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QUANTUM MEASUREMENT: A SHORT REVIEW

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ABSTRACT. Quantum measurement problem is a long-standing problem at the heart of the foundations of modern physics and nascent technology, such as quantum technology. The last forty years brought a renowned interest in this problem with an emphasis on the new foundations of quantum theory and unpredicted formulation of some new quantum disciplines, such as the open quantum systems theory and the quantum information and computation science. A state of the art of the problem of measurement is shortly presented, including a brief historical view, with a review of the authors' original contributions.

Keywords: quantum physics, quantum foundations, quantum measurement.

INTRODUCTION

The standard postulates of quantum dynamics (DIRAC, 1930; VON NEUMANN, 1932; HERBUT, 1984; NIELSEN and CHUANG 2000; DUGIĆ and JEKNIĆ-DUGIĆ, 2024) introduce *unitary dynamics* for *closed* quantum systems (that include the isolated systems), which do not interact with any other quantum system, but can be exposed to external fields, such as magnetic or electric field. Dynamically, such systems are described by the Schrödinger law (Schrödinger equations). Simultaneously, there is a separate *postulate about the probabilities of the quantum measurement outcomes*, also known as the "Born's rule". The object of quantum measurement is not closed but an *open* system in (typically strong) interaction with the measurement instrument ("apparatus"). The Born's rule is not a dynamical description of quantum measurement but a rule that kinematically describes the measurement outcomes—more precisely, describes the sole effect ("output") of the measurement (that is sometimes assumed instantaneously to occur). This is a peculiar situation. Indeed, *instead of a general dynamical law for both closed and open quantum systems*, there is a gap between the two kinds of dynamical changes of states of quantum systems. This gap is *the basic formulation of the quantum measurement problem* (QMP). Numerous alternative formulations of QMP represent

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different *aspects* of the problem, whose treatments provide a whole library of published papers and books. Some insight in the related bibliography can be made, starting from WHEELER and ZUREK (1983) or SCHLOSSHAUER (2004), or DUGIĆ (2004).

The *physical nature of the measurement instrument ("apparatus")* is one of the central aspects of QMP. In a laboratory, the apparatus is a device fully described by the standard laws of classical physics. Thence numerous related questions that go deep into the problem: (i) is there an *a priori*, more-or-less sharp line dividing "quantum" from "classical"; (ii) is it allowed to describe the apparatus quantum-mechanically; (iii) the apparatus is built of numerous quantum systems, so how and "where" it comes to abandoning the quantum and to a transition to the classical behavior of the apparatus (in the words of Einstein: "Is the Moon there when nobody looks?"); (iv) is it possible, at least in principle, to observe certain quantum behavior of the macroscopic systems such as the apparatus, etc.? All those questions are recognized as different (mutually related) aspects of "*the problem of the transition from quantum to classical*" (JOOS *et al.*, 2003; DUGIĆ, 2004), as one of the basic aspects of QMP.

Formal description of the quantum measurement process on the *level of the* single *object of measurement* is known as the "*state collapse*" (or "*state reduction*") *problem*. According to the standard description, an initial state $|\psi\rangle$ "collapses" to a state $|n\rangle$, which corresponds to a value a_n obtained in a *single run* of measurement of some observable $\hat{A}$:

$$|\psi\rangle = \sum_p c_p\,|p\rangle \rightarrow\ |n\rangle,\ \text{with probability } |c_n|^2. \tag{1}$$

In every single run of the measurement, one value of the measured observable $\hat{A}$ is obtained, in general described by Eq. (1) with the probability $|c_n|^2$ of obtaining the result a_n. On this level of description, quantum measurement is a *stochastic* (quantum) *process*. Then the whole set (statistical ensemble, henceforth "ensemble") of the obtained results is described by the state transition,

$$|\psi\rangle = \sum_p c_p\,|p\rangle \rightarrow \hat{\rho} = \sum_p \left|c_p\right|^2 |p\rangle\langle p|, \tag{2}$$

where the statistical operator ("density matrix") $\hat{\rho}$ carries all the information about the measurement and the output states of the objects of measurement.

The fact that neither Eq. (1) nor Eq. (2) can be described by the Schrödinger unitary (linear) dynamics is *another aspect* of QMP. Expectably, it raises numerous (mutually related) questions and ramifications, e.g.: (a) could the transitions (1) and/or (2) be derived from the unitary quantum mechanics; (b) may it be the case that a new dynamical law, instead of the Schrödinger unitary law, is needed to reproduce the state collapse; (c) is there a unifying scheme for both quantum and classical, such as the object of measurement and the measurement apparatus; (d) may it be the case, that the collapse is a matter of interpretation and not so much of the theory building process etc.? All those questions remain unaltered within the theory of the so-called generalized quantum measurements (NIELSEN and CHUANG 2000).

In this review, we aim to discuss the physically essential aspects of the quantum measurement problem as briefly described above. For this reason, our review is neither extensive nor exhaustive. Rather, we briefly present and discuss the main, widely recognized interpretations and alternative quantum theories, while focusing on the current state of the art in the field of quantum foundations without complicated mathematical apparatus.

A VIEW ON THE HISTORY OF QMP

There is a sharp line dividing the QMP community from those who claim that there is not any problem whatsoever. As regard to the latter, such claims are based on some interpretational ground (BELL, 1990) or within a purely operational approach to physics and science in general (FUCHS and STACEY, 2019), that is essentially just an *interpretation* of the scientific work generally.

