Kragujevac J. Sci. 47 (2025) 3 *Special Issue* 163-172 UDC 530.145:113/119
doi: 10.5937/KgJSci2503163D *Review*

MODERN COSMOLOGY – 100 YEARS OF QUANTUM SCIENCE

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(Received December 05, 2025; Accepted December 20, 2025)

ABSTRACT. In this article, we contribute to the special issue of the Kragujevac Journal of Science devoted to the International Year of Quantum Science and Technology (IYQ2025). We discuss several connections between Quantum Theory and Modern Cosmology. We begin with a brief overview of the major theoretical discoveries of Quantum Theory and Quantum Mechanics, proceed to Quantum Field Theory (QFT), and conclude with aspects of modern, quantum cosmology.

Keywords: International Year of Quantum Science and Technology, quantum mechanics, quantum field theory, modern cosmology.

INTRODUCTION

On 7 June 2024, United Nations General Assembly adopted Resolution No 78/287 and proclaimed 2025 the International Year of Quantum Science and Technology, IYQ. The Resolution contains two parts emphasizing one of the most important roles of science and its purpose of existence (https://docs.un.org/en/A/RES/78/287):

> *"...Emphasizing that quantum science and technology is vital for economic advancement and that its potential applications could address basic needs such as food, health, sustainable cities and communities, communications, clean water and energy, and support climate action...",*

as well as

> *"...Recognizing that the year 2025 coincides with the 100th anniversary of the development of the methods of quantum mechanics that have led to its prominence in both science and technology today...".*

One of the authors of this article was delighted and very proud when the Resolution was first announced. The other experienced similar emotions once he became aware of the Resolution, the IYQ program, and aspects of its implementation. The entire world recognized the importance and the tremendous, century-long effort of scientists in defining and

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developing this fundamental theory – the essence and one of the cores of what we now call Modern Physics. And not scientists in general, but more explicitly **physicists**. And not physicists in general, but, in particular, **theoretical physicists**.

Of course, as theoretical physicists, we fully acknowledge the great efforts and contributions of experimental physicists and many others. We simply wish to emphasize the fundamental contribution of theoretical physics to the foundations and development of Quantum Theory.

This article is written to celebrate 100 years of Quantum Science, and the narrative we choose, and use is intended primarily for young physicists, students, and all readers interested in various aspects of Quantum Theory and its applications.

The rest of the paper is organized as follows. In the next section, we review the major theoretical discoveries of Quantum Theory and Quantum Mechanics. The following two sections are devoted to the shortcomings of Quantum Mechanics and to Quantum Field Theory, respectively. After that, we discuss a portion of Modern Cosmology that involves the use of QFT in curved spacetime. We conclude with comments on Quantum Theory at the Planck scale, the wave function of the universe, the path-integral formulation of Quantum Theory, and our final thoughts. Throughout the paper, we use natural units i.e. $\hbar = c = 1$.

THANKS TO …

Quantum theory was not created all at once. According to Ball (BALL, 2025), "The creation of modern quantum mechanics was a messy business in which many of the participants did not grasp the significance of their own discoveries."

At the beginning of the 20th century, the limitations of the classical physical approach in describing processes and phenomena in the micro-world were becoming increasingly apparent. It was at this time that the first successful steps toward non-classical, i.e., quantum, descriptions began to emerge.

Thanks to Max Planck, we now know that light can be emitted in discrete quanta, i.e., as particles. This breakthrough occurred in 1900 and was published the following year (PLANCK, 1901).

Thanks to Albert Einstein, we now know that light (waves) can also be absorbed as particles. This discovery was made in 1905 (EINSTEIN, 1905).

Thanks to Louis de Broglie, we now understand that this dual nature of light-behaving as a wave in some circumstances and as a particle in others – is a general property of all micro-objects. This work was completed in 1924 and published in 1925 (DE BROGLIE, 1925).

Thanks to Werner Heisenberg, we now have a formal mathematical framework for the new physics-quantum physics. His work in 1925 (HEISENBERG, 1925) introduced matrix mechanics, which enabled the prediction of quantum properties of micro-objects, such as the emission spectra of atoms.

Thanks to Erwin Schrödinger and the circumstances of the time, we also have an alternative, and ultimately more popular, framework called wave mechanics. Schrödinger was inspired by de Broglie's largely overlooked suggestion. This work was completed in 1926 (SCHRÖDINGER, 1926).

