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Original scientific paper

BUOYANCY DRIVEN FLOW OF A REACTIVE HYDROMAGNETIC HEAT GENERATING FLUID THROUGH A POROUS CHANNEL

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ABSTRACT. The survey of buoyancy driven flow of a reactive hydromagnetic Poiseuille fluid flowing within porous channel with respect to a heat source produced internally is investigated. The coupled differential equations regulating the fluid flow are simplified by seeking the use of modified Adomian decomposition method (MADM) which is later presented in tables and graphs to show the effects of buoyancy force and various parameters included in the flow regime.

Keywords: Buoyancy force, reactive fluid, porous plates and modified Adomian decomposition method *(MADM).*

INTRODUCTION

Significant interest from researchers to investigate the reactive hydromagnetic fluid flow has been on the increase due to its broad demands in design engineering and industrial phenomenon, such as petroleum industries, chemical engineering, polymer extrusion, etc., as mentioned (MAKINDE and BEG, 2010; HASSAN and GBADEYAN, 2014; 2015a,b; HASSAN and MARITZ, 2016a,b). Reactive hydromagnetic fluid flow is mainly connected with heat transfer in many engineering applications, most especially, when the fluid flows within porous media like in the transportation and purification of petroleum products, lubrication, power generators and pumps, to mention a few, as expressed in BEAR (1972), BADRUDDIN *et al.* (2006), ATTIA (2007) and HASSAN and MARITZ (2016 b). In many aspects of these studies, various physical aspects order the fluid behaviour flowing through a porous medium which measures the capacity and ability of the formation to transmit fluid. For instance, HASSAN *et al*. (2017) recently examined the survey of a radiative energy transfer of a reactive hydromagnetic fluid flowing in the midst of parallel porous plates with convective boundary status obeying the cooling law of Newton.

Moreover, the combination of different properties of basic heat transfer and fluid flow mechanisms show the practical and scientific relevance in hydromagnetic flows and energy transfer which has further change significantly in recent years because of numerous essential procedures in engineering and industries. In specific conditions, agents dealing with chemical compositions and separate fluids, heat sink or source may become necessary which at the

same time may be accounted for by adding its effect to the energy equation. In addition to that, buoyancy effect can become more stressed in fluid motion and energy transfer over a reactive fluid flow due to its nature to either increase the speed of the fluid or a rise in the temperature of the fluid emission to the layers.

Meanwhile, when fluid experiences a gravitational force and variations in density caused by temperature gradient in free convective flow, then a buoyancy force is produced and cannot be neglected. To show the significant effect of buoyancy force, CHINYOKA and MAKINDE (2015) examined the compound reaction of buoyancy force and asymmetric convective cooling on unsteady magnetohydrodynamic boundary layer flow and thermal energy in a reactive third grade fluid. Also, MAKINDE (2006) analysed the effects of thermal buoyancy on the boundary layer flow over a vertical plate with convective surface boundary conditions. In addition to that, MALIK *et al*. (2013) presented the measure of the buoyancy driven flow in between a bottom - heated vertical coordinated cylindrical enclosures. Other related studies showing the importance of studying the effect of buoyancy force can be found in existing literature (EPSTEIN, 1988; COOPER, 1995; INATOMI, 2006; RAPPOLDT *et al.*, 2003*;* MAKINDE and CHINYOKA, 2013; FENUGA *et al.*, 2015).

Accordingly, the current survey intends to extend the recent work of HASSAN and MARITZ (2016a) and HASSAN *et al.* (2017), in order to analyse the striking effect of buoyancy force and porosity on a reactive hydromagnetic heat source of fluid flow with parallel fixed plates with respect to Arrhenius chemical kinetics which was not accounted for in the previous study. The importance in several engineering and industrial applications has already been highlighted above. The problem is strongly nonlinear with coupled differential equations governing the momentum and energy distributions obtained using a rapidly convergent modified Adomian decomposition method (MADM). The proposed modification method is seen to be more reliable if analysed with the well-known Adomian decomposition method (ADM). The major progress is presented in WAZWAZ (1999), WAZWAZ and SAYED (2001), BABOLIAN and BIAZAR (2002) and RAY (2014), as the series converge with fewer iterations. The effects of the Grashof number (Gr), which is a function of buoyancy force and Darcy's porous permeability coefficients on momentum and energy distributions, together with entropy production and Bejan number are considered and demonstrated.