Nowadays, there are dozens of different interpretations, and the list is not complete yet. Owing to this endeavor, we witness formulation of some new quantum sciences, such as the Open quantum systems theory (BREUER and PETRUCCIONE 2002; RIVAS and HUELGA 2012; JEKNIĆ-DUGIĆ*et al.*, 2024), Quantum information and computation science (NIELSEN and CHUANG 2000; DUGIĆ, 2009) as well as Quantum metrology (NAWROCKI, 2019) and Quantum thermodynamics (BINDER *et al.*, 2018), along with the worldwide made efforts in realizing the goals of the so-called Quantum Technology, https://fizika.pmf.kg.ac.rs/pages/KFKT /Osnovna.htm. Quantum interpretations proved to be one of the most productive activities in physics research so far.

The best-known and widely supported *Copenhagen Interpretation* assumes a sharp line dividing quantum from classical systems. Division of a quantum-measurement setup into the quantum object of measurement and the classical apparatus is subject to the *principle of complementarity*: certain measurements exclude each other. The consistency of the interpretation requires a *movable line* that separates "quantum" from "classical", when, in certain situations, the classical apparatus for one procedure becomes a part of the quantum object of measurement for some other situation. However, this dichotomic treatment of the apparatus remains unexplained, and in any case without a clear rule—everything is left to the *ad hoc* choice for every measurement procedure separately.

The first consistent quantum measurement theory is due to VON NEUMANN (1932). The apparatus (A) is treated fully quantum-mechanically in interaction with the object (O) of measurement and formation of entanglement in the *unitary* O+A system is the very basis of successful measurement. The (unitary) state transition due to the measurement

$$|\psi\rangle_O \otimes |0\rangle_A = \sum_p c_p \, |p\rangle_O \otimes |0\rangle_A \to |\Psi\rangle = \sum_p c_p \, |p\rangle_O \otimes |p\rangle_A, \tag{3}$$

introduces the quantum correlations (entanglement) that provides a one-to-one relation of the obtained (non-degenerate) value a_p of a measured observable $\hat{A}$ (with the eigenstate $|p\rangle_O$) with the apparatus state $|p\rangle_A$ as an eigenstate of some apparatus observable, whose reading off the apparatus completes the measurement process. However, it turns out that the bipartite system O+A should be extended with the *observer* at the end of the chain. Von Neumann and supporters, e.g. WIGNER (1961) and STAPP (2009), distinguish the role of the observer's consciousness, which should bring the final decision in measurement as described by Eq. (1). Necessity of the conscious observer is known as "*psycho-physical parallelism*": it completes the measurement by *providing the consciousness-induced "collapse"* into a state $|p\rangle_O \otimes |p\rangle_A$ that corresponds to a *single outcome* a_p of the measurement—in accordance with Eq. (1).

The assumption that all the quantum particles possess definite positions and momenta in every instant of time is characteristic of the de Broglie-Bohm theory (BOHM, 1952; BELL, 1990; BOHM and HILEY, 1993; DÜRR *et al.*, 2012). However, the particles' positions and momenta are assumed to be *experimentally inaccessible*—i.e., the *hidden variables* of the theory. All the particles are imagined to be immersed in a quantum field ("potential"), which drives the movement of particles and their dynamics. Thence, the quantum uncertainty is present only due to the quantum potential—whose consequences can be observed by measurements performed on the particles, in full agreement with the standard quantum theory. In this context, QMP does not present a problem on the ontological level (BELL, 1990; MAUDLIN, 2016).

Taking the right-hand-side (rhs) of Eq. (3) literally is characteristic of Everett's "*relative state interpretation of quantum mechanics*" (EVERETT, 1957) — also known as Everettian Quantum Mechanics or Many Worlds Interpretation (*MWI*) of quantum mechanics (WHEELER,

1957; DEWITT, 1970; DEUTSCH, 1996; SAUNDERS *et al.*, 2010; WALLACE, 2012). The Entangled state in equation (3) is assumed to present the quantum ontology, which is inaccessible to a local observer, and which belongs to one "branch" (one "Everett World") indexed by "p" in Eq, (3). In every measurement-like situation, formation of quantum entanglement is followed by a split of the "branches", where a branch "i" carries a definite quantum state $|i\rangle_O \otimes |i\rangle_A$ of both (quantum) systems O and A, and hence a definite outcome of a single act of measurement as presented by Eq. (1). Mathematical formalism of the interpretation is a variant (GELL-MANN and HARTLE, 1990) of the so-called Consistent History Approach (CHA) to quantum mechanics (GRIFFITHS, 2002) and of the effect of decoherence (JOOS *et al.*, 2003; DUGIĆ, 2004). However, this interpretation remains silent regarding some severe criticism (to be presented below).

Table 1 briefly summarizes the answers provided by the main interpretations regarding some basic foundational questions.

Table 1. Answers of the main interpretations to some basic foundational questions; "Y" stands for "yes", while "N" stands for "no".

