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EFFECT OF TEMPERATURE DEPENDENT VISCOSITY AND THERMAL CONDUCTIVITY ON THE UNSTEADY MHD COUETTE-POISEULLE FLOW

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ABSTRACT. The unsteady magnetohydrodynamic MHD Couette-Poiseulle flow and heat transfer of an electrically conducting fluid is studied in the presence of a transverse uniform magnetic field with temperature dependent viscosity and thermal conductivity. The fluid is subjected to a constant pressure gradient and an external uniform magnetic field perpendicular to the plates which are kept at different but constant temperatures. The effect of the magnetic field, the temperature dependent viscosity and thermal conductivity on both the velocity and temperature fields is reported.

List of Symbols:

a: viscosity parameter,
b: thermal conductivity parameter,
B_o: magnetic induction,
c_p: specific heat at constant pressure,
Ec: Eckert number,
Ha: Hartmann number,
J: current density,
k: thermal conductivity,
P: pressure gradient,
Pr: Prandtl number,
T: temperature of the fluid,
T_1: temperature of the lower plate,
T_2: temperature of the upper plate,
u: velocity component if the x-direction,
U_o: velocity of the upper plate,
x: axial direction,
y: distance in the vertical direction,
μ: viscosity of the fluid,
ρ: density of the fluid,
σ: electrical conductivity of the fluid

INTRODUCTION

The flow with heat transfer of a viscous incompressible electrically conducting fluid between two parallel plates is a classical problem that has important applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry. This problem has been considered by many researchers under different physical effects [1-5]. Most of these studies are based on constant physical properties, although some physical properties are varying with temperature and assuming constant properties is a good approximation as long as small differences in temperature are involved [6]. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of these physical properties with temperature. Klemp et al. [7] studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. The MHD fully developed flow and heat transfer of an electrically conducting fluid between two parallel plates with temperature dependent viscosity is studied in [8,9] without taking the Hall effect into consideration.

In the present work, the Transient Couette-Poiseulle flow of a viscous incompressible electrically conducting fluid with heat transfer between two electrically insulating plates is studied in the presence of uniform magnetic field. The upper plate is moving with a constant speed and the lower plate is kept stationary while the fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected [1,5]. The two plates are kept at two constant but different temperatures while the viscosity and thermal conductivity of the fluid are assumed to vary with temperature. Thus, the coupled set of the nonlinear equations of motion and the energy equation including the viscous and Joule dissipations terms is solved numerically using finite differences to obtain the velocity and temperature distributions at any instant of time.

FORMULATION OF THE PROBLEM

The fluid is assumed to be flowing between two infinite horizontal plates located at the $y=\pm h$ planes. The upper plate moves with a uniform velocity U_o while the lower plate is stationary. The two plates are assumed to be electrically insulating and kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2>T_1$. A constant pressure gradient dP/dx is applied in the x-direction. A uniform magnetic field $\mathbf{B}_o$ is applied in the positive y-direction which is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [1,5]. The viscosity of the fluid is assumed to vary exponentially with temperature while the thermal conductivity is assumed to depend linearly on temperature. The viscous and Joule dissipations are taken into consideration. The flow of the fluid is governed by the Navier-Stokes equation which has the form [1,5],

$$\rho\frac{\partial u}{\partial t}=-\frac{dP}{dx}+\mu\frac{\partial^2 u}{\partial y^2}+\frac{\partial\mu}{\partial y}\frac{\partial u}{\partial y}-\sigma B_o^2 u \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity of the fluid, σ is the electric conductivity of the fluid, and $u=u(y,t)$ is the velocity component of the fluid in the x-direction. It is assumed that the pressure gradient is applied at $t=0$ and the fluid starts its motion from rest. Thus

$$t=0:\ u=0 \tag{2a}$$

For $t>0$, the no-slip condition at the plates that

$$y=-h:\ u=0,\quad y=h:\ u=U_o \tag{2b}$$

The energy equation describing the temperature distribution for the fluid is given by [1,10]

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) + \mu(\frac{\partial u}{\partial y})^2 + \sigma B_o^2 u^2 \tag{3}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The last two terms in the left-hand side of Eq. (3) represent, respectively, the viscous and Joule dissipations. The temperature of the fluid must satisfy the boundary conditions,