MATHEMATICAL FORMULATION

Taking into account the steady flow of an incompressible and electrically conducting reactive fluid driven by buoyancy force running through in between parallel porous plates located at $y = -a$ and $y = a$ as shown in figure 1.

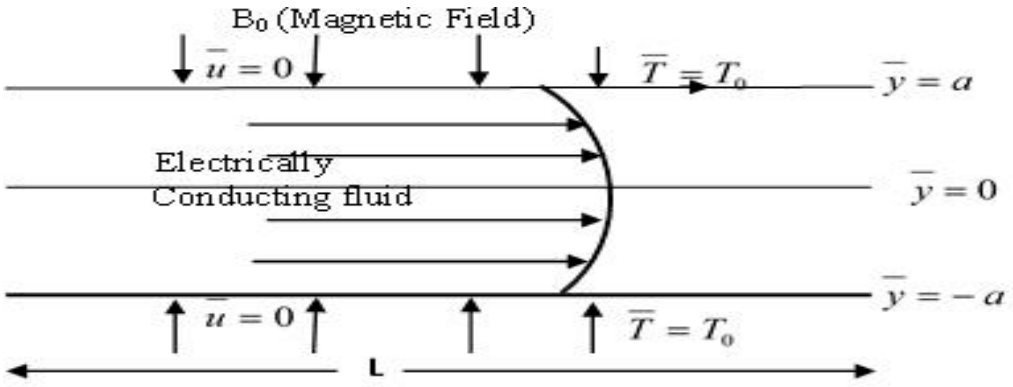

Figure 1: Schematic diagram of the problem.

Both plates are fixed with a process taking place at constant wall temperature (T_0) subject to the impact of a transverse magnetic terrain (B_0). The internal heat expression is assumed to be a linear relation of temperature. Under the foregoing assumptions and ignoring the dissipation of the reactant, the governing boundary layer equations as recorded in MAKINDE and BEG (2010), HASSAN and GBADEYAN (2014, 2015a) and HASSAN and MARITZ (2016) gives on Figure 1.

In momentum equation (1) each member corresponds to the following transports: pressure, viscous, magnetic, porosity and buoyancy drive while the energy equation (2) respectively comprises: heat transfer, viscous dissipation, magnetic, porosity, reactivity and internal heat source.

$$-\frac{\mathrm{d}\bar{p}}{\mathrm{d}\bar{x}}+\mu\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}\bar{y}^2}-\sigma {B_0}^2\bar{u}-\frac{\mu}{K}\bar{u}+\rho g\alpha(\bar{T}-T_0)=0 \tag{1}$$

$$k\frac{\mathrm{d}^2\bar{T}}{\mathrm{d}\bar{y}^2}+\mu\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}\bar{y}}\right)^2+\sigma {B_0}^2\bar{u}^2+\frac{\mu}{K}\bar{u}^2+QC_0Ae^{-\frac{E}{R\bar{T}}}+Q_0(\bar{T}-T_0)=0 \tag{2}$$

The flow is regular about the vertical *x – axis* with the matching boundary conditions along the channel centreline are given as:

$$\frac{\mathrm{d}\bar{u}}{\mathrm{d}\bar{y}}=\frac{\mathrm{d}\bar{T}}{\mathrm{d}\bar{y}}=0 \ \text{ on } \ \bar{y}=0 \ \text{ and } \ \bar{u}=\bar{T}=0 \ \text{ on } \ \bar{y}=\pm a\,. \tag{3}$$

Furthermore, the regular equation for the entropy production per unit volume accompanied with magnetic field strength and porous medium is hereby stated thus:

$$S^m=\frac{k}{T_0^2}\left(\frac{\mathrm{d}\bar{T}}{\mathrm{d}\bar{y}}\right)^2+\frac{\mu}{T_0}\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}\bar{y}}\right)^2+\frac{\sigma B_0^2\bar{u}^2}{T_0}+\frac{\mu\bar{u}^2}{KT_0} \tag{4}$$