	Copenhagen	**Von Neumann**	**Bohm**	**Everett**
Is there a dividing line between quantum and classical?	Y	N	N	N
Is there a state collapse?	Y	N (Psycho-physical parallelism instead)	Ignorant (QMP is not recognized as a problem)	N (Branching of the Worlds instead)
Are there hidden variables?	N	N	Y	N
Is dynamics of the O+A system unitary?	N	Y	Y	Y

The 1980s' substantial increase of interest in quantum foundations can be recognized in introducing some new aspects and deeper questions raised by QMP. Those include:

- Leggett's program of investigating the so-called *macroscopic quantum phenomena of the second kind* (LEGGETT, 1980; LEGGETT *et al.*, 1987; DUGIĆ, 2004). That is a search for the suspected "borderline" between quantum and classical as a part of the problem of the transition from quantum to classical.
- With the same aim as Leggett's program, a renewed interest in the consequences of quantum entanglement, equation (3), is set as a general program beyond the scope of QMP. *Quantum decoherence* (ZUREK, 1982; JOOS *et al.*, 2003; SCHLOSSHAUER, 2004) is recognized as a genuine quantum-mechanical effect (process) that brings the *effectively classical-like behavior of open systems* exposed to certain influence of its environment (see also DUGIĆ, 2004).
- In search for the quantum coherence and the split (branching) of the Everett Worlds in a tabletop experiment, David Deutsch (DEUTSCH, 1985) introduced a concept, formulated a formalism, and gave an example of the first-ever *quantum-computation algorithm*—the famous Deutsch's algorithm (see also NIELSEN and CHUANG, 2000; DUGIĆ, 2009).
- Introducing *a new fundamental dynamical law* instead of the unitary Schrödinger law (HAWKING, 1983; BANKS, 1984) is at the root of all the *objective collapse theories*. On the

more elaborate ground, the so-called *spontaneous wave function collapse* theories introduce a new conceptual as well as formal layer in the foundations of quantum mechanics (GHIRARDI *et al.*, 1986; DIOSI, 1986). The time-continuous variants soon appeared and are known as *continuous spontaneous localization* (GHIRARDI *et al.,* 1990; GHIRARDI *et al.*, 1995) and *gravity-related spontaneous collapse theory* (DIOSI, 1987; DIOSI, 1989; PENROSE, 1996) a review can be found in BASSI *et al.* (2013).
- A paradigm shift appears in quantum foundations due to the use of "information" and recognition of "quantum information" as the very basis of every operational approach to physics generally, and in particular to quantum mechanics. Thence the Physical World is a subject of the (quantum) information-theoretic description (VEDRAL, 2010).
- Zeilinger's group in Vienna challenges the quantum/classical borderline in investigating quantum coherence versus decoherence of large quantum systems such as some large molecule species (ARNDT *et al.,* 1999), HACKERMULLER *et al*., 2004) and investigate quantum entanglement as a manipulable quantum-information/computation resource that includes quantum teleportation (BOWMEESTER *et al*., 1997), quantum entanglement swapping (PAN *et al*., 1998), the Bell inequality test (WEIHS *et al*., 1998), etc.
- The work on the quantum foundations with QMP brought about quantum physics (https://fizika.pmf.kg.ac.rs/pages/KFKT/Osnovna.htm), as a broad field of research and development of the desired quantum technology worldwide. Currently, there is extensive technological research and development in mastering quantum and some related technologies, such as nanotechnology.

CURRENT VERSIONS OF SOME INTERPRETATIONS

In this section, we briefly outline certain criticisms and the thereafter reformulations of some prominent interpretations of quantum mechanics.

QBism: a modern Copenhagen interpretation

The lack of a clear and consistent rule for dividing "quantum from classical" and of a theoretical explanation of the state collapse, as presented by Eqs. (1) and (2), make the CI a purely operational, almost a phenomenological, description of quantum measurement. Hence CI is an option that operationally can hardly be discarded but is virtually useless in the deeper theoretical considerations. In other words, CI offers a solution to QMP "by definition".

Within the quantum information/computation community, a leading paradigm on QMP is essentially shared with CI. Quantum measurement is regarded as an elementary process that does not require any explanation. Formally, quantum measurement is a "black box" ("oracle") that is a part of the elementary set of "quantum gates" (quantum logical operations) which is known to be non-unitary and always used at the end of the protocol/algorithm (NIELSEN and CHUANG, 2000). In this context, QMP would read (DUGIĆ, 2009): which logical gates constitute the "measurement gate"? However, since oracles are generally used without a question regarding their operation, QMP is not a part of the current quantum information/computation science (QICS).

Furthermore, a leading paradigm in QICS is that quantum mechanics is all about updating the probability distributions regarded as a carrier of information, thus reducing science and scientific endeavor to more-or-less collecting and systematization of the collected data. Such a view of the general scientific attitude is known as QBism ("quantum Bayesianism", FUCHS and STACEY, 2019). Its fully pragmatic attitude and ignorance about the deeper foundational questions in the foundations of quantum mechanics make QBism virtually indistinguishable from the Copenhagen Interpretation (CI). It is the concept of "information" that QBism adds to CI and perhaps makes it more consistent, but equally useless for the foundations of quantum mechanics. Certainly, it can be adopted as a matter of personal taste or a worldview, which may include discarding, whatever meaning, “physical reality” (VEDRAL, 2010).