$t=0$: $T=T_1$ (4a)

$t>0$: $T=T_1$, $y=-h$, $T=T_2$, $y=h$ (4b)

The viscosity of the fluid is assumed to vary with temperature and is defined as, $\mu=\mu_o f_1(T)$ and μ_o is the viscosity of the fluid at $T=T_1$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form [7], $f_1(T)=\exp(-a_1(T-T_1))$, a_1 is a constant takes positive or negative values [10]. In some cases a_1 may be negative, i.e. the coefficient of viscosity increases with temperature [8,9]. Also, the thermal conductivity of the fluid is assumed to vary with temperature as $k=k_o f_2(T)$ and k_o is the thermal conductivity of the fluid at $T=T_1$. We assume linear dependence for the thermal conductivity upon temperature in the form $k=k_o[1+b_1(T-T_1)]$ [10], where the parameter b_1 may be positive or negative [10].

The problem is simplified by writing the equations in the non-dimensional form. To achieve this, we define the following non-dimensional quantities,

$$(\hat{x},\hat{y}) = \frac{(x,y)}{h}, \hat{t} = \frac{tU_o}{h}, \hat{P} = \frac{P}{\rho U_o^2}, \hat{u} = \frac{u}{U_o}, \theta = \frac{T-T_1}{T_2-T_1}, G = -\frac{d\hat{P}}{d\hat{x}}$$

$\hat{f}_1(\theta) = \exp(-a_1(T_2-T_1)\Theta) = \exp(-a\theta)$, "a" is the viscosity exponent,

$\hat{f}_2(\theta) = 1+b_1(T_2-T_1)\,\theta = 1+b\theta$, "b" is the thermal conductivity parameter,

$Rr=\rho U_o h/\mu_o$, is the Reynolds number,

$Ha^2=\sigma B_o^2 h^2/\mu_o$, Ha is the Hartmann number,

$Pr=\mu_o c_p/k_o$ is the Prandtl number,

$Ec=U_o^2/c_p(T_2-T_1)$ is the Eckert number,

$Nu_L=(\partial\theta/\partial\hat{y})_{\hat{y}=-1}$ is the Nusselt number at the lower plate,

$Nu_U=(\partial\theta/\partial\hat{y})_{\hat{y}=1}$ is the Nusselt number at the upper plate.

In terms of the above non-dimensional quantities Eqs. (1) to (4) read (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t} = G + \frac{1}{\mathrm{Re}} f_1(\theta)\frac{\partial^2 u}{\partial y^2} + \frac{1}{\mathrm{Re}}\frac{\partial f_1(\theta)}{\partial y}\frac{\partial u}{\partial y} - \frac{1}{\mathrm{Re}} Ha^2 u \tag{5}$$

$t=0$: $u=0$ (6a)

t>0: u=0, y=-1, u=0, y=1 (6b)

$$\frac{\partial\theta}{\partial t}=\frac{1}{\mathrm{Re}\,\mathrm{Pr}}f_2(\theta)\frac{\partial^2\theta}{\partial y^2}+\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial f_2(\theta)}{\partial y}\frac{\partial\theta}{\partial y}+\frac{Ec}{\mathrm{Re}}f_1(\theta)(\frac{\partial u}{\partial y})^2+\frac{Ec}{\mathrm{Re}}Ha^2u^2 \tag{7}$$

t=0: θ=0 (8a)

t>0: θ=0, y=-1, θ=1, y=1 (8b)

Equations (5) and (7) represent coupled system of non-linear partial differential equations which are solved numerically under the initial and boundary conditions (6) and (8) using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method is used at two successive time levels [11]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas-algorithm [11]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the *y*-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. The computational domain is divided into meshes each of dimension Δt and Δy in time and space, respectively. We define the variables $v=\partial u/\partial y$ and $H=\partial\theta/\partial y$ to reduce the second order differential Eqs. (5) and (7) to first order differential equations. The finite difference representations for the resulting first order differential Eqs. (5) and (7) take the form