here $\bar{p}$ stands for pressure, $\bar{u}$ represents the fluid velocity and μ stands for fluid viscosity. Furthermore, σ stands for electrical conductivity, ρ is the density estimated at the mean temperature, where g is the gravitational constant, α denotes the coefficient of thermal expansion, k represents the thermal conductivity coefficient, $\bar{T}$ is the fluid temperature and Q is the heat of the reaction term. Moreover, C_0, A, E, R, Q_0, K and T_0 respectively represent the reactant species initial concentration, reaction rate constant, activation energy, the universal gas constant, the dimensional heat generation coefficient, the Darcy permeability coefficient and the wall temperature. Lastly, S^m represents the entropy generation number in non - dimensionless form. Convincingly, let it be known that the last term in equation (1) is the additional term to extend the study of HASSAN and MARITZ, 2016 a; in examining the impact of buoyancy force as in MAKINDE and CHINYOKA, 2013 and the fourth term in both equations (1) and (2) is to analyse the effectiveness of the porosity due to the similar modelling done in MAKINDE, 2006 and HASSAN and MARITZ, 2016 b.

Bringing up the subsequent non -dimensional measures:

$$y=\frac{\bar{y}}{a},\ x=\frac{\bar{x}}{a},\ u=\frac{\bar{u}}{U},\ T=\frac{E(\bar{T}-T_0)}{R{T_0}^2},\ Br=\frac{E\mu U^2}{kR{T_0}^2},\ \Omega=\varepsilon=\frac{RT_0}{E},$$

$$\gamma=\frac{\mu U^2}{QAa^2C_0}e^{\frac{E}{RT_0}},\ H^2=\frac{\sigma {B_0}^2a^2}{\mu},\ G=-\frac{dp}{dx},\ p=\frac{a\bar{p}}{\mu U},\ \delta=\frac{a^2}{K}$$

$$Gr = \frac{\rho g \alpha a^2}{U\mu}\left(\frac{RT_0^2}{E}\right),\ \lambda = \frac{QEAa^2C_0}{kRT_0^2}e^{-\frac{E}{RT_0}} \text{ and } \beta = \frac{Q_0RT_0^2}{QEAC_0}e^{\frac{E}{RT_0}}. \quad (5)$$

Therefore, the dimensionless regulating equations for the momentum and energy with appropriate boundary conditions are written as follows:

$$\frac{d^2u}{dy^2} + G - (H^2 + \delta)u + GrT = 0 \quad (6)$$

$$\frac{d^2T}{dy^2} + \lambda\left[e^{\frac{T}{1+\varepsilon T}} + \gamma\left(\left(\frac{du}{dy}\right)^2 + (H^2 + \delta)u^2\right) + \beta T\right] = 0 \quad (7)$$

subject to the boundary conditions

$$\frac{du}{dy} = \frac{dT}{dy} = 0 \text{ on } y = 0 \text{ and } u = T = 0 \text{ on } y = \pm 1 \quad (8)$$

Also, the expression for the entropy production number in dimensionless state using the existing dimensionless variables and parameter is given as:

$$N_s = \frac{S^m a^2 E^2}{kR^2T_0^2} = \left(\frac{dT}{dy}\right)^2 + \frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2 + (H^2 + \delta)u^2\right] \quad (9)$$

here, G stands for pressure gradient and U is the mean velocity. Also, Gr, H, and Br are numbers for Grashof, Hartmann and Brinkman. In addition to that $\lambda, \varepsilon, \gamma, \beta, \delta$ and Ω are respectively parameters for critical explosion named after Frank-Kamenettski, activation energy, viscous heating, heat source, porous medium permeability and the wall temperature. Also, the entropy production rate in dimensionless form is represented with N_s. The physical significance of the dimensionless parameters and their range of variation of nonlinear terms are taken to be very small because of the uniqueness and the nature of flow regime.