As an argument against the reduction of physics to information, let us address a classic lesson on the “macroscopic measurements” within the quantum-mechanical theory (VON NEUMANN, 1932). Consider a one-dimensional system described by the position $\hat{x}$ and the momentum $\hat{p}$ observables, which do not commute, i.e. $[\hat{x},\hat{p}] = i\hbar$. Those observables can be approximated by the respective observables $\hat{\xi}$ and $\hat{\pi}$, so that $[\hat{\xi},\hat{\pi}] = 0$. Hence a practice in which $\hat{x}$ is *operationally* indistinguishable from $\hat{\xi}$ and $\hat{p}$ is indistinguishable from $\hat{\pi}$ would provide *information* about the system that does not leave a room for the *true physical contents* of the quantum non-commutativity and of the therein derived operational limitations imposed by the uncertainty relation $\Delta\hat{x}\Delta\hat{p} \geq \hbar/2$. That is, the reduction of all knowledge to the accessible information (here: simultaneous measurements of both $\hat{\xi}$ and $\hat{\pi}$), can hide the *real* quantum-mechanical nature of the system’s position and momentum variables. Hence, we conclude that purely information-theoretical thinking can hardly substitute theoretical thinking and model-based research.

Modern Everretian wisdom: emergent branching worlds

All the versions of the Everett MWI suffer from the problem of interpreting Born’s rule. Namely, it is not clear how to incorporate the concept of probability in a *single* branch (KENT, 1990). Thence escaped into a “murky water” of decision problem in “deriving” the Born’s rule (DEUTSCH, 1999; WALLACE, 2003). Another, only recently recognized, problem deals with the exactness of the decoherence-induced classicality as a very basic of modern MWI (WALLACE, 2012). Again, escape from the problem is proposed based on vague concepts, such as “emergent decoherence” (SAUNDERS *et al*., 2010; WALLACE, 2012). Perhaps, the term “emergent” stands for the degrees of freedom that approximate a set of some “microscopic” degrees of freedom, which are simultaneously subjects of the “local” decoherence processes. However, the problems multiply.

The occasionally re-discovered “entanglement relativity” (ER), (DUGIĆ and JEKNIĆ, 2006; DUGIĆ and JEKNIĆ-DUGIĆ, 2008) imposes the following rule: if a composite system is in a tensor-product state for one bipartition (split into a pair of subsystems), the actual state of the total system is typically entangled for virtually every alternative bipartition of the total system. Thence a basic *problem regarding the very concept of the world branching*. Since every branching gives rise to a tensor-product state in one World, due to ER, there cannot be branching for any other bipartition of the total system. The problem consists in that that quantum mechanics does not single out any partition of the Universe (ZUREK, 1998; DUGIĆ and JEKNIĆ 2006; DUGIĆ and JEKNIĆ-DUGIĆ, 2008; DUGIĆ and JEKNIĆ-DUGIĆ, 2012; JEKNIĆ-DUGIĆ *et al*., 2014a; NELSON and RIEDEL, 2017). Hence the conclusion: consistency of MWI requires a preferred structure of the Universe. Seemingly, as an escape from the problem may be, as emphasized above, to invoke “emergentism”: microscopically different decohered degrees of freedom, which pertain to different partitions, may result in *emergent decoherence*, *which supports* “branching”. While different degrees of freedom of a total system may indeed decohere in parallel (DUGIĆ and JEKNIĆ-DUGIĆ, 2012), there is a relevant model that does not support any emergent degrees of freedom whatsoever (JEKNIĆ-DUGIĆ *et al*., 2014a). Hence, at least one important model (of a quantum Brownian particle) that does not support “emergent decoherence”—and the problem remains unsettled. Therefore, we conclude that even modern Everett MWI is not yet an acceptable option as a solution to QMP.

Quantum decoherence program

Quantum decoherence is an essential part of the modern MWI (see previous subsection) and of certain cosmological programs (GELL-MANN and HARTLE, 1990). As an independent program, it has occasionally been regarded as a basis of a solution to QMP.

Quantum decoherence is an orthodox quantum-mechanical theory (ZUREK, 1982; JOOS *et al.*, 2003; SCHLOSSHAUER, 2004). Its central part (ZUREK, 1982) is essentially an extension and elaboration of the von Neumann's quantum measurement theory (DUGIĆ, 2004). It regards an open system whose interaction with its environment implies dynamical formation of entanglement, cf. Eq. (3), and thence dynamics of the open system obtained via the "tracing out" operation. In Eq. (3), the object's "reduced state"

$$\hat{\rho}_O = tr_A(|\Psi\rangle\langle\Psi|) = \sum_p \left|c_p\right|^2 |p\rangle_O \langle p|, \tag{4}$$

which carries all the necessary information about the measurement—the final states $|p\rangle_O$ uniquely linked with the measurements results a_p as well as the related probabilities $\left|c_p\right|^2$. Nevertheless, in order to completely define what is measured, it is often argued that another, a third, system in the chain of Eq. (3) is needed. It is proposed (ZUREK, 1982) that the apparatus environment can close the "circle" thus providing a complete description of the measurement process as described by Eq. (4)–when tracing out encompasses the apparatus environment.

The reduced state $\hat{\rho}_O$ is an "improper mixture" (D'ESPAGNAT, 1999), i.e., it does not represent a state of the object, but a mathematical artifact sufficient for describing the information obtained during the measurement. If it were a state, the state of the O+A system would read $\hat{\rho}_0 \otimes \hat{\rho}_A$, instead of the pure state $|\Psi\rangle$. Therefore, quantum decoherence *cannot solve* QMP—which is the reason that von Neumann resorted to psycho-physical parallelism, or the Everett MWI to "branching".

