

HALL AND ION-SLIP EFFECTS ON MHD FREE CONVECTIVE FLOW OF A VISCOELASTIC FLUID THROUGH POROUS REGIME IN AN INCLINED CHANNEL WITH MOVING MAGNETIC FIELD

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ABSTRACT. This paper consists of a mathematical analysis of MHD free convective flow of viscoelastic fluid through a porous regime in an inclined channel. The flow system is permeated by a uniform moving magnetic field with strong magnetic intensity to produce Hall and ion-slip effects. The flow governing equations are obtained from the suitable field and constitutive equations and solved analytically. To accentuate the consequences of various flow controlling parameters to the nature of the flow, numerical results are discussed in assistance with graphs and tables. An important fact noted from the study that Hall current generates the flow in the direction perpendicular to the main flow while ion-slip current reduces the flow in the direction perpendicular to the main flow. It is also seen that the moving magnetic field produces less rigidity in the flow in comparison to the stationary magnetic field.

Mathematics Subject Classification: 76W05, 76R10, 76S05, 76U05.

Keywords: Hall and ion-slip effects, free convection, viscoelastic fluid, porous regime, moving magnetic field.

INTRODUCTION

In recent years, theoretical and experimental investigation of rotating flows has attracted the attention of the research community because free convection can be also set up in the rotating flows by the action of centrifugal force and this is proportional to the fluid density. One of the excellent examples of such a situation is flows and heat transfer in a gas turbine. Heat and mass transfer due to free convection is one of the scientific phenomena which is used in many industrial and technological applications particularly in the chemical manufacturing industries and designing of the control system of the heat exchangers. Diverse applications of the heat and mass transfer due to free convection have encouraged the research scientists (KHAN *et al.*, 2011; CHAMKHA and AL-RASHIDI, 2013; BARIK *et al.*, 2013; MANGLESH and GORLA, 2013; NARAHARI and DEBNATH, 2013; IBRAHIM *et al.*, 2015; SETH and SARKAR, 2015; BUT and ALI, 2016; IQBAL *et al.*, 2016; SINGH *et al.*, 2016, 2018a, 2019b; FALADE *et al.*, 2017) to examine the flow behaviour of such flows under various configurations. The analysis of flow behaviour in a porous regime has been given more importance by the researchers in the last few

decades because the permeability of the porous regime produces the rigidity in the flow like the presence of a magnetic field in the flow of electrically conducting fluid. The simultaneous study of both the magnetic drag force and the drag force induce due to porous regime is significant due to its enormous applications in the geothermal energy plant and petrochemical industries. In recent years researchers (IQBAL *et al.*, 2016; SINGH *et al.*, 2016, 2018a, 2019a, 2019b, 2020; FALADE *et al.*, 2017; KRISHNA *et al.*, 2020) have investigated the simultaneous impacts of magnetic and Darcian drag forces on the hydromagnetic flows.

In an ionized fluid, the drifting velocity of the electrons is much higher than the drifting velocity of the ions (CRAMER and PAI, 1973). Therefore, in most of the investigations of the hydromagnetic fluid flows the drifting velocity of ions is neglected from the generalised Ohm's law for a moving conductor. However, in many industrial processes in which the strength of the applied magnetic field is very strong in such a situation the drifting velocity of ions cannot be neglected and there induces two currents called Hall and ion-slip current. In such a situation both the influence and Hall and ion-slip currents are considered on the hydromagnetic flows. Many investigations have been performed to notice the consequences of Hall and ion-slip currents on the flow. Some relevant contributions to the literature are due to the researchers RAM and TAKHAR (1993), TAKHAR and JHA (1998), ABO-ELDAHAB and AZIZ (2000), ELSHEHAWAY *et al.* (2004), HOSSAIN *et al.* (2015), SINGH *et al.* (2017a, 2017b, 2019b, 2020) and KRISHNA *et al.* (2020).

Nowadays the research community is paying more attention to the investigation of non-Newtonian fluid dynamics because most of the fluids are of non-Newtonian in nature and it may find a huge number of industrial applications such as lubricants, paints and polymer industries. The modelling of non-Newtonian fluid flow problems is more sophisticated than the modelling of Newtonian fluid flow problems. The study of impacts of the magnetic field on the flow of electrically conducting non-Newtonian flow may find tremendous applications in biomedical engineering, chemical engineering, food processing and metallurgy. The impact of the magnetic field on the flow of non-Newtonian fluid was firstly investigated by SARPKEYA (1961). He investigated the MHD flow of non-Newtonian fluid within two parallel non-conducting plates by considering two models, namely, the Bingham plastic model and the power law model. Moreover, SAPUNKOV (1970), DJUKIC (1974) and COBBLE (1980) discussed the dynamics of non-Newtonian fluid in the presence of the magnetic field. Soon after, some eminent researchers (CHAMKHA *et al.*, 2011; SHEHZAD *et al.*, 2013; KHALID *et al.*, 2015; NEJAD *et al.*, 2015, ZHAO *et al.*, 2017; HUANG *et al.*, 2018) discussed the influence of the magnetic field to the hydromagnetic flow of non-Newtonian fluids. Non-Newtonian fluids are divided into many categories, one of which is viscoelastic fluid. In a practical point of view, the study of the motion of viscoelastic fluid under the influence of electromagnetic force is of great importance due to its various applications such as petroleum drilling, gas reservoirs and in the production of complex multiphase products such as paints, inks and ceramic pastes. These diverse applications stimulated the scientists (GHASEMI *et al.*, 2011; UWANTA *et al.*, 2011; GARG *et al.*, 2014; SINGH *et al.*, 2018b, 2018c; TURKYILMAZOGLU, 2014; RAMESH and DEVAKAR, 2016; NAYAK *et al.*, 2016; BAAG *et al.*, 2017; MAJEED *et al.*, 2018) to investigate the dynamics of viscoelastic fluid under in the influence of the magnetic field considering various physical models and techniques.