$$\left(\frac{u_{i+1,j+1}-u_{i,j+1}+u_{i+1,j}-u_{i,j}}{2\Delta t}\right)=G+\left(\frac{\bar{f}_1(\theta)_{i,j+1}+\bar{f}_1(\theta)_{i,j}}{2\,\mathrm{Re}}\right)\left(\frac{(v_{i+1,j+1}+v_{i,j+1})-(v_{i+1,j}+v_{i,j})}{2\Delta y}\right)+ \\ \left(\frac{\bar{f}_1(\theta)_{i,j+1}-\bar{f}_1(\theta)_{i,j}}{\Delta y}\right)\left(\frac{v_{i+1,j+1}+v_{i,j+1}+v_{i+1,j}+v_{i,j}}{4\,\mathrm{Re}}\right)-Ha^2\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{4\,\mathrm{Re}}\right)=0 \tag{9}$$

$$\left(\frac{\theta_{i+1,j+1}-\theta_{i,j+1}+\theta_{i+1,j}-\theta_{i,j}}{2\Delta t}\right)=\left(\frac{\bar{f}_2(\theta)_{i,j+1}+\bar{f}_2(\theta)_{i,j}}{2\,\mathrm{Re}\,\mathrm{Pr}}\right)\left(\frac{(H_{i+1,j+1}+H_{i,j+1})-(H_{i+1,j}+H_{i,j})}{2\Delta y}\right)+ \\ \left(\frac{\bar{f}_1(\theta)_{i,j+1}-\bar{f}_1(\theta)_{i,j}}{\Delta y}\right)\left(\frac{H_{i+1,j+1}+H_{i,j+1}+H_{i+1,j}+H_{i,j}}{4\,\mathrm{Re}\,\mathrm{Pr}}\right)-Ec\left(\frac{\bar{f}_1(\theta)_{i,j+1}+\bar{f}_1(\theta)_{i,j}}{2\,\mathrm{Re}}\right). \\ \left(\frac{\bar{v}_{i+1,j+1}+\bar{v}_{i,j+1}+\bar{v}_{i+1,j}+\bar{v}_{i,j}}{2}\right)\left(\frac{v_{i+1,j+1}+v_{i,j+1}+v_{i+1,j}+v_{i,j}}{2}\right)+ \\ EcHa^2\left(\frac{\bar{u}_{i+1,j+1}+\bar{u}_{i,j+1}+\bar{u}_{i+1,j}+\bar{u}_{i,j}}{2\,\mathrm{Re}}\right)\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{2}\right) \tag{10}$$

The variables with bars are given initial guesses from the previous time steps and an iterative scheme is used at every time to solve the linearized system of difference equations. Computations have been made for G=5, Re=1, Pr=1 and Ec=0.2. Grid-independence studies show that the computational domain $0<t<\infty$ and $-1<y<1$ can be divided into intervals with step sizes Δt=0.0001 and Δy=0.005 for time and space respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u, v, θ and H for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1<y<1$ at every time step. Less than 7 approximations are required to satisfy this convergence criteria for all ranges of the parameters studied here.

RESULTS AND DISCUSSION

Figures 1a and b present the velocity and temperature distributions as functions of y for various values of time t starting from t=0 up to the steady-state. The figures are evaluated for Ha =1, a=0.5, and b=0.5. The velocity component u reaches the steady state faster than θ. This is expected as u is the source of θ. Figure 1b shows that the temperature θ inside the fluid may exceed the value 1, which is the temperature of the hot plate, especially at large times. This is due to the Joule and viscous dissipations.

Figures 2a and b present the time development of the velocity component u at the center of the channel (y=0), for various values of the parameters "a" and Ha and for b=0. Figures 2 show that increasing the parameter "a" decreases u for all values of Ha. It is also shown that the steady state time of u is not greatly affected by changing "a". Comparing Figs. 1a and b indicates the damping effect of the magnetic field which decreases u for all values of "a". Figures 3a and b present the time development of the temperature θ at the center of the channel (y=0), for various values of the parameters "a" and Ha and for b=0. The figures show that increasing "a" decreases θ for all values of Ha as a result of decreasing the velocity u and its gradient the function f_1 which decreases the viscous and Joule dissipations. It is also shown that the steady state value of θ is not greatly affected by changing "a". The comparison between Figs. 2a and b shows that increasing Ha increases θ, for all values of a, due to the increase in the Joule dissipations.