"Quantum decoherence" is a phrase used in different contexts and with the different meanings (JOOS *et al.*, 2003; DUGIĆ, 2004)). The central part of the theory introduces the so-called "pointer basis" (such as the basis $|p\rangle_O$ in Eq. (4))—more precisely the "[the environment-induced] superselection sectors", which are simply eigenspaces of the "measured" observable with the basis $\{|p\rangle_O, p = 1,2,3, \ldots\}$ adapted to the superselection sectors. For large systems, decoherence is exceedingly fast so effectively providing a classical-like behavior determined by the superselection sectors of the open system.

Consistent histories approach

Consistent histories approach (CHA) is a framework aiming at unifying quantum dynamics (GRIFFITHS, 2002). It introduces a chain of projectors $\ldots \hat{P}_{t_{i+1}}(t_{i+1})\hat{P}_{t_i}(t_i) \ldots \hat{P}_{t_{j+1}}(t_{j+1})\hat{P}_{t_j}(t_j) \ldots$, where $t_{i+1} \geq t_i \geq \cdots \geq t_{j+1} \geq t_j$. The projectors $\hat{P}$ are indexed by the time instants, where the different time instants distinguish different complete sets of orthogonal projectors, while time dependence distinguishes dynamics (unitary or not) of the system of interest. If the initial state is pure, then every "history" is simply a trajectory in the Hilbert state space of the system.

Certain histories can fulfill a condition of consistency ("consistent histories"), which may be interpreted as a condition of classical-like dynamics of the system (closed or open). A subclass of "consistent histories" is known under the name of "decoherent histories" as a basis of certain cosmology programs (GELL-MANN and HARTLE, 1990) as well as of the modern Everett MWI (SAUNDERS *et al.*, 2010; WALLACE, 2012). However, conditions for decoherent histories are not the conditions for the occurrence of decoherence, i.e. for the effective superselection rules (sectors). On the other hand, projectors distinguished by decoherence may be used to introduce consistent histories and thus for "emergent classicality" as it is introduced by the very existence of the dynamically chosen and environmentally induced superselection sectors. In any case, neither the cosmological quasiclassical behavior nor the consistency of the *modern* MWI can be established without the concept of decoherent histories. Nevertheless, there is a caveat to this program.

It has recently been shown (JEKNIĆ-DUGIĆ *et al.*, 2024) that the histories consisting exclusively of the pure states *cannot be justified*, at least for the widely known Markovian (JEKNIĆ-DUGIĆ *et al.*, 2023a) processes. Therefore, if the formalism should apply to the Universe describable by a history of pure states, the CHA formalism *may not be useful*.

CURRENT RESEARCH

Most of the original interpretations turned into distinctive theories. In this section, we briefly outline a few such theories under current scrutiny.

Bohmian theory

Von Neumann's "impossibility proof" (VON NEUMANN, 1932) on the existence of hidden variables soon turned out to be incorrect. Of course, it was correct mathematically. The point is that von Neumann introduced unreasonable assumptions in regard to where to look for the hidden variables. Already Bohm's initial interpretation (BOHM, 1952) completely ignored von Neumann's "proof" and exhibited there is room for the subquantum realm that is *kinematically* describable by the classical-like variables. On that basis, BELL (1964) formulated his famous inequality that initiated a search for testable completeness of quantum mechanics. Soon after Bohm's pioneer work, it was clear that hidden variables, which serve to "complete" the standard quantum mechanical theory (DIRAC, 1930; VON NEUMANN, 1932; HERBUT, 1984; DUGIĆ and JEKNIĆ-DUGIĆ, 2024), cannot be of the kind known for the variables of the standard classical physics. Indeed, it turned out that there is a certain kind of *nonlocality* of those variables as clearly suggested by the standard quantum-mechanical theory and experimental evidence.

Nowadays, positions, as well as momenta of the quantum particles, are regarded as hidden variables that underlie the quantum theory but for the price that, first, they are hidden—meaning *unobservable*—and second, carrying *nonlocality*. As to the latter, relativistic causality quantified by the light speed c in the vacuum is not jeopardized. What is jeopardized is our intuition that seemingly independent systems may react to the actions locally performed to some another system, even if mutually arbitrarily spatially distant—first expressed by the "EPR paradox" (EINSTEIN *et al.*, 1935) and soon after by the Schrödinger's cat (SCHRÖDINGER, 1935). This is precisely the situation in quantum measurement as presented by the rhs of Eq. (3)—correlations expressed by the same index p provide *instantaneous information* on the final state (after the measurement) *of both systems* O and A. Violation of the Bell inequalities is regarded as a quantitative measure of quantum nonlocality (NIELSEN AND CHUANG, 2000).

Since the hidden variables cannot, even in principle, be measured, the whole of the Bohmian theory remains speculative and, so far, has not offered any significant improvements to quantum theory, which nowadays hugely exceeds quantum mechanics (https://fizika.pmf.kg.ac.rs/files/%D0%9D%D0%BE%D0%B1%D0%B5%D0%BB%D0%BE%D0%B2%D0%B0.pdf; https://fizika.pmf.kg.ac.rs/pages/KFKT/Osnovna.htm).