METHOD OF SOLUTION

The fluid velocity and energy equations are couple equations that need to be integrated twice to obtain the following:

$$u(y) = a_0 - \frac{Gy^2}{2} + (H^2 + \delta)\int_0^y\int_0^y u(y)dYdY - Gr\int_0^y\int_0^y T(y)dYdY \quad (10)$$

$$T(y) = b_0 - \lambda\int_0^y\int_0^y\left[e^{\frac{T}{1+\varepsilon T}} + \gamma\left(\left(\frac{du}{dy}\right)^2 + (H^2 + \delta)u^2\right) + \beta T\right]dYdY \quad (11)$$

where a_0 and b_0 are respectively equal to $u(0)$ and $T(0)$ to be determined by other boundary conditions stated in equation (8). In order to find the solutions of the coupled equations (10) and (11), we appropriate an infinite series results in the form of:

$$u(y) = \sum_{n=0}^{\infty} u_n(y) \text{ and } T(y) = \sum_{n=0}^{\infty} T_n(y) \quad (12)$$

with the series solution (12) in (10) and (11), we have,

$$u(y) = a_0 - \frac{Gy^2}{2} + (H^2 + \delta)\int_0^y\int_0^y \sum_{n=0}^{\infty} u_n(y)dYdY - Gr\int_0^y\int_0^y \sum_{n=0}^{\infty} T_n(y)dYdY \tag{13}$$

$$T(y) = b_0 - \lambda\int_0^y\int_0^y \left[e^{\frac{\sum_{n=0}^{\infty} T_n(y)}{1+\varepsilon(\sum_{n=0}^{\infty} T_n(y))}} + \gamma\left(\left(\frac{d(\sum_{n=0}^{\infty} u_n(y))}{dy}\right)^2 + (H^2+\delta)(\sum_{n=0}^{\infty} u_n(y))^2\right) + \beta(\sum_{n=0}^{\infty} T_n(y))\right] dYdY \tag{14}$$

We let

$$\sum_{n=0}^{\infty} A_n(y) = e^{\frac{\sum_{n=0}^{\infty} T_n(y)}{1+\varepsilon(\sum_{n=0}^{\infty} T_n(y))}},\ \sum_{n=0}^{\infty} B_n(y) = \left(\frac{d(\sum_{n=0}^{\infty} u_n(y))}{dy}\right)^2 \text{ and } \sum_{n=0}^{\infty} C_n(y) = \left(\sum_{n=0}^{\infty} u_n(y)\right)^2 \tag{15}$$

where the respective components A_0, A_1, A_2, …, B_0, B_1, B_2, …, and C_0, C_1, C_2, …, are called Adomian polynomials. With that, (15) is thereby amplified in a manner that:

$$\begin{aligned}
A_0 &= e^{\frac{T_0(y)}{\epsilon T_0(y)+1}},\\
A_1 &= \frac{T_1(y)e^{\frac{T_0(y)}{\epsilon T_0(y)+1}}}{(\epsilon T_0(y)+1)^2},\\
A_2 &= \frac{e^{\frac{T_0(y)}{\epsilon T_0(y)+1}}\left(T_1(y)^2\left(-2\epsilon^2 T_0(y) - 2\epsilon + 1\right) + 2T_2(y)(\epsilon T_0(y)+1)^2\right)}{2(\epsilon T_0(y)+1)^4}, \ldots,\\
B_0 &= u_0'(y)^2, \qquad B_1 = 2u_0'(y)u_1'(y), \qquad B_2 = u_1'(y)^2 + 2u_0'(y)u_2'(y), \ldots,\\
C_0 &= u_0(y)^2, \qquad C_1 = 2u_0(y)u_1(y), \qquad C_2 = u_1(y)^2 + 2u_0(y)u_2(y), \ldots,
\end{aligned} \tag{16}$$

Therefore, the velocity and momentum equations (13) and (14) are reduced to:

$$u(y) = a_0 - \frac{Gy^2}{2} + (H^2 + \delta)\int_0^y\int_0^y \sum_{n=0}^{\infty} u_n(y)dYdY - Gr\int_0^y\int_0^y \sum_{n=0}^{\infty} T_n(y)dYdY \tag{17}$$

$$T(y) = b_0 - \lambda\int_0^y\int_0^y \left[\sum_{n=0}^{\infty} A_n(y) + \gamma\left((\sum_{n=0}^{\infty} B_n(y)) + (H^2+\delta)(\sum_{n=0}^{\infty} C_n(y))\right) + \beta(\sum_{n=0}^{\infty} T_n(y))\right] dYdY \tag{18}$$

