

TREATMENT OF THE TURNING POINT IN ADK-THEORY INCLUDING NON-ZERO INITIAL MOMENTA

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ABSTRACT. The ADK-theory usually has been used to describe the ionization which occurs when the initial momentum of the ejected electrons is zero. There has been one try to include a nonzero initial momentum of electrons [7], but it did not draw much attention at the moment, as this effect had not had great impact on the results of the theory because the laser intensities in the experiments were giving the fields up to 10^{12} Wcm⁻² (atomic field is approximately 10^{16} Wcm⁻²). Now, with ADK-theory dealing with field strengths of the order of atomic, and even higher [6, 8], this effect may be expected to gain in its importance, so we shall consider the corrections to transition rate in ADK-theory up to the terms with the field strength to minus fifth as in [9], but with nonzero initial momentum, which is the first time ever to include this effect in so high orders of corrections.

1. INTRODUCTION

In last two decades of XX Century tunneling ionization (Keldysh's [1] parameter $\gamma \ll 1$) of atoms and ions by a strong low-frequency electromagnetic field has become the subject of intensive research. The theory which was one of the most propulsive ones in the field was the Ammosov-Delone-Krainov (ADK) theory [2-5]. This theory uses the concept of a quasi-stationary external field which is ejecting valence electrons via tunneling. It has been, at the end of this period [6], extended to the barrier-suppression ionization of complex atoms and atomic ions, i.e. to the case of super-strong fields.

But the ADK-theory usually has been used to describe the ionization which occurs when the initial momentum of the ejected electrons is zero. There has been one try to include a nonzero initial momentum of electrons [7], but it did not draw much attention at the moment, as this effect had not had great impact on the results of the theory because the laser

intensities in the experiments were giving the fields up to 10^{12} Wcm^{-2} (atomic field is approximately 10^{16} Wcm^{-2}). Now, with ADK-theory dealing with field strengths of the order of atomic, and even higher [6, 8], this effect could have been expected to gain in its importance, so we considered the corrections to transition rate in ADK-theory up to the terms with the field strength to minus fifth as in [9], but with nonzero initial momentum, which is the first time ever to include this effect in so high orders of corrections.

2. TRANSITION RATE IN GENERAL

As already mentioned we are dealing with tunneling ionization ($\gamma \ll 1$, [1, 12]). In this case the transition rate is given by the expression [11, 12]:

$$W_{if} = \left| e^{i \int_0^{\tau} [E_f(t) - E_i(t)] dt} \right|^2 \equiv |e^{iS(\tau)}|^2. \quad (1)$$

Here $S(\tau)$ is the classical action and $E_i(t)$ and $E_f(t)$ are energies of the initial and final states as functions of the time parameter t . The condition $\gamma \ll 1$ implies that the frequency ω of the external (laser) field is much smaller than atomic frequency $\omega \ll E_i$ (atomic units system: $e = m_e = \hbar = 1$ is used throughout this paper), which justifies using the adiabatic approximation in obtaining (1). Upper limit in the integral in (1) is the complex turning point obtained from relation

$$E_i(\tau) = E_f(\tau). \quad (2)$$

Here τ is being complex, as it being obtained from equation (2) corresponds to "turning point" of Classical Mechanics, but this kind of transition is classically forbidden which produces the complexity of the roots of equation (2).

In the case of linear field polarization the Keldysh approximation [1] gives $E_i(\tau) = -E_i$, where E_i is the ionization potential of the ground state of valence electron and

$$E_f(\tau) = \frac{1}{2} \left[\vec{p} + \frac{1}{c} \vec{A}(\tau) \right]^2 = \frac{1}{2} \left[\vec{p} - \frac{\vec{F}}{\omega} \sin \omega t \right]^2, \text{ where } \vec{F} \text{ is the field strength vector and } \vec{p} \text{ is the}$$

momentum of an ejected electron.

As the transition rate is equal to (see Equation 1 and [11,12])

$$W_{if} = e^{-2\text{Im}S(\tau)}, \quad (3)$$

one has to calculate the imaginary part of the integral

$$S(\tau) = \int_0^\tau \left[\frac{1}{2} \left(\vec{p} - \frac{\vec{F}}{\omega} \sin \omega \tau \right)^2 + E_i \right] dt. \quad (4)$$

If we take into account that the probability of ionization is the biggest if the vector \vec{p} were directed along the field vector \vec{F} [see 10,12], and having in mind that the lower limit of integration is real, we have

$$\text{Im} S = \text{Im} \left\{ \left(\frac{p^2}{2} + E_i + \frac{F^2}{4\omega^2} \right) \tau + \frac{pF}{\omega^2} \cos \omega \tau - \frac{F^2}{8\omega^3} \sin 2\omega \tau \right\}. \quad (5)$$

Usually here momentum p was considered zero, to make the calculations simpler, but as in reality there should be some nonzero distribution of electron momenta, we shall now take into account this correction.