Quantum stochastic processes

Assumption of the universal validity of the Schrödinger unitary law underlies most of modern quantum physics (NIELSEN AND CHUANG, 2000; BREUER AND PETRUCCIONE 2002; JOOS *et al.*, 2003; DUGIĆ, 2004; RIVAS AND HUELGA, 2012). This useful hypothesis is at the root of the formation of quantum entanglement as presented by the rhs of Eq. (3).

However, different programs were started, some of them even built, bearing in mind some specific problems, such as the construction of a quantum theory of gravitation (HAWKING, 1983; BANKS, 1984) or the theory of quantum measurement. Those programs aim at a formulation of a *new fundamental quantum dynamical law*, instead of the unitary Schrödinger law. However, such attempts may fall out without a clear program framework.

The so-called *stochastic quantum equations* have a clear methodological basis. Every mixed quantum state can be considered as an "average state" of a statistical ensemble—formally compared to the mathematical definition of the average (mean) value, this is actually the case. For example, a mixed ensemble described by a collection of pure subensembles described by the pure states $|p\rangle$ with the related statistical weights (probabilities) $|c_p|^2$ can be presented by "averaging" the states $|p\rangle$, thus giving rise to Eq. (4). Then it is perfectly imaginable that the pure states $|p\rangle$ are *consequences of a stochastic choice* (cf. Eq. (1)) and thence the time-determined trajectories $|p(t)\rangle$, for every p separately. In other words, a series of the stochastic choices of an action exerted on a single system in an initial state $|p\rangle$ can lead to a trajectory $|p(t)\rangle$, while summing up overall ps would lead to Eq. (4)—with the time-dependent terms in the sum in Eq. (4).

The constraint on obtaining Eq. (4) for the purely pure-state dynamics has been a prominent problem of quantum theory. It was conjectured, that the only physically allowed stochastic dynamics must be nonlinear (GISIN, 1989; BASSI and HEJAZI, 2015). Nowadays, it is widely accepted such a position and, at least formal, related models of the open quantum systems theory (BREUER and PETRUCCIONE, 2002), quantum information/computation science (NIELSEN and CHUANG, 2000), and the theories of quantum measurement, have been devised. However, the fundamental distinction emerges. Within the open systems theory [which assumes unitary dynamics for the total system], this is a mathematical description that can be reduced to environmental influence. In the alternative theories aiming at *solving QMP*, there is not any environment whatsoever—the system is isolated, but not unitary, and the source of the stochastic behavior is, so to say, within the system itself, not an externally-induced process. In the stochastic basis of the GRW theory (GHIRARDI *et al.*, 1986), the so-called CSL theory (GHIRARDI *et al.*, 1990), the effect of "spontaneous localization" is effective only for large bodies (for the "collective"—such as the center-of-mass—degrees of freedom) thus describing the macroscopic apparatus of the von Neumann's quantum measurement theory in a manner proposed by the Copenhagen interpretation of quantum mechanics, but *without* invoking sharp line dividing quantum from classical.

The not-so-long history of those attempts and the related results, successes, and falls can be found, e.g., in DIOSI (2018) and BASSI *et al.* (2023). The main objection to the whole program is due to DIOSI (2018): (i) the single trajectories are experimentally unobservable, and (ii) the equations of the program can be described by the standard unitary theory, except for the gestural claims of the intrinsic stochasticity (for a similar point of view, see PRIGOGINE, 1997).

SOME PROSPECTS

In this section, we briefly present certain programs, which encapsulate and essentially go beyond the quantum measurement problem. To this end, the motivations can be anticipated by the contents of Table 2, which collects and briefly presents certain weaknesses of the interpretations and alternative quantum theories presented in the previous sections of this review. The full presentation in this regard would occupy a whole library of the books and scientific papers that have appeared in the last, approximately, one hundred years, and particularly in the last 25 years. Unfortunately, there is not a single source, including the electronic repositories, that could be reliably and straightforwardly used. It is beyond any doubt that the contents of Table 2 reflect the personal views of the authors.

Table 2. The main weaknesses of the interpretations and alternative theories presented in this work.

Copenhagen interpretation & QBism	Purely operational and thus conceptually poor; poorly justified resorting physics to information.
Bohm's theory	Experimentally unattainable ontological level; the lack of predictions beyond the standard quantum theory.
Everett many-worlds interpretation	Artificial role of probability; the world branching requires a preferred structure (decomposition into subsystems) of the Universe.
Quantum decoherence program	A direct consequence of the standard quantum (measurement) theory without any elements for solving the quantum measurement problem.
Consistent history approach	The histories typically might not consist of exclusively pure quantum states.
Stochastic equations program	Experimentally attainable results concern the mixed states—hence virtually indistinguishable from the quantum decoherence theory.

Can quantum mechanics be falsified?

True scientific progress typically occurs when a fresh theoretical perspective is provided on the known (typically numerous) collected data and experimental evidence (as it was the case with the Maxwell equations and Einstein's formulation of Special Relativity) or in a clash of the existing knowledge with the new experimental findings (as it was the case in formulation of quantum mechanics). Unfortunately, neither is foreseen for modern quantum science. Hence a search for the *possibly wrong predictions of quantum theory*.