Then, respective iterative function with the zeroth portion as previously specified in WAZWAZ, 1999; WAZWAZ and SAYED, 2001; BABOLIAN and BIAZAR, 2002; RAY, 2014 as:

$$u_0 = a_0 - \frac{Gy^2}{2} \qquad T_0 = 0 \tag{19}$$

$$u_1 = (H^2+\delta)\int_0^y\int_0^y u_0(y)dYdY - Gr\int_0^y\int_0^y T_0(y)dYdY$$

$$T_1 = b_0 - \lambda\int_0^y\int_0^y\left[A_0(y)+\gamma\left((B_0(y))+(H^2+\delta)(C_0(y))\right)+\beta(T_0(y))\right]dYdY \tag{20}$$

$$u_{n+1} = (H^2+\delta)\int_0^y\int_0^y u_n(y)dYdY - Gr\int_0^y\int_0^y T_n(y)dYdY$$

$$T_{n+1} = -\lambda\int_0^y\int_0^y\left[A_n(y)+\gamma\left((B_n(y))+(H^2+\delta)(C_n(y))\right)+\beta(T_n(y))\right]dYdY,\ n\geq 1 \tag{21}$$

Equations (19) – (21) are then programmed in the application package to derive the approximate solutions adopted and deliberated upon in the subsequent sections as:

$$u(y)=\sum_{n=0}^{m}u_n(y) \quad \text{and} \quad T(y)=\sum_{n=0}^{m}T_n(y) \tag{22}$$

However, in order to resolve the entropy production, we let the first term of N_s in equation (9) ,that is, $\left(\frac{dT}{dy}\right)^2$ be assigned N_1 which is the irreversibility with respect to energy transfer while the second term $\frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2+(H^2+\delta)u^2\right]$ referred to as N_2 is the confined entropy production due to the compound consequences of the viscous distribution, the magnetic field and porosity of the flow system. Moreover, the irreversibility dispersion is defined as ϕ and is given as:

$$\phi = \frac{N_2}{N_1} \tag{23}$$

which indicates that energy transfer exercises control over the fluid friction when $0\leq\phi<1$ and otherwise when $\phi > 1$. Importantly, this is used to reach a decision on safety measures especially in numerous engineering schemes. Alternatively, another option known to determine the irreversibility distribution ratio is named Bejan number *Be*, which can be stated as:

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\phi} \quad where \quad 0\leq Be\leq 1. \tag{24}$$

RESULTS AND DISCUSSION

This part presents the results of the impact of buoyancy on a reactive hydromagnetic fluid flowing in between parallel porous channels with constant wall temperature. The fluid is under the influence of internal heat source accompanied with other significant flow variables. Interestingly, our new result shall be equivalent to that of HASSAN and MARITZ, 2016 a; when the porous medium permeability parameter (γ) and buoyancy effect parameter known as Grashof number (Gr) are both zero to validate the solutions obtained.

Table 1 displays the rapid convergence of the series solution for the constants a_0 and b_0 in equations (10) and (11) which shows the efficiency and reliability of the modified ADM. It really signifies that the series converge with sizeable iterations.

Table 1: Rapid convergence of the series solution for the constants a_0 and b_0

$[G = H = 1, \varepsilon = 0.1, \beta = \delta = Gr = 0.5]$

n	a_0	b_0
0	0. 5	0
1	0. 321429	0.256657
2	0.352677	0.381090
3	0. 356170	0.389705
4	0. 355650	0.386856
5	0.355642	0.386840
6	0.355650	0.386899
7	0.355650	0.386899
8	0.355650	0.386899

Table 2 indicates the evaluation of numerical results of temperature distributions between the previous results in HASSAN and MARITZ, 2016 a, where Adomian decomposition method (ADM) and the new result from modified Adomian decomposition method (MADM) were used. The buoyancy effect parameter (Gr) and porous medium permeability parameter (δ) are both zero in the previously obtained result in HASSAN and MARITZ, 2016 a, showing the efficiency and the reliability of the new method. The absolute error is with average order of 10^{-3} which is due to the variations in the number of iterations done in both results.

Table 2: Comparison of numerical results of the temperature profile.