Figures 4a and b present the time development of the temperature θ at the center of the channel (y=0), for various values of the parameters "b" and Ha and for a=0. The figures show that the variation of the temperature θ with the parameter "b" depends on t where a crossover in θ-t charts occurs. The effect of "b" on θ depends on t and increasing "b" increases θ at small times, but decreases θ when t is large. This occurs because, at low times, the center of the channel acquires heat by conduction from the hot plate, but after large time, when u is large, the Joule dissipation is large at the center and center looses heat by conduction. It is noticed that the parameter "b" has no significant effect on u in spite of the coupling between the momentum and energy equations. It is also shown in the figures that increasing the parameter "b" decreases the steady state time of θ. Figure 3b indicates that increasing H_a increases θ as the Joule dissipation increases and decreases the time at which the crossover in θ-t charts occurs.

Tables 1a and 1b present the variation of the steady state Nusselt numbers at both walls, Nu_L and Nu_U, respectively, with the parameters "a" and "b" for Ha =1. Increasing "a" increases Nu_L and the magnitude of Nu_U for all values of "b". However, increasing "b" decreases the magnitude of Nu_U for all "a". For small values of "a", increasing "b" decreases Nu_L and increasing "b" more increases Nu_L. On the other hand, for moderate and higher values of "a", increasing "b" increases Nu_L steadily.

Table 1a Variation of the steady state Nusselt number at the lower plate Nu_L for various values of "a" and "b" (Ha=1)

Nu_L	a=-0.5	a=-0.1	a=0.0	a=0.1	a=0.5
b=-0.5	1.7357	1.8706	1.9067	1.9429	2.0566
b=-0.1	1.7309	1.8817	1.9236	1.9674	2.1571
b=0.0	1.7347	1.8863	1.9282	1.9721	2.1640
b=0.1	1.7397	1.8918	1.9337	1.9776	2.1704
b=0.5	1.7645	1.9178	1.9595	2.0030	2.1949

Table 1b Variation of the steady state Nusselt number at the upper plate Nu_U for various values of "a" and "b" (Ha=1)

Nu_U	a=-0.5	a=-0.1	a=0.0	a=0.1	a=0.5
b=-0.5	-0.8129	-1.1897	-1.2915	-1.3936	-1.6899
b=-0.1	-0.4381	-0.6731	-0.7407	-0.8127	-1.1479
b=0.0	-0.3959	-0.6079	-0.6687	-0.7333	-1.0345
b=0.1	-0.3629	-0.5558	-0.6109	-0.6694	-0.9417
b=0.5	-0.2844	-0.4255	-0.4652	-0.5071	-0.7004

CONCLUSIONS

The transient Couette-Poiseulle flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied with temperature dependent viscosity and thermal conductivity in the presence of an external uniform magnetic field. It was found that the magnetic field or the viscosity exponent has a damping effect on the velocity component u while the effect of the parameter "b" on u can be entirely neglected. It is also shown that increasing the magnetic field increases the temperature θ, however, increasing the viscosity exponent decreases θ. It is of interest to find that the effect of the parameter "b" on the temperature θ depends on time.

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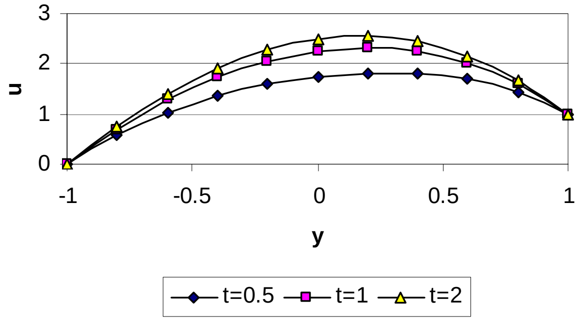
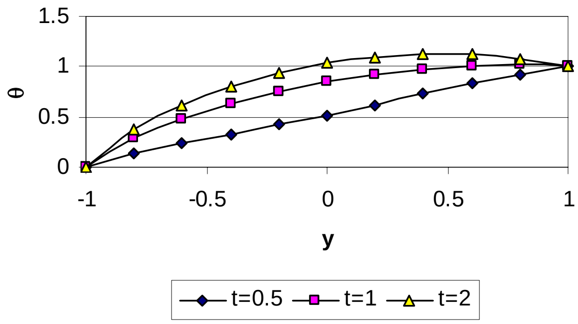
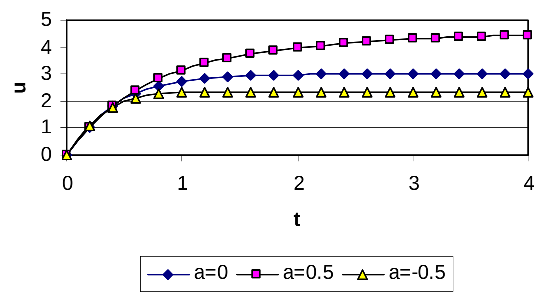