Some proponents of the Bell tests assume that non-violation of a Bell inequality should be regarded as experimental evidence of the existence of the system's hidden variables. However, this does not seem to be viable, for the simple reason that the hidden variables are unreachable by any measurement whatsoever. Therefore, an apparently inescapable conclusion: at their best, the Bell tests should be regarded as the tests of the classical *versus* the quantum character of the *measured physical quantities* — it is all appearance that any measurement performed on the Moon would lead to the validity of *every* Bell inequality.

In the early 1980s, Anthony Leggett launched a program of looking for quantum effects on some many-particle systems, which are well described by the Josephson effects (LEGGETT, 1980; LEGGETT and GARG, 1985; LEGGETT *et al*., 1987). The point was in the fact that those systems are sufficiently well described by some semi-phenomenological, classical-physics equations (see also DUGIĆ (2004) for some details). The sought for effects regarded the quantities for which, due to their classical equations, carry what is expected to be *classical reality*. Then the possible violations of the quantum-mechanical predictions could serve as a mark of the wrong theoretical predictions of quantum mechanics—truly inspiring and provocative. Soon, the first quantum effects of *macroscopic quantum tunneling*, and *macroscopic quantum coherence* were justified. As (hopefully!) the last and decisive step of the program, Leggett and Garg (LEGGETT AND GARG, 1985) proposed a Bell-like inequality to be experimentally tested. We're still waiting for such a test. To this end, the main reason is the fact that, in general, Bell tests are conditioned with the so-called loopholes, which have only recently been claimed to be closed (HENSEN *et al*., 2015; SHALM *et al*.; 2015; GIUSTINA *et al*., 2015). Nevertheless, the proposed test remains an interesting and provocative task and a promising perspective on going deeper into quantum-mechanical theory.

A search for the wrong predictions of quantum theory has recently been launched in different directions, NAVASCUES GROUP in Vienna, https://www.iqoqi-vienna.at/research/navascues-group. The main virtue of the program is the possibility to act in the so-called device-independent manner (POPESCU and ROHRLICH, 1994). Theoretical considerations allow for the use of the theory-indifferent models, thus in principle encapsulating predictions of the classical, quantum, and the so-called post-quantum theory; the latter being a desired "descendant" of quantum theory. This is another interesting prospect that is not directly linked with the truly deep problem of quantum measurement and still does not offer a hint on the post-quantum theory likewise.

In the quantum-to-classical border territory

Different interpretational descriptions of the measurement apparatus can be found only partially linked to each other. The original Copenhagen interpretation postulates the purely classical-physics nature of the apparatus. Within the decoherence program, as an extension of von Neumann's measurement theory, the apparatus is assumed to be a quantum many-particle system in unavoidable interaction with its (quantum) environment, which induces effectively a classical-like behavior of the apparatus. In the GRW theory, apparatus is a many-particle system subject to the objective state collapse, thus providing a rather fast collapse of the state of the composite system "object of measurement+apparatus". Hence a closer look into the quantum-mechanical behavior of the many-particle systems may be a useful tool in investigating the "border territory" between the "quantum" and "classical".

Fascinating experiments with mesoscopic systems (ARNDT *et al.*, 1999; HORNBERGER *et al.*, 2003; HACKERMUELLER *et al.*, 2004; SHAYEGHI *et al.*, 2020) clearly demonstrated unsharp line dividing quantum coherence from the effect of decoherence for some large molecules. This is a convincing demonstration of the *nonexistence of the sharp line dividing quantum from classical*–in contrast to the original Copenhagen interpretation. The experiments were performed with both C_{60} and C_{70} fullerene molecules, with the tetraphenylporphyrin and fluorofullerene C60F48 molecules as well as with the gramicidin molecule (that is composed of 15 amino acids). The moral from those experiments is rather fascinating: it appears that only the technical obstacles keep us from observing the genuine quantum nature of the physical world. Hence one important step towards the deep secrets of Nature is made.

Leggett's program sets a framework for investigating the border territory between quantum and classical—dynamics and behavior of "mesoscopic" systems: apply the standard rules of quantization of the classical models. This attitude has been put forward in investigating quantum corrections to the classical models of the molecular cogwheels (MC), which are recognized as one of the basic elements of the desired nanotechnology and typically treated as the classical-physics systems, which are subject to *dissipation*. Quantum dissipation is a basic problem in the foundations of quantum mechanics and quantum physics generally, with the main method developed by CALDEIRA and LEGGETT (1983). Different *statistical methods* have been employed to investigate quantum corrections to dynamical stability of the propeller-shaped MCs (that are described by the Caldeira-Leggett master equation): dynamics of the standard deviations (JEKNIĆ-DUGIĆ *et al*., 2018; PETROVIĆ *et al*., 2020; PETROVIĆ *et al*., 2022; JEKNIĆ-DUGIĆ *et al.*, 2023b), and the mean first passage time (PETROVIĆ *et al.*, 2020) of the angle and the angular momentum observables of a one-dimensional harmonic and non-harmonic rotator. Interestingly enough, the obtained results do not point out any simple or straightforward rules for achieving sufficient dynamical stability (and thence satisfactory dynamical control) of the cogwheel rotations. Rather, *a need for optimization*, in the manner of the engineering tasks (RAVINDRAN *et al*., 2006), has been put forward. Not only that the *realistically non-negligible quantum corrections* have been recognized, but practical utilization of the results must follow from an optimization that, in turn, cannot be performed without scrutinizing experimental tests of the theoretical results. Therefore, not only that *non-sharp border territory* has been justified, but the treatment of the quantum corrections *relies on the classical procedures* that are typical for complex macroscopic systems. Hence submission of the classical optimization under the quantum rules remains an important and mind-provoking task.