$[G = H = \varepsilon = \gamma = 1, \lambda = \beta = 0.5, \delta = G_r = 0]$

y	$T(y)$ **ADM** **[**HASSAN and MARITZ, 2016a**]**	$T(y)$ **MADM**	**Absolute Error**
-1	0.0003681026	-1.50704×10^{-17}	3.68103×10^{-4}
-0.75	0.1771365903	0.1757301606	1.40643×10^{-3}
-0.50	0.3046016196	0.3028513826	1.75024×10^{-3}
-0.25	0.3819272135	0.3800505753	1.87664×10^{-3}
0	0.4078774354	0.4059636879	1.91375×10^{-3}
0.25	0.3819272135	0.3800505753	1.87664×10^{-3}
0.50	0.3046016196	0.3028513826	1.75024×10^{-3}
0.75	0.1771365903	0.1757301606	1.40643×10^{-3}
1	0.0003681026	-1.50704×10^{-17}	3.68103×10^{-4}

Figures 2 and 3 respectively display the buoyancy and porosity effects on the velocity distribution. On a general note, the highest limit of velocity is attained at the centreline within the plates. The effect of buoyancy is significant on the fluid flow in figure 2, as it increases the fluid velocity which implies that, the greater the buoyancy force, the faster

the motion of the fluid. While it is clearly noticed in figure 3 that the fluid motion reduces with rising values of porous term (δ) which is due to the retarding effects of porosity in nature and magnetic force present in the flow channel.

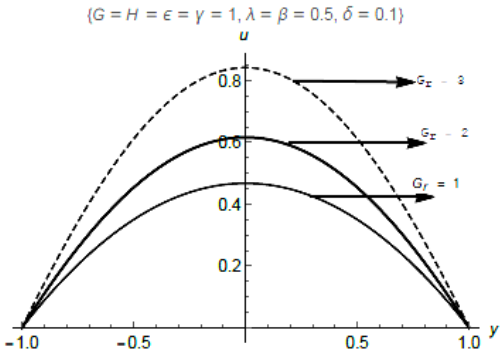

Figure 2: Effects of Gr on $u(y)$

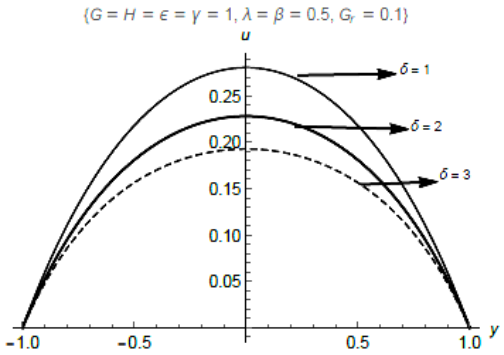

Figure 3: Effects of δ on $u(y)$

The temperature profiles for variations in the Grashof number (Gr) and porosity permeability term (δ) are respectively depicted in figures 4 and 5. The effect of buoyancy force is noticed in figure 4 and it shows that the rising temperature occurs due to increase in buoyancy force in the presence of internal energy produced during fluid interactions while on the other hand, in figure 5, the plot clearly shows a reduction in temperature as the porosity term increases which is due to the high presence of Darcy permeability content in the fluid flow, hence resulting in a reduction in temperature.