(a)

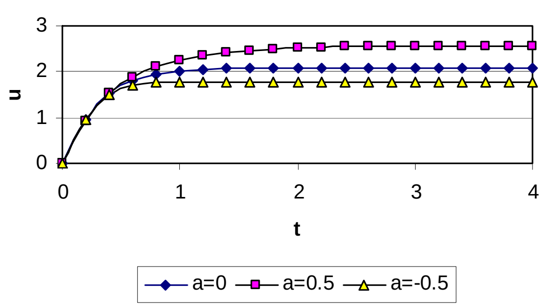


(b)

Fig. 1 Time development of the profile of: (a) u; (b) θ. (Ha =1, a=0.5, b=0.5)

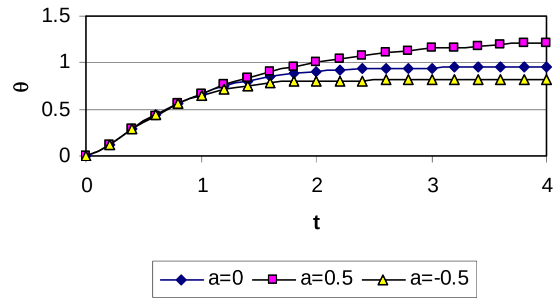


(a)

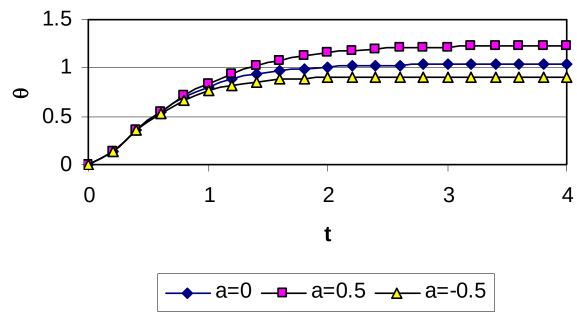


(b)

Fig. 2 Time development of u at y=0 for various values of “a”: (a) Ha =0; (b) Ha =1. (b=0)

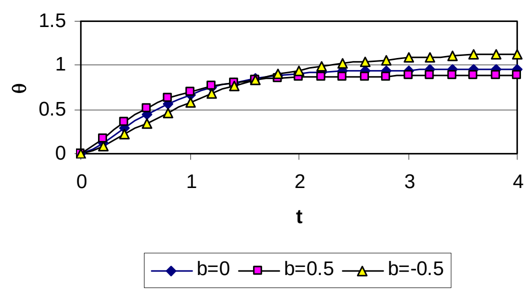


(a)

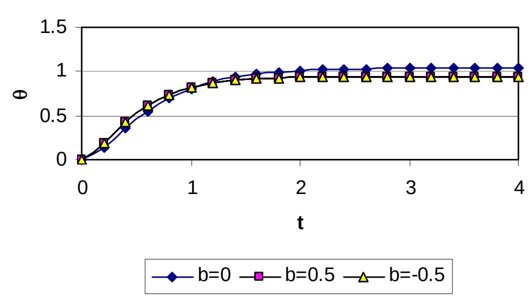


(b)

Fig. 3 Time development of θ at y=0 for various values of “a”: (a) Ha =0; (b) Ha =1. (b=0)



(a)



(b)

Fig. 4 Time development of θ at y=0 for various values of “b”: (a) Ha =0; (b) Ha =1. (a=0)