Heisenberg's 1925 work is considered the first published article in which the theoretical foundations of Quantum Mechanics were defined. That is why 2025 marks the celebration of 100 years of Quantum Theory and Quantum Technology, recognized as the International Year of Quantum Science and Technology (IYQ2025).

Thanks to Niels Bohr, Max Born, Wolfgang Pauli, Arnold Sommerfeld, John von Neumann, and many others, we now have a much deeper understanding of the quantum nature of Nature.

BUT …

From the very beginning, Quantum Mechanics was unable to address relativistic phenomena, which are treated at the classical (non-quantum) level by the Special Theory of Relativity (STR). Extending the theory to encompass relativistic phenomena at the quantum level became both necessary and unavoidable. This challenge was ultimately addressed through the development of the Quantum Theory of Fields, now known as Quantum Field Theory (QFT).

How can relativistic considerations be implemented at the quantum level? The first step was achieved using symmetries. A consistent theory must be Lorentz invariant, i.e., invariant under the Lorentz group – or more precisely, the Poincaré group – which ensures that it is relativistic. These symmetries, which form the core of STR, must be incorporated into Quantum Mechanics.

In theory, additional symmetries simplify physical problems. For example, rotational invariance greatly simplifies scattering problems at both the classical and quantum levels. However, the inclusion of relativistic considerations through Lorentz invariance complicates quantum mechanical analysis. The primary reason is that, in relativistic systems, the number of particles need not be conserved.

In standard Quantum Mechanics, the dynamics of a particle are described by its wave function, i.e., the time evolution of the associated wave function, which is governed by the Schrödinger equation.

$$i\hbar\frac{\partial}{\partial t}\psi(t,\vec{x})=\hat{H}\psi(t,\vec{x}), \tag{1}$$

where Hamiltonian of the system is

$$\hat{H}=\frac{\hat{\vec{p}}^{2}}{2m}+V(\hat{\vec{x}}). \tag{2}$$

In order to incorporate Lorentz invariance even for a free-particle system, the first approach is to adopt the relativistic expression for the Hamiltonian.

$$\hat{H}=\sqrt{\hat{\vec{p}}^{2}c^{2}+m^{2}c^{4}}, \tag{3}$$

However, this is not sufficient. To merge Quantum Mechanics with Lorentz invariance, it is necessary to abandon the single-particle paradigm of Quantum Mechanics. In relativistic phenomena, the number of particles need not be conserved, because the expression for a particle's energy

$$E^{2}=\vec{p}^{2}c^{2}+m^{2}c^{4}, \tag{4}$$

shows that energy can be converted into particles (i.e., mass) and vice versa. This implies that a multi-particle paradigm is required. Another, more mathematically rigorous explanation is that causality and unitarity cannot be simultaneously accommodated within the single-particle framework of Quantum Mechanics. For example, according to standard Quantum Mechanical procedures, the probability amplitude for a particle to propagate from one point to another is given by

$$K(\vec{x}_2,\vec{x}_1)=\left\langle \vec{x}_2 \left| \exp\{-i\hat{H}t\} \right| \vec{x}_1 \right\rangle, \tag{5}$$

where $\hat{H}$ can be either non-relativistic or relativistic. However, the probability amplitude will have non-zero value even if position 4-vectors $x_1=(ct,\vec{x}_1)$ and $x_2=(ct,\vec{x}_2)$ are space-like separated. This implies a violation of causality, i.e. $K(\vec{x}_2,\vec{x}_1)\neq 0$ is not physically acceptable result for space-like separated points.

In conclusion, there is no consistent quantum theory that is both relativistic and restricted to a single particle. At the quantum level, this has profound implications, as we will revisit. Quantum Field Theory (QFT) is the only framework capable of reconciling Quantum Mechanics with the Special Theory of Relativity (STR).

QUANTUM FIELD THEORY

Quantum Field Theory is necessary to describe nature at its most fundamental level, at the shortest distances accessible to us. The physical phenomena under consideration involve relativistic (fundamental) particles and their interactions, and QFT provides an elegant framework for understanding them (RATTAZZI, 2024).

The unification of Quantum Mechanics with the Special Theory of Relativity was not an easy task. Paul Dirac was among the first physicists to successfully describe the electron using only its quantum behaviour and the basic principles of STR (DIRAC, 1928). This work laid the foundation for Werner Heisenberg and Wolfgang Pauli to extend the description to particles of arbitrary mass, charge, or spin using quantum fields (HEISENBERG AND PAULI, 1929; HEISENBERG AND PAULI, 1930). They also introduced the Lagrangian formulation of Quantum Field Theory. For more details, see a recent reference by BERNARDEZ (2023).