3. THE CORRECTIONS TO THE TRANSITION RATE DUE TO NON-ZERO MOMENTA OF EJECTED ELECTRONS

Explicit calculations of Equation (2) give us

$$\begin{aligned} \sin \omega \tau &= \omega \frac{p + i\kappa}{F} \equiv \alpha, \\ \text{or,} \\ \omega \tau &= \arcsin \alpha, \end{aligned} \quad (6)$$

where $\kappa = \sqrt{2E_i}$.

As frequency ω is very small in the case of the low frequency field, we can consider the newly introduced parameter α a small quantity, and expand the above expression in the following manner

$$\omega\tau \approx \alpha + \frac{\alpha^3}{6} + \frac{3\alpha^5}{40}. \quad (7)$$

Also, in equation (5), we may expand in powers cosine and sine under the same conditions, and obtain

$$\begin{aligned} \cos \omega\tau &\approx 1 - \frac{\omega^2\tau^2}{2} + \frac{\omega^4\tau^4}{24} - \frac{\omega^6\tau^6}{720} \\ \sin 2\omega\tau &\approx 2\omega\tau - \frac{4}{3}\omega^3\tau^3 + \frac{4}{15}\omega^5\tau^5 - \frac{8}{315}\omega^7\tau^7. \end{aligned} \quad (8)$$

After somewhat lengthy, but straightforward calculations, substituting (7) and (8) into (5) we obtain

$$\text{ImS}(\tau) = \frac{\kappa^3}{3F} + \left(\frac{p^2\kappa^3}{6} - \frac{\kappa^5}{30} \right) \frac{\omega^2}{F^3} + \left(\frac{\kappa^3 p^4}{8} - \frac{3\kappa^5 p^2}{20} + \frac{3\kappa^7}{280} \right) \frac{\omega^4}{F^5}, \quad (9)$$

where we have collected all the terms corresponding to field power up to -5, because it was shown earlier [10] that these are the only relevant corrections to the basic factor $\frac{\kappa^3}{3F}$, which in the expression for probability gives the well known short range potential term $-2(2E_i)^{3/2}/3F$ [12].

The first and third term of expression (9) could be combined to give [10]:

$$\frac{\kappa^3}{3F} - \frac{\kappa^5\omega^2}{30F^3} = \frac{\kappa^3}{3F} \left(1 - \frac{\kappa^2\omega^2}{10F^2} \right) = \frac{\kappa^3}{3F} \left(1 - \frac{\gamma^2}{10} \right), \quad (10)$$

where we used the fact that Keldysh's parameter is given by [12]: $\gamma = \frac{\kappa\omega}{F}$. So, since $\gamma \ll 1$ we can neglect the second term in Eq. (10).

As it can be shown that atomic field strength is proportional to κ^3 [12], for very strong fields one should not take into account none of the terms in parenthesis before F^{-5} , thus the only term of any relevance with nonzero momentum being

$$K_1 = \frac{p^2 \kappa^3 \omega^2}{6F^3} = \frac{p^2 \gamma^3}{6\omega}, \quad (11)$$

because the small parameters γ^3 and ω may cancel each other and leave some space for influence of nonzero initial momentum of the electron, which is not new [10], but it can now give greater impact on probability w , because the fields are much stronger. And the influence of the field on initial momentum can be seen from the expression for the final energy E_f of the electron, in a long laser pulse approximation:

$$E_f = p^2 / 2 + F^2 / 4\omega^2. \quad (11)$$

So, finally,

$$p^2 / 2 = E_f - F^2 / 4\omega^2, \quad (12)$$

we see that the influence of the field strength on the initial momentum of the electron is considerable, and some effects in the measured probabilities could be expected.

4. CONCLUSION

Finally, we can conclude that including non-zero initial momenta of ejected electrons in the case of ionization of atoms and ions by the strong laser fields to the higher order of correction than before [10], gives the results that are already obtained in earlier works [7,10], but as they were estimated for the strong fields, and now superstrong fields strengths [6] can be used this should influence their impact on the electron ejection probabilities. But exact estimations we leave for future work.

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