A general characteristic of “classical thinking” is to *deal with a special set of degrees of freedom* (DoF), which describe a composite system as a whole—the so-called *collective* DoFs. Furthermore, the related experience and experimental evidence generally support the wisdom: different DoFs give rise to “different physics”—starting already from the symmetry rules. Already in 2006, this path has been put forward (DUGIĆ and JEKNIĆ, 2006; JEKNIĆ-DUGIĆ and DUGIĆ, 2008). This is a unique program that has led to numerous results of importance in the quantum foundations. To this end, among else, the list involves the discovery of relativity of quantum correlations (entanglement and discord (DUGIĆ AND JEKNIĆ, 2006; DUGIĆ AND JEKNIĆ-DUGIĆ, 2008; DUGIĆ *et al.*, 2013), a method for avoiding quantum decoherence (JEKNIĆ-DUGIĆ and DUGIĆ, 2008), preferred structure for a two-mode system (ARSENIJEVIĆ *et al*., 2013), the parallel occurrence of decoherence (DUGIĆ and JEKNIĆ-DUGIĆ, 2012) and thence incompleteness of the Everett MWI (JEKNIĆ-DUGIĆ *et al*., 2013) etc. As an original approach to quantum foundations, this program of investigating different partitions into subsystems of the composite quantum systems has been recognized as a new path for going deeper into the “world of quantum puzzles” (JEKNIĆ-DUGIĆ *et al.*, 2013; KASTNER *et al*., 2017).

Emergent physical time

A recent proposal on the *dynamically emergent nature of physical time* introduces a new fundamental law as an extension of the standard unitary Schrödinger law. Time is not any of the fundamental physical notions, but a consequence of quantum dynamics for the local (approximately isolated) quantum systems. A pioneering proposal of KITADA (1994) has been extended and generalized to introduce a fundamental uncertainty of the emergent local time—the so-called Local Time Scheme (JEKNIĆ-DUGIĆ *et al.*, 2014b; JEKNIĆ-DUGIĆ *et al.*, 2016). The basic rule for introducing local time is a rather simple consequence of minimalist thinking (KITADA *et al.*, 2016): a local system’s Hamiltonian uniquely determines the system’s local time, and *vice versa*. Within the statistical-ensemble description, this Local Time Scheme (LTS) provides a number of interesting results, among else: offers a natural (unsharp) delimitation of the small and many-particle systems, reproduces some basic rules of the open quantum systems theory, introduces (possibly the first ever) non-differential (yet unital) dynamical map, naturally introduces the so-called Born approximation for the open quantum systems, introduces effective (emergent) Markovianity and complete positivity etc. While there remains a lot of work yet to be done within LTS as a candidate for a reformulation of quantum theory, the existing results are encouraging.

CONCLUSIONS

The quantum measurement problem remains unsettled. However, the progress seen in this regard in the last forty years moved the focus from purely interpretational or formal mathematical research. Most of the original interpretations obtained the outlines of the self-consistent quantum theories that can be more deeply investigated, particularly by tackling the assumptions they rely on. Furthermore, new theoretical tools are developed within the new quantum sciences that have been formulated, such as the open quantum systems theory, quantum information/computation science, quantum thermodynamics, quantum metrology, etc. Closely related technological progress in measurement procedures and techniques is encouraging and currently relies on the development of the once not foreseen technologies, such as quantum and nanotechnology. It is all appearance that the related practice and experience may change our understanding and therefore views on the quantum and mesoscopic physical world, possibly opening new routes in approaching the quantum measurement problem.

So far, we have learned that the quantum world is *nonlocal*—as undoubtedly emphasized by violations of the Bell inequalities. The underlying non-classical correlations between the quantum systems are believed to be forbidden in the classical world.

We have also learned that *there is not a sharp line dividing quantum from classical*. Then a sharpened problem of the transition from quantum to classical.

Already dynamics of mesoscopic systems are physically rich and emphasize the nontrivial role of the *separation of composite systems into subsystems*. Local actions (including quantum measurements) performed on subsystems of different structures (partitions into subsystems) of the total system, at least theoretically, reveal different physical contents and conclusions endowed by a genuine lesson that resorting to *sub*systems unavoidably discards certain information of the total system. Investigating quantum non-locality in such systems is another challenge taking its own time.

Validity and the expected limits of the standard laws of *thermodynamics* in the quantum and mesoscopic systems is another challenge that, in turn, regards the foundations of the standard (Gibbsian) statistical mechanics. Another layer of the current research that is also not presented in this short review regards the *intersection of quantum theory with the theory of relativity*. To this end, some inspirations and tasks come from the research devoted to nonlocality, i.e. to the Bell inequalities. Nevertheless, that is a broad new “territory” and a nascent “hybrid” quantum-relativity field that strongly relates to *cosmological problems* and the problem of *quantum gravity*. All those research programs deserve special and separate reviews to be fairly presented.

It is beyond doubt that the scientific progress launched from within the quantum foundations studies will bring new physics and a solution to the quantum measurement problem. At this point of scientific progress appears the question of *which price will be paid*—it may happen that the new quantum theory (expectably with a new fundamental dynamical law) will be even weirder than the one we know and extensively use and investigate.

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