For example, using the Lagrangian formalism the action functional for a real scalar field $\phi(t,\vec{x})$ (in Minkowski spacetime) is

$$S[\phi]=\frac{1}{2}\int dtd^3x\left[\dot{\phi}^2-(\nabla\phi)^2-m^2\phi^2\right], \tag{6}$$

where dot denotes partial derivative with respect to time, and dynamical equation for the field is Klein-Gordon equation

$$\ddot{\phi}(t,\vec{x})-\nabla^2\phi(t,\vec{x})+m^2\phi(t,\vec{x})=0. \tag{7}$$

Solving this equation, and after canonical quantization procedure, we end up with quantum field $\hat{\phi}(t,\vec{x})$, whose mode expansion is

$$\hat{\phi}(t,\vec{x})=\frac{1}{\sqrt{2}}\frac{1}{(2\pi)^{3/2}}\int d^3\vec{k}\left[\hat{a}_{\vec{k}}v^*_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}}+\hat{a}^\dagger_{\vec{k}}v_{\vec{k}}(t)e^{-i\vec{k}\cdot\vec{x}}\right], \tag{8}$$

where $v_{\vec{k}}(t)$ are mode functions, while $\hat{a}_{\vec{k}}$ and $\hat{a}^\dagger_{\vec{k}}$ are time-independent annihilation and creation operators, respectively. Each of these operators will annihilate or create one-particle state, which means that we now have in our hand tools to describe phenomena in which particles are destroyed or created.

The mode functions satisfy the equation

$$\ddot{v}_{\vec{k}}(t) + \omega_{\vec{k}}^2 v_{\vec{k}}(t) = 0, \tag{9}$$

where $\omega_{\vec{k}}$ is energy of a particle with mass m and momentum vector $\vec{k}$

$$\omega_{\vec{k}} = +\sqrt{\vec{k}^2 + m^2}. \tag{10}$$

Mode functions satisfy normalization conditions

$$W(v_{\vec{k}}, v_{\vec{k}}^*) \equiv \dot{v}_{\vec{k}}(t) v_{\vec{k}}^*(t) - v_{\vec{k}}(t) \dot{v}_{\vec{k}}^*(t) = 2i, \tag{11}$$

where $W(v_{\vec{k}}, v_{\vec{k}}^*)$ is Wronskian. Non-vanishing commutation relation for time-independent annihilation and creation operators is then

$$\left[\hat{a}_{\vec{k}}, \hat{a}_{\vec{q}}^\dagger\right] = \delta^{(3)}(\vec{k} - \vec{q}). \tag{12}$$

Quantum Field Theory thus addresses both challenges by introducing a new paradigm: the fundamental entities are not particles, but fields-abstract objects defined at every point in spacetime. Particles then emerge as excitations of these fields. From this perspective, there are fields corresponding to all types of particles, whether massive or massless, charged or neutral, and with spin (in unit of $\hbar$) zero, one half, one, one and a half, two, and so on. The framework for QFT is four-dimensional Minkowski spacetime, where all calculations are done.

A natural next step in the development of the quantum program is the formulation of QFT on curved spacetime.

QFT ON EXPANDING SPACETIME

Modern Cosmology is based on Einstein's Theory of General Relativity (CARROLL, 2019) and deals with phenomena in expanding spacetime. In expanding spacetime, such as spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime (probably the most used spacetime by cosmologists), fields behave differently from what we know in the Minkowski spacetime of particle physics.

Spatially flat FLRW spacetime metric is of the form

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \tag{13}$$

where t is cosmological time, $\vec{x}$ are comoving spatial coordinates and $a(t)$ is the scale factor of the Universe. We also use another time coordinate, the conformal time η defined as

$$dt = a(\eta) d\eta, \tag{14}$$

and the metric is now

$$ds^2 = a(\eta)^2 \left(-d\eta^2 + d\vec{x}^2\right). \tag{15}$$

For example, the action functional for real scalar field $\phi(\eta, \vec{x})$ (in FLRW spacetime) is (MUKHANOV AND WINITZKI, 2007)

$$S[\phi]=\frac{1}{2}\int d\eta d^3\vec{x}a^2\left[\phi'^2-(\nabla\phi)^2-m^2a^2\phi^2\right], \tag{16}$$

where prime denotes derivative with respect to conformal time. Redefining scalar field

