out that although the Hall effect is considered ( $m=3$ ), the overshooting in  $u$  and  $w$  appears only when  $Ha$  is large ( $Ha=3$ ). This emphasizes the fact that the Hall effect becomes more pronounced in case the magnetic field is high. Figure 10 shows that the effect of  $Ha$  on the temperature  $T$  depends on  $t$ . If  $Ha$  is small ( $0 < Ha < 1$ ), then increasing  $Ha$  increases  $T$  for all values of  $t$  as a result of increasing the Joule dissipation. In case of large  $Ha$ , for small values of  $t$ , the velocity components  $u$  and  $w$  are small and increasing  $Ha$ , although it decreases  $u$  and  $w$  and their gradients, increases the Joule dissipations and then increases  $T$ . However, for large values of  $t$  increasing  $Ha$  decreases  $T$  due to the reduction in the Joule and viscous dissipations. The figure also shows that the influence of  $Ha$  on  $T$  becomes more pronounced for the case of small  $K$  due to the increase in the velocity components and their gradients which results in increasing the Joule and viscous dissipations.

Figures 11-13 show the effect of the suction parameter  $\$$  on the time development of  $u$ ,  $w$  and  $T$  at  $y=0$  with time, respectively, for various values of the suction parameter  $\$$  and for  $K=0, 0.5$ , and  $1$ . In these figures  $Ha=3$  and  $m=3$ . Figure 11 shows that  $u$  at the centre decreases with increasing  $\$$  for all values of  $K$  due to the convection of the fluid from regions in the lower half to the centre, which has higher fluid speed. Figure 12 shows that  $w$  decreases with increasing  $\$$  for all values of  $K$  as a result of decreasing  $u$  which affects the source term of  $w$ . The figure presents also the influence of  $\$$  on the reduction of the overshooting in  $w$  for all values of  $K$ . Figure 13 indicates that increasing  $\$$  decreases the temperature at the centre of the channel for all values of  $K$ . This is due to the influence of the convection in pumping the fluid from the cold lower half towards the centre of the channel.

## CONCLUSIONS

The unsteady Couette flow of a viscoelastic fluid under the influence of an applied uniform magnetic field is studied considering the Hall effect. The effects of the non-Newtonian fluid behavior (the parameter  $K$ ), the magnetic field (Hartmann number  $Ha$ ), the Hall effect (Hall parameter  $m$ ), and the suction and injection velocity (suction parameter  $\$$ ) are studied. The Hall term affects the main velocity component  $u$  in the  $x$ -direction and gives rise to another velocity component  $w$  in the  $z$ -direction. An overshooting in the velocity components  $u$  and  $w$  with time due to the Hall effect is observed for all values of  $K$  and high values of the magnetic field. The non-Newtonian fluid characteristics have an apparent effect in controlling the overshooting in  $u$  or  $w$  and the time at which it occurs. The results show

that the influence of the parameter  $K$  on  $u$  and  $w$  depends on time and the Hall current. When  $t$  is small, increasing  $K$  decreases  $u$  and  $w$  for all values of  $m$ , but when  $t$  is large, the parameter  $K$  has a significant effect on  $u$  and  $w$  only when  $m$  is large. It is found also that the effect of the Hall term on  $w$  depends on time and the non-Newtonian fluid characteristics. Unless  $K$  is large, increasing  $m$  decreases  $w$  when  $t$  is small but increases it when  $t$  is large. However, for large values of  $K$  and  $m$ , increasing  $m$  decreases  $w$  for all values of  $t$ . It is detected also that the behavior of  $T$  for different values of  $m$  or  $Ha$  depends on  $t$ . When  $t$  is small,  $T$  decreases with increasing  $m$ , but when  $t$  is large, it increases with increasing  $m$ . On the other hand, unless  $Ha$  is small, increasing the magnetic field increases  $T$  for small  $t$  and decreases it for large  $t$ . The effect of the Hall term or the magnetic field on  $T$  depends on the characteristics of the non-Newtonian fluid and becomes more pronounced in case of small  $K$ .

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