In most of the research investigations, the applied magnetic field is considered to be stationary. However, the motion of the applied magnetic field is significantly affecting the flow behaviour. Motivated from the described literature survey we propose to examine the unsteady MHD free convective flow of a viscoelastic fluid through a porous regime in an inclined channel. The flow permeated to pass through a moving magnetic field and the intensity of the applied magnetic field is very strong. Thus, we considered the Hall and ion-slip effects also on the fluid flow.

MATHEMATICAL ANALYSIS

We considered the fully developed MHD laminar flow of a viscoelastic fluid through a porous regime in an inclined channel. The plates of the channel are inclined through an angle γ with the vertical. The flow system is permeated by a uniform moving magnetic field \vec{B} $(0, 0, B_0)$ applied along a direction normal to the plane of the channel walls and the flow system is in the action of rigid body rotation about z' -axis with angular velocity $\vec{\Omega}$ $(0, 0, \Omega)$. The left wall of the channel $z' = -z_0/2$ is stationary while the right wall of the channel $z' = z_0/2$ is executing oscillations in its own plane with velocity $U_0 f(t') = U_0(1 + \varepsilon \cos(\omega t'))$. The magnetic field is moving with the equal velocity as that of right wall of the channel in x' -direction. The temperature and concentration of the left wall is kept fixed while the temperature and concentration of the right wall is fluctuating. The configuration of the physical model is exhibited in Figure 1.

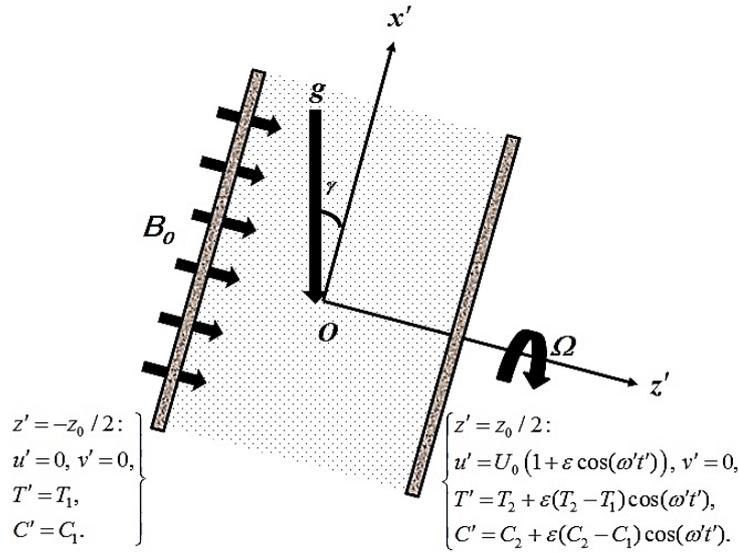


Figure 1. Configuration of the physical model.

The flow within the channel is developed due to the appearance of the gravitational field and motion of one of the walls of the channel. Induced magnetic field and polarization current are neglected in this problem. Boundary layer and Boussinesq approximations are assumed to be holding good.

The flow governing equations are obtained from the suitable field and constitute equations and described as

$$\frac{\partial u'}{\partial t'} - 2\Omega v' = \nu_e \frac{\partial^2 u'}{\partial z'^2} + \frac{\beta'}{\rho} \frac{\partial^3 u'}{\partial z'^2 \partial t'} - \frac{\sigma B_0^2}{\rho(\alpha_e^2 + \beta_e^2)} [\alpha_e u' - \beta_e v' - K_1 U_0 f(t')] - \frac{\nu_f u'}{k'} + g\beta_T(T' - T_1) \cos \gamma + g\beta_C(C' - C_1) \cos \gamma, \quad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega u' = \nu_e \frac{\partial^2 v'}{\partial z'^2} + \frac{\beta'}{\rho} \frac{\partial^3 v'}{\partial z'^2 \partial t'} - \frac{\sigma B_0^2}{\rho(\alpha_e^2 + \beta_e^2)} [\beta_e u' + \alpha_e v'] - \frac{\nu_f v'}{k'}, \quad (2)$$

where $\alpha_e = 1 + \beta_e \beta_i$. $K_1 = 1$ when the magnetic field is moving with the equal velocity as that of right wall of the channel in x' -direction while $K_1 = 0$ when the magnetic field is stationary. The energy equation with heat source is

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - S'(T' - T_1). \quad (3)$$

The concentration equation with first order chemical reaction is given by

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} - K'(C' - C_1). \quad (4)$$

The conditions at the boundary walls of the channel for the specified problem are

$$\left. \begin{array}{l} \text{At } z' = z_0 / 2 : \left\{ \begin{array}{l} u' = U_0(1 + \varepsilon \cos(\omega t')), v' = 0, \\ T' = T_2 + \varepsilon(T_2 - T_1) \cos(\omega t'), \\ C' = C_2 + \varepsilon(C_2 - C_1) \cos(\omega t'). \end{array} \right. \\ \text{At } z' = -z_0 / 2 : \left\{ \begin{array}{l} u' = v' = 0, T' = T_1, C' = C_1. \end{array} \right. \end{array} \right\} \quad (5)$$

In order to write the flow governing equations in non-dimensional form, we introduce the following non-dimensional quantities

$$\left. \begin{array}{l} z = z' / z_0, u = u' / U_0, v = v' / U_0, \tau = t' U_0 / z_0, \omega = \omega' z_0 / U_0, \\ T = (T' - T_1) / (T_2 - T_1), C = (C' - C_1) / (C_2 - C_1). \end{array} \right\} \quad (6)$$

Making use of the non-dimensional quantities and $q = u + iv$ the flow governing equations assume the form

$$\text{Re} \frac{\partial q}{\partial \tau} - \beta \frac{\partial^3 q}{\partial z^2 \partial \tau} - \frac{\partial^2 q}{\partial z^2} + X_0 q = G_T T \cos \gamma + G_C C \cos \gamma - K_1^* f(\tau), \quad (7)$$

$$\text{RePr} \frac{\partial T}{\partial \tau} = \frac{\partial^2 T}{\partial z^2} - S T, \quad (8)$$

$$\text{ReSc} \frac{\partial C}{\partial \tau} = \frac{\partial^2 C}{\partial z^2} - K_2 C, \quad (9)$$

where

$$\left. \begin{array}{l} \text{Re} = U_0 z_0 / \nu_e, E = \Omega z_0^2 / \nu, \beta = \beta' / \rho z_0^2, M^2 = \sigma B_0^2 z_0^2 / \rho \nu_e, K = k' \nu_e / \nu_f z_0^2, \\ K_1^* = (K_1 M^2) / (\alpha_e^2 + \beta_e^2), X_0 = 2iE + (1/K) + (M^2(\alpha_e + i\beta_e) / (\alpha_e^2 + \beta_e^2)), \\ G_T = g \beta_T z_0^2 (T_2 - T_1) / \nu_e U_0, G_C = g \beta_C z_0^2 (C_2 - C_1) / \nu_e U_0, \\ \text{Pr} = \nu_e \rho C_p / k, S = S' z_0^2 / k, \text{Sc} = \nu_e / D, K_2 = K' z_0^2 / D. \end{array} \right\}$$

The non-dimensional conditions at the boundary walls of the channel are

$$\left. \begin{array}{l} \text{At } z = 1/2 : q = T = C = f(\tau) = 1 + \varepsilon \cos(\omega \tau). \\ \text{At } z = -1/2 : q = T = C = 0. \end{array} \right\} \quad (10)$$

In order to find the solution of the resulting equations, the fluid velocity, temperature and concentration are assumed as

$$q(z, \tau) = q_0(z) + (\varepsilon / 2)(q_1(z)e^{i\omega\tau} + q_2(z)e^{-i\omega\tau}), \quad (11)$$

$$T(z, \tau) = T_0(z) + (\varepsilon / 2)(T_1(z)e^{i\omega\tau} + T_2(z)e^{-i\omega\tau}), \quad (12)$$

$$C(z, \tau) = C_0(z) + (\varepsilon / 2)(C_1(z)e^{i\omega\tau} + C_2(z)e^{-i\omega\tau}), \quad (13)$$

Making use of equations (11)-(13) to resulting flow representing equations (7)-(9), yield

$$q_0'' - X_0 q_0 = -K_1^* - G_T T_0 \cos \gamma - G_C C_0 \cos \gamma, \quad (14)$$

$$(1 + i\omega \operatorname{Re} \beta) q_1'' - (X_0 + i\omega \operatorname{Re} \beta) q_1 = -K_1^* - G_T T_1 \cos \gamma - G_C C_1 \cos \gamma, \quad (15)$$

$$(1 - i\omega \operatorname{Re} \beta) q_2'' - (X_0 - i\omega \operatorname{Re} \beta) q_2 = -K_1^* - G_T T_2 \cos \gamma - G_C C_2 \cos \gamma. \quad (16)$$

$$T_0'' - S T_0 = 0, \quad (17)$$

$$T_1'' - (S + i\omega \operatorname{Re} \operatorname{Pr}) T_1 = 0, \quad (18)$$

$$T_2'' - (S - i\omega \operatorname{Re} \operatorname{Pr}) T_2 = 0, \quad (19)$$

$$C_0'' - K_2 C_0 = 0, \quad (20)$$

$$C_1'' - (K_2 + i\omega \operatorname{Re} \operatorname{Sc}) C_1 = 0, \quad (21)$$

$$C_2'' - (K_2 - i\omega \operatorname{Re} \operatorname{Sc}) C_2 = 0, \quad (22)$$

With the help of equations (11)-(13), the conditions at the boundary walls of the channel become

$$\left. \begin{aligned} \text{At } z = 1/2 : q_0 = q_1 = q_2 = 1, T_0 = T_1 = T_2 = 1, C_0 = C_1 = C_2 = 1. \\ \text{At } z = -1/2 : q_0 = q_1 = q_2 = 0, T_0 = T_1 = T_2 = 0, C_0 = C_1 = C_2 = 0. \end{aligned} \right\} \quad (23)$$

The solutions of equations (14)-(22) with the boundary conditions (23) are expressed as follows

$$q_i = \frac{\sinh(x_i(z + (1/2)))}{\sinh(x_i)} + \frac{K_1^*}{X_i} \left[1 - \frac{\cosh(x_i z)}{\cosh(x_i / 2)} \right] - \frac{G_{iT}}{(r_i^2 - x_i^2)} \left[\frac{\sinh(r_i(z + (1/2)))}{\sinh(r_i)} - \frac{\sinh(x_i(z + (1/2)))}{\sinh(x_i)} \right] - \frac{G_{iC}}{(s_i^2 - x_i^2)} \left[\frac{\sinh(s_i(z + (1/2)))}{\sinh(s_i)} - \frac{\sinh(x_i(z + (1/2)))}{\sinh(x_i)} \right], \quad i = 0, 1, 2, \quad (24)$$

$$T_i = \frac{\sinh(r_i(z + (1/2)))}{\sinh(r_i)}, \quad i = 0, 1, 2, \quad (25)$$

$$C_i = \frac{\sinh(s_i(z + (1/2)))}{\sinh(s_i)}, \quad i = 0, 1, 2, \quad (26)$$

where

$$\left. \begin{aligned} X_{1,2} &= X_0 \pm i\omega \text{Re}, \quad x_0 = \sqrt{X_0}, \quad x_{1,2} = \sqrt{\frac{X_0 \pm i\omega \text{Re}}{1 \pm i\omega \text{Re} \beta}}, \quad G_{0T} = G_T \cos \gamma, \\ G_{1T,2T} &= \frac{G_T \cos \gamma}{1 \pm i\omega \text{Re} \beta}, \quad G_{0C} = G_C \cos \gamma, \quad G_{1C,2C} = \frac{G_C \cos \gamma}{1 \pm i\omega \text{Re} \beta}, \quad r_0 = \sqrt{S}, \\ r_{1,2} &= \sqrt{S \pm i\omega \text{Re} \text{Pr}}, \quad s_0 = \sqrt{K_2}, \quad s_{1,2} = \sqrt{K_2 \pm i\omega \text{Re} \text{Sc}}. \end{aligned} \right\}$$

The solution for velocity field, fluid temperature and concentration can be obtained by substitution of q_i , T_i and C_i , $i = 0, 1, 2$ from equations (24)-(26) to the equations (11)-(13) respectively.

Skin friction coefficient at the stationary and moving boundary walls of the channel in the direction of main flow and normal to the main flow are expressed as

$$\left. \begin{aligned} \tau_1 &= \tau_{x1} + i\tau_{y1} = \left[q'_0 + (\varepsilon/2)(q'_1 e^{i\omega\tau} + q'_2 e^{-i\omega\tau}) \right]_{z=-1/2}, \\ \tau_2 &= \tau_{x2} + i\tau_{y2} = \left[q'_0 + (\varepsilon/2)(q'_1 e^{i\omega\tau} + q'_2 e^{-i\omega\tau}) \right]_{z=1/2}. \end{aligned} \right\} \quad (27)$$

The heat and mass transfer rates at the stationary and moving boundary walls of the channel is expressed as Nusselt and Sherwood numbers in the following form

$$\left. \begin{aligned} Nu_1 &= Nu_{10} + (\varepsilon/2)(Nu_{11} e^{i\omega\tau} + Nu_{12} e^{-i\omega\tau}), \\ Nu_2 &= Nu_{20} + (\varepsilon/2)(Nu_{21} e^{i\omega\tau} + Nu_{22} e^{-i\omega\tau}), \end{aligned} \right\} \quad (28)$$

$$\left. \begin{aligned} Sh_1 &= Sh_{10} + (\varepsilon/2)(Sh_{11} e^{i\omega\tau} + Sh_{12} e^{-i\omega\tau}), \\ Sh_2 &= Sh_{20} + (\varepsilon/2)(Sh_{21} e^{i\omega\tau} + Sh_{22} e^{-i\omega\tau}), \end{aligned} \right\} \quad (29)$$

where

$$\left. \begin{aligned} Nu_{1i} &= r_i / \sinh(r_i), \quad Nu_{2i} = r_i \cosh(r_i) / \sinh(r_i), \\ Sh_{1i} &= s_i / \sinh(s_i), \quad Sh_{2i} = s_i \cosh(s_i) / \sinh(s_i), \quad i = 0, 1, 2. \end{aligned} \right\}$$

Analysis of the numerical results

In order to examine the flow behaviour corresponds to various flow controlling parameters, we performed the numerical computation and results are demonstrated through graphs and tables. For the computational purpose, the default values of flow controlling parameters are fixed as $\beta_e = 0.5$, $\beta_i = 0.5$, $E = 1$, $M^2 = 9$, $K = 0.3$, $\gamma = \pi/6$, $\beta = 0.25$, $\omega = 3$, $G_T = 2$, $G_C = 3$, $\text{Re} = 2$, $\text{Pr} = 0.71$, $S = 2$, $\text{Sc} = 0.22$, $K_2 = 0.2$, $K_1^* = 1$ and $\omega\tau = \pi/2$. Figures 2-10 demonstrate the velocity distribution while Figures 11 and 12 are respectively representing the temperature and concentration profiles. The skin friction coefficient and heat and mass transfer rates at the channel walls are tabulated in tables 1 and 2.

Velocity field

Figures 2 and 3 are respectively representing the responses of Hall and ion-slip currents on the velocity distributions. Growing values of Hall current bring increment in the fluid flow. This is because it is generated due to spiralling of electrons around magnetic field lines. The

nature of this current is to generate more flow in the direction normal to the main flow. Ion-slip current has similar behaviour as that of Hall current on the main flow while opposite behaviour on the flow perpendicular to the main flow. This current is generated when diffusion velocity of ions is not neglected in comparison to that of electrons because the ions are heavier particles than the electrons. In the absence of Hall current there is no effect of ion-slip current on the fluid flow. Action of rotation on the flow is displayed in Figure 4. Rotation leads to induce the flow in the direction normal to the main flow while it brings the rigidity in the main flow. This due to the reason that rotation produce Coriolis force in the direction normal to the main flow. Magnetic field and permeability of the porous regime effects are respectively shown in the Figures 5 and 6. Both the magnetic force and permeability of the porous regime lead to generate a drag force which resists the fluid flow. The graph of velocity showing the consequence of angle of inclination is represented in Figure 7. Rising values of angle of inclination leads to decrement in the fluid flow i.e. there is more flow in a vertical channel in comparison to an inclined channel. Figures 8 and 9 respectively plot the behaviour of fluid flow for viscoelastic and frequency parameters. Drop in velocity is observed on rising the elastico-viscous nature of the fluid while grow in velocity is seen on rising the oscillation frequency. The effects of viscoelastic parameter on the flow behaviour in the cases of both the moving and stationary magnetic fields are exhibited in the Figure 10. It is revealed from this figure that moving magnetic field produces less rigidity in the flow in compare to the stationary magnetic field.

Temperature and concentration

It is noticed from the Figures 11 and 12 that increment in the fluctuation frequency of temperature and concentration of the moving wall of the channel leads to raise the fluid temperature and concentration.

Quantities of physical interest

The change in skin friction coefficients at the boundary walls of the channel corresponds to different values of flow influencing parameters are tabulated in table 1. It can be easily seen from table 1 that in the direction of the main flow, Hall and ion-slip currents and frequency of oscillations of the moving wall lead to an increment in the skin friction coefficient at the stationary wall while rotation, magnetic and Darcian drag forces, angle of inclination and viscoelastic parameter bring a decrement in the skin friction coefficient at the stationary wall. In the direction normal to the main flow, Hall current, rotation and frequency of oscillations of the moving wall of the channel grow the skin friction coefficient at the stationary wall while ion-slip current, magnetic and Darcian drag forces, angle of inclination and viscoelastic parameter tend to bring the fall in the skin friction coefficient at the stationary wall. At the moving wall, rotation, magnetic and Darcian drag forces, angle of inclination and viscoelastic parameter lead to grow the skin friction coefficient in the direction of main flow while Hall and in-slip currents and frequency of oscillation of the moving wall lead to fall in the skin friction coefficient in the direction of main flow. In the direction perpendicular to the main flow, Hall current, rotation, magnetic drag force and frequency of oscillations of the moving wall tend to raise the skin friction coefficient while ion-slip current, Darcian drag force, angle of inclination and viscoelastic parameter bring fall in the the skin friction coefficient at the moving wall. Table 2 reflects that the growing values of fluctuation frequency of temperature of the moving wall of the channel bring enhancement in the heat transfer rate at the stationary wall of the channel. This nature becomes opposite at the moving wall of the channel. Fluctuation frequency of concentration of the moving wall of the channel leads to an increment in the mass transfer rate at the stationary wall of the channel while this effect overturned at the moving wall of the channel.

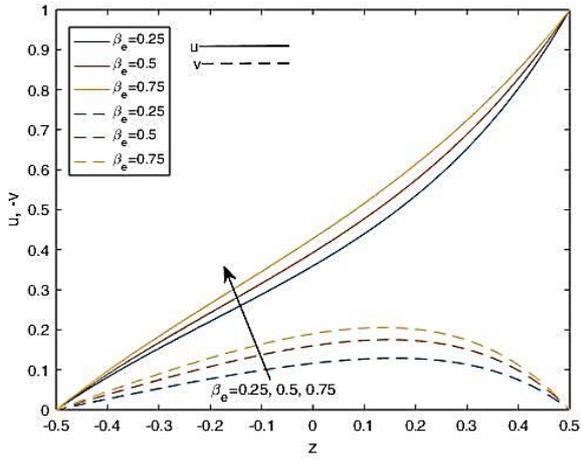


Figure 2. Velocity distribution for β_e .

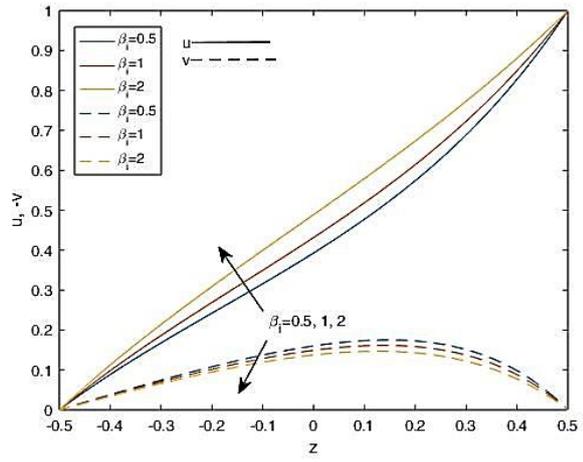


Figure 3. Velocity distribution for β_l .

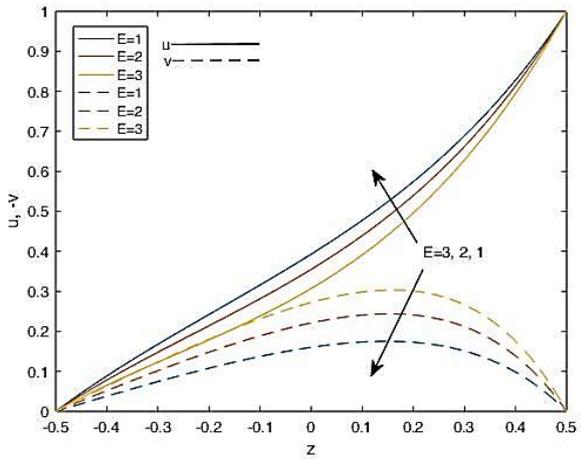


Figure 4. Velocity distribution for E .

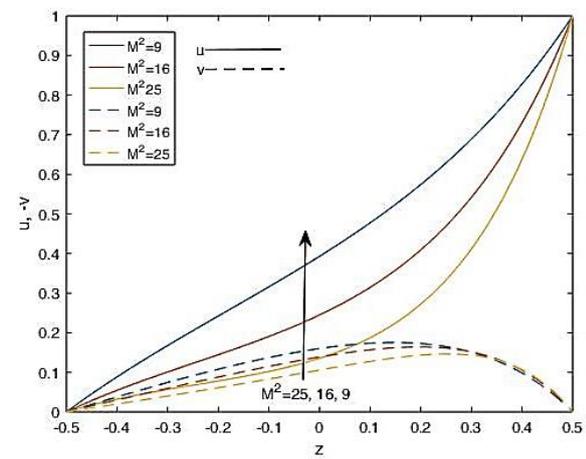


Figure 5. Velocity distribution for M^2 .

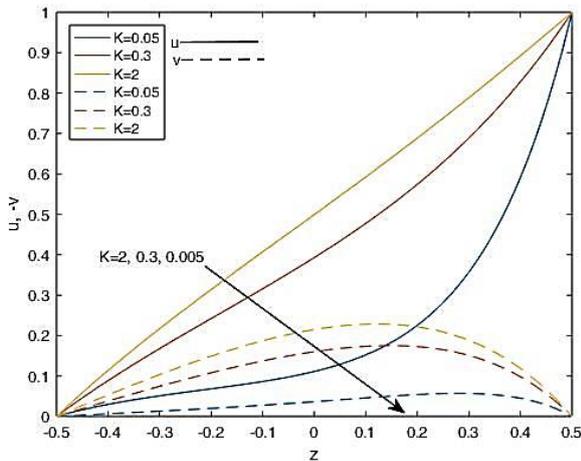


Figure 6. Velocity distribution for K .

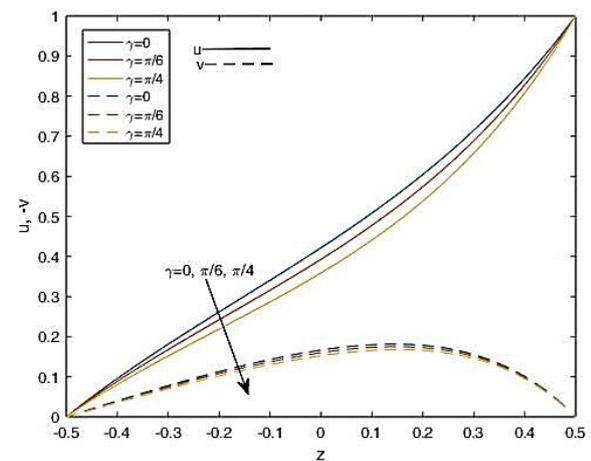


Figure 7. Velocity distribution for γ .

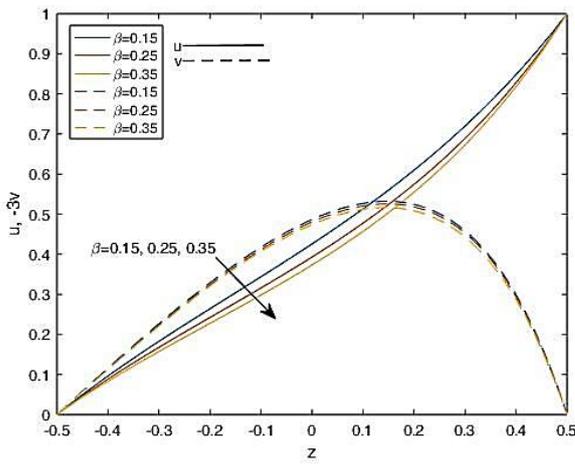


Figure 8. Velocity distribution for β .

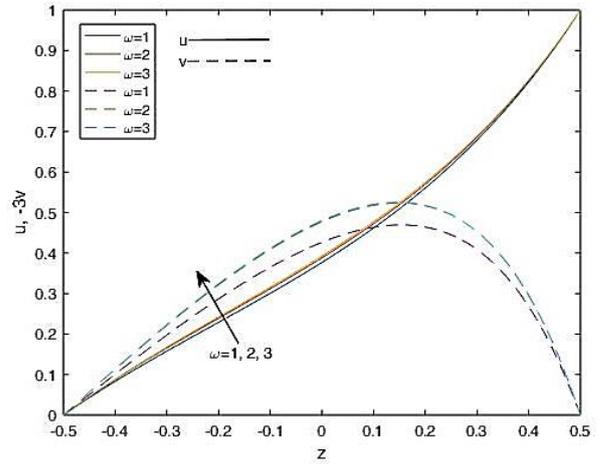
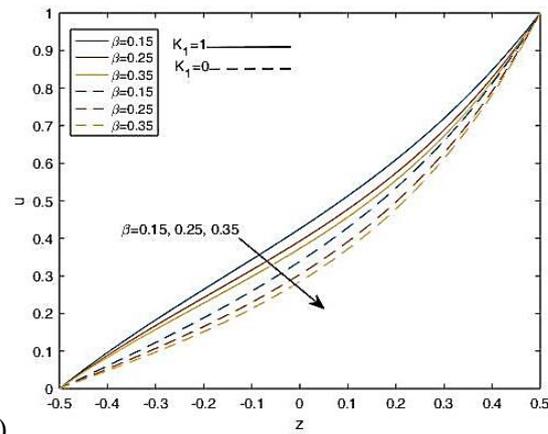
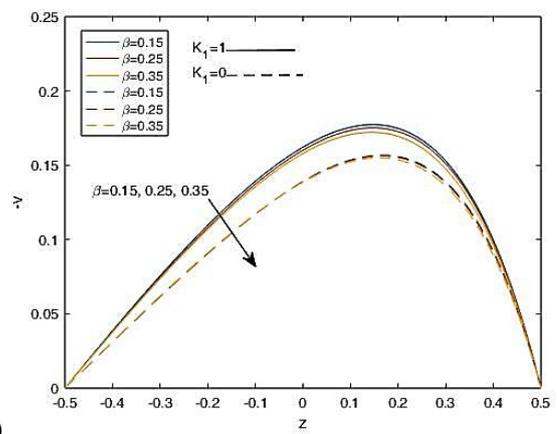


Figure 9. Velocity distribution for ω .



(a)



(b)

Figure 10. (a) Primary and (b) secondary velocity distribution for β .

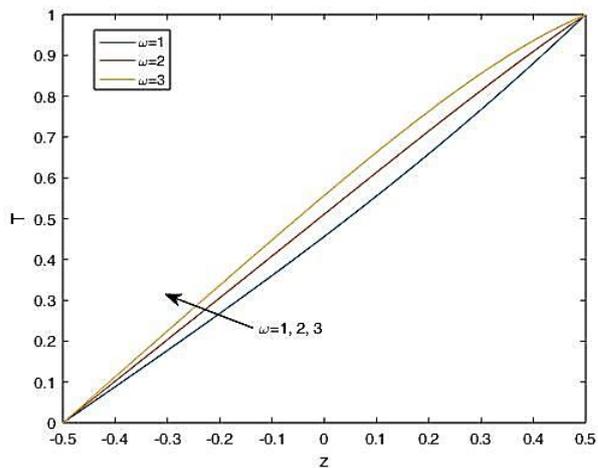


Figure 11. Temperature distribution for ω .

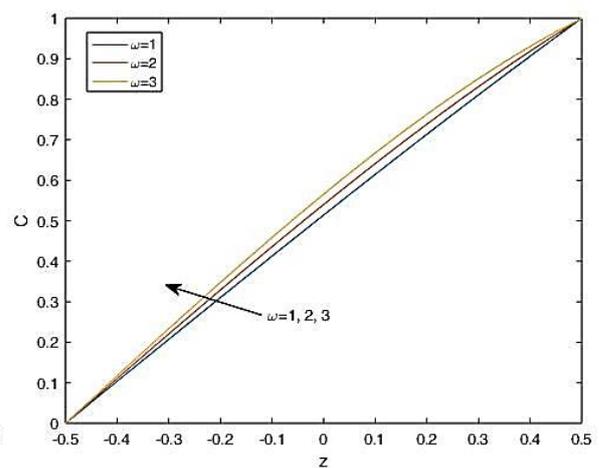


Figure 12. Concentration distribution for ω .

Table 1. Skin friction coefficient at the channel walls.

β_e	β_i	E	M^2	K	γ	ω	β	τ_{x1}	$-\tau_{y1}$	τ_{x2}	τ_{y2}
0.5	0.5	1	9	0.3	$\pi/6$	3	0.25	0.9490	0.3817	1.9181	1.2721
0.25	0.5	1	9	0.3	$\pi/6$	3	0.25	0.8760	0.2717	2.2528	0.9836
0.75	0.5	1	9	0.3	$\pi/6$	3	0.25	1.0291	0.4603	1.6115	1.4288
0.5	1	1	9	0.3	$\pi/6$	3	0.25	1.0445	0.3611	1.6489	1.1305
0.5	2	1	9	0.3	$\pi/6$	3	0.25	1.1873	0.3392	1.2559	0.9761
0.5	0.5	2	9	0.3	$\pi/6$	3	0.25	0.8450	0.5212	2.0706	1.8042
0.5	0.5	3	9	0.3	$\pi/6$	3	0.25	0.7153	0.6318	2.2657	2.3023
0.5	0.5	1	16	0.3	$\pi/6$	3	0.25	0.6080	0.3025	3.2020	1.4487
0.5	0.5	1	25	0.3	$\pi/6$	3	0.25	0.3856	0.2007	4.5130	1.6080
0.5	0.5	1	9	0.05	$\pi/6$	3	0.25	0.3590	0.0639	5.3063	0.7119
0.5	0.5	1	9	2	$\pi/6$	3	0.25	1.2021	0.5341	1.0878	1.4848
0.5	0.5	1	9	0.3	0	3	0.25	1.0176	0.3994	1.7077	1.2970
0.5	0.5	1	9	0.3	$\pi/4$	3	0.25	0.8675	0.3607	2.1677	1.2425
0.5	0.5	1	9	0.3	$\pi/6$	2	0.25	0.9392	0.3791	1.9378	1.2829
0.5	0.5	1	9	0.3	$\pi/6$	1	0.25	0.8912	0.3384	1.9739	1.1699
0.5	0.5	1	9	0.3	$\pi/6$	3	0.35	0.8901	0.3777	2.0362	1.2324
0.5	0.5	1	9	0.3	$\pi/6$	3	0.15	1.0273	0.3886	1.6580	1.2919

Table 2. Heat and mass transfer rates at the channel walls.

ω	Nu_1	Nu_2	Sh_1	Sh_2
1	0.8817	1.2193	1.0373	0.9231
2	1.0170	0.8631	1.1062	0.7814
3	1.1253	0.5358	1.1732	0.6416

CONCLUSIONS

In this paper, MHD free convective flow of a viscoelastic fluid through a porous regime in an inclined channel with a moving magnetic field is modelled and mathematically analysed. The consequences of various flow influencing parameters are numerically analysed. The novel results presented may find significant applications in chemical manufacturing industries, biofluid dynamics etc. Some important observations are summarised below:

- Both the Hall current and ion-slip currents brought increment in the main flow while on the flow perpendicular to the main flow their nature is opposite. Hall current generates the flow in the direction perpendicular to the main flow while ion-slip current reduces the flow in the direction perpendicular to the main flow.
- An increment in the angle of inclination leads to decrement in the fluid flow i.e. there is more flow in a vertical channel in comparison to an inclined channel.
- Fall in flow is observed on raising the viscoelastic nature of the fluid while rise in flow is seen for raising values of the oscillation frequency.
- It is noticed that the moving magnetic field produces less rigidity in the flow in comparison to the stationary magnetic field.

Nomenclature

B_0	applied magnetic field (T)
C	non-dimensional species concentration

C'	species concentration (mol / m^3)
C_1	concentration at left wall (mol / m^3)
C_2	a constant concentration (mol / m^3)
C_p	specific heat at constant pressure ($J / kg.K$)
D	chemical molecular diffusivity (m^2 / s)
E	rotation parameter
g	acceleration due to gravity (m / s^2)
G_C	solatal Grashof number
G_T	thermal Grashof number
k	thermal conductivity of the fluid ($W / m.K$)
K	permeability parameter
k'	permeability (m^2)
K'	chemical reaction constant
K_2	chemical reaction parameter
M	magnetic parameter
Pr	Prandtl number
Re	Reynolds number
S	heat source parameter
Sc	Schmidt number
t'	time (s)
T	non-dimensional fluid temperature
T'	fluid temperature (K)
T_1	Temperature at left wall (K)
T_2	a constant temperature (K)
U_0	characteristic velocity (m / s)
(u', v', w')	velocity components along coordinate axes (m / s)
(x', y', z')	rectangular Cartesian coordinates

Greek Symbols

β	viscoelastic parameter
β'	viscoelastic constant
β_C	volumetric coefficient of concentration expansion
β_e	Hall parameter
β_i	ion-slip parameter
β_T	volumetric coefficient of thermal expansion (K^{-1})
γ	angle of inclination
ν_e	effective coefficient of viscosity (m^2 / s)
ν_f	coefficient of viscosity (m^2 / s)
Ω	angular velocity
ω'	frequency of oscillation
ω	frequency parameter (s^{-1})
ρ	fluid density (kg / m^3)

σ	electrical conductivity (S / m)
τ	non-dimensional time.

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