$$\chi(\eta,\vec{x})=a(\eta)\phi(\eta,\vec{x}), \tag{17}$$

the action will look almost the same as the action for Minkowski spacetime

$$S[\chi]=\frac{1}{2}\int d\eta d^3\vec{x}\left[\chi'^2-(\nabla\chi)^2-m_{eff}^2\chi^2\right], \tag{18}$$

where, now, we have time dependent effective mass squared

$$m_{eff}^2(\eta)=m^2a^2(\eta)-\frac{a''(\eta)}{a(\eta)}. \tag{19}$$

This time dependence of effective mass accounts for the interaction of the field with the gravitational background. Dynamical equation for the field $\chi(\eta,\vec{x})$ is, without surprise, again of Klein-Gordon type

$$\ddot{\chi}(\eta,\vec{x})-\nabla^2\chi(\eta,\vec{x})+m_{eff}^2(\eta)\chi(\eta,\vec{x})=0. \tag{20}$$

Solving this equation, and after canonical quantization procedure, we end up with quantum field $\hat{\chi}(\eta,\vec{x})$, whose mode expansion is

$$\hat{\chi}(\eta,\vec{x})=\frac{1}{\sqrt{2}}\frac{1}{(2\pi)^{3/2}}\int d^3\vec{k}\left[\hat{a}_{\vec{k}}v_{\vec{k}}^*(\eta)e^{i\vec{k}\cdot\vec{x}}+\hat{a}_{\vec{k}}^\dagger v_{\vec{k}}(\eta)e^{-i\vec{k}\cdot\vec{x}}\right]. \tag{21}$$

The mode functions satisfy the equation

$$v''_{\vec{k}}(\eta)+\Omega_{\vec{k}}^2(\eta)v_{\vec{k}}(\eta)=0, \tag{22}$$

where $\Omega_{\vec{k}}$ is now

$$\Omega_{\vec{k}}(\eta)=+\sqrt{\vec{k}^2+m_{eff}^2(\eta)}. \tag{23}$$

Mode functions satisfy the same normalization conditions as in Minkowski spacetime

$$W(v_{\vec{k}},v_{\vec{k}}^*)\equiv v'_{\vec{k}}(\eta)v_{\vec{k}}^*(\eta)-v_{\vec{k}}(\eta)v'^*_{\vec{k}}(\eta)=2i, \tag{24}$$

and non-vanishing commutation relation for time-independent operators $\hat{a}_{\vec{k}}$ and $\hat{a}_{\vec{q}}^\dagger$ is again

$$\left[\hat{a}_{\vec{k}},\hat{a}_{\vec{q}}^\dagger\right]=\delta^{(3)}(\vec{k}-\vec{q}). \tag{25}$$

This means that $\hat{a}_{\vec{k}}$ and $\hat{a}_{\vec{q}}^\dagger$ can be again interpreted as the annihilation and creation operators.

Now that we know how to quantize scalar fields in expanding spacetime and interpret results, we can use it to study cosmological perturbations.

COSMOLOGICAL PERTURBATIONS

Real scalar fields are used most of the time in cosmology. We know how to mimic various cosmological fluids using scalar fields (MUKHANOV, 2005). We also know how to use scalar fields to study cosmological inflation (STAROBINSKY, 1980; GUTH, 1981; LINDE, 1982,) and to interpret the Cosmic Microwave Background Radiation (CMBR) spectrum (LYTH AND LIDDLE, 2009). We are also able to address the problem of primordial black hole formation as a dark matter candidate (BILIĆ *et al.*, 2025). All of this is done using cosmological perturbation theory.

Cosmological perturbations of spacetime are introduced through the metric

$$g_{\mu\nu}(\eta,\vec{x}) = \bar{g}_{\mu\nu}(\eta) + \delta g_{\mu\nu}(\eta,\vec{x}), \tag{26}$$

where $\bar{g}_{\mu\nu}(\eta)$ are components of non-perturbed spatially homogenous and flat background FLRW metric tensor, while $\delta g_{\mu\nu}(\eta,\vec{x})$ are metric perturbations. Explicitly,

$$\begin{aligned} ds^2 = {} & a^2\left[-(1+2A)d\eta^2 + 2(\partial_i B)d\eta dx^i\right] + \\ & a^2\left[\left(1-2(D+\frac{1}{3}(\partial_k\partial_k E))\right)\delta_{ij} + 2(\partial_i\partial_j E)\right]dx^i dx^j, \end{aligned} \tag{27}$$

where $A(\eta,\vec{x})$, $B(\eta,\vec{x})$, $D(\eta,\vec{x})$ and $E(\eta,\vec{x})$ are the scalar type metric perturbations. Perturbations of vector and tensor type are also present but are not explicitly written here. Perturbations $\delta\phi(\eta,\vec{x})$ of the scalar field are introduced in the same way

$$\phi(\eta,\vec{x}) = \phi_0(\eta) + \delta\phi(\eta,\vec{x}), \tag{28}$$

where $\phi_0(\eta)$ is non-perturbed solution of Klein-Gordon equation for spatially homogenous and flat background FLRW spacetime. The next step is to engage in the perturbation theory.