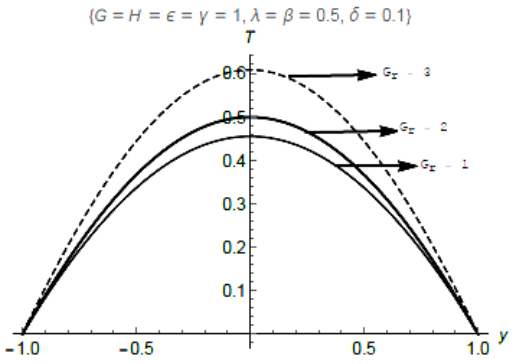

Figure 4: Effects of Gr on $T(y)$

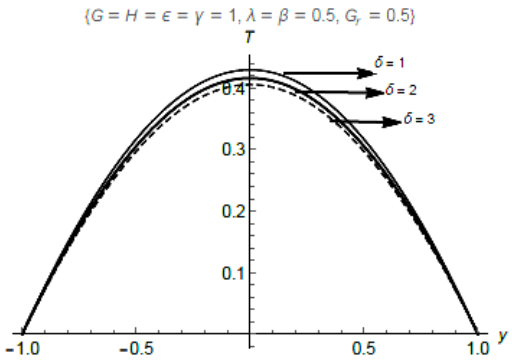

Figure 5: Effects of δ on $T(y)$

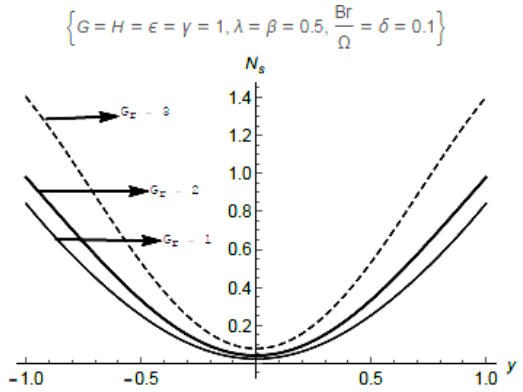

Figure 6: Effects of Gr on N_s

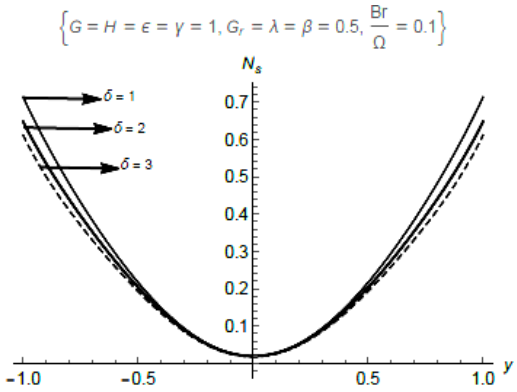

Figure 7: Effects of δ on N_s

Figures 6 and 7 display results for entropy generation rate versus the channel width for Grashof number (Gr) and porous permeability term (δ). Generally, we note that the entropy generation rate is at a minimum value around the core region of the channel and rises to a maximum value around the plate surfaces. In figure 6, an increase in (Gr) results in an increase in the entropy generation rate, while it is observed that an increase in the porous permeability term (δ) results in a decrease in the entropy generation rate in figure 7.

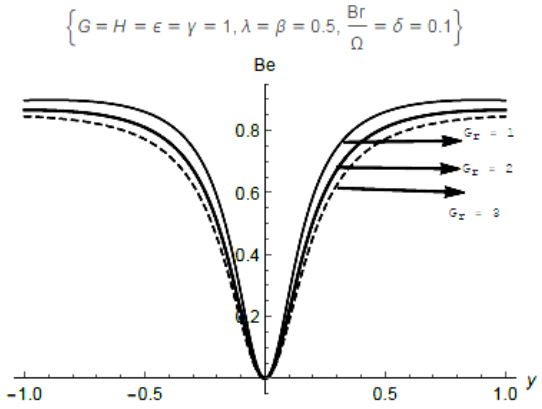

Figure 8: Effects of Gr on Be

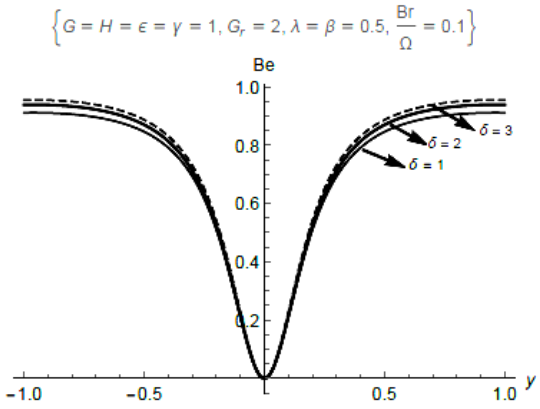

Figure 9: Effects of δ on Be

Figures 8 and 9 show the relationship of Bejan number (Be) across parallel plates. Notably, the energy transfer is in control at both lower and upper surfaces while the fluid friction irreversibility is in control around the core region. The regulatory effects of energy transfer irreversibility at the bottom plates reduce with rising values of (Gr) in figure 8, while an increase is noticed with the increasing values of porous permeability (δ) around the core region in figure 9.

CONCLUSION

The study investigated the consequence of buoyancy force on a hydromagnetic Poiseuille reactive fluid flow within porous plates. The results obtained for the differential equations regulating the fluid flow which are strongly nonlinear are acquired employing the modification of an infinite series solution (MADM). The speedy convergence of the series solution was presented and supported in the comparison of the new result with the formerly acquired result. The effects of buoyancy force is noticed to increase the fluid motion and heat transfer, while the porosity retards the fluid motion and also reduces the fluid heat within the plates.

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