Kragujevac J. Sci. 39 (2017) 37-46. UDC 532.546:537.638.5

THE EFFECT OF INTERNAL HEAT GENERATION ON A STEADY HYDROMAGNETIC POISEUILLE FLUID FLOW BETWEEN TWO PARALLEL POROUS PLATES

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(Received April 22, 2016; Accepted July 21, 2016)

ABSTRACT. This paper investigates the effects of internal heat generation on the steady flow of an incompressible and electrically conducting fluid flowing through a channel made up of two parallel porous plates with isothermal wall temperature, under the influence of a transverse magnetic field strength and temperature dependent heat source. The variable of an