Before we enter into the perturbation theory, it is important to stress that neither $\delta\phi(\eta,\vec{x})$ nor $\delta g_{\mu\nu}(\eta,\vec{x})$ would be physical variables. They depend on choice of what coordinates are in use. The change of the coordinate system will change their values. A gauge-invariant definition of cosmological perturbations is unavoidable in order to make connection with physical observables, which are gauge-invariant. We need to define gauge-invariant perturbations by constructing quantities that remain invariant under coordinate transformations.

One such quantity (of scalar type) is Burdeen gauge invariant potential (BURDEEN, 1982)

$$\psi(\eta,\vec{x}) \equiv D + \frac{1}{3}(\partial_k\partial_k E) - H(B - \frac{\partial E}{\partial\eta}), \tag{29}$$

where $H(\eta) = a'/a$ is the Hubble parameter. We can also mention a very useful Mukhanov-Sasaki gauge invariant variable (MUKHANOV *et al.*, 1992) using Burdeen potential

$$v(\eta,\vec{x}) \equiv a(\eta)\delta\phi(\eta,\vec{x}) + \frac{a(\eta)\phi_0'(\eta)}{H(\eta)}\psi(\eta,\vec{x}), \tag{30}$$

Mukhanov-Sasaki variable will be our real scalar field to be quantized.

The action functional for this real scalar field is

$$S = \frac{1}{2}\int d\eta d^3\overline{x}\left((v')^2 - (\nabla v)^2 + \frac{z''}{z}v^2\right), \tag{31}$$

where the so-called pump field $z(\eta)$ depends only on the background quantities

$$z(\eta) = \frac{a(\eta)\phi_0'(\eta)}{H(\eta)}. \tag{32}$$

Using the quantization procedure outlined in the previous section, we are able to quantize cosmological perturbations, compute the relevant quantities, and connect the resulting theoretical predictions with concrete observational data (MUKHANOV, 2005).

Let us mention and emphasize here a few very important applications of Quantum Theory in Cosmology, as well as an alternative, intuitive, and very useful formulation of Quantum Mechanics and QFT – the path-integral formulation (FEYNMAN AND HIBBS, 1965). It contributes to the introduction of the wave function of the universe (HARTLE AND HAWKING, 1983), one of the most widely studied approaches to Quantum Cosmology and, more broadly, Quantum Gravity.

There is no doubt that on the horizon of recent, modern, and, in particular, forthcoming applications of Quantum Mechanics, we find physics at the Planck scale. This line of research continues to challenge our fundamental understanding of spacetime at extremely small distances. Non-Archimedean spacetime (DRAGOVICH *et al.*, 2017) and the path-integral formulation of quantum dynamics on ultrametric spaces (DJORDJEVIC AND DRAGOVICH, 1997), together with Quantum Theory on non-commutative spacetime (SZABO, 2003), represent new and intriguing directions for Quantum Theory in its second century.

FINAL TOUGHTS

It is logical, and history of science shows it, that scientific development is very important, at least for later practical purposes, such as "making life easier." This article represents an attempt to connect various phases in the development of the quantum paradigm, following only one of the many branches that scientists carefully developed, starting from the micro-world and extending to the entire Universe.

Finally, we can proudly state that Quantum Theory is a highly successful and fruitful physical theory. According to Görnitz (GÖRNITZ, 2010), "About one third of the gross national product in the developed countries results from its applications. The applications of this theory range from nuclear power to all tools for computing, lasers, solar cells, and so on."

If the Universe is the Answer, what is the Question? (LEDERMAN, 2006).

Acknowledgments

We thank our colleague Miroljub Dugić, as well as our colleagues from the University of Kragujevac, for giving us the opportunity to participate in and contribute to the celebration of the 2025 International Year of Quantum Science and Technology with this article. This research was partially supported by ICTP–SEENET-MTP project NT-03 in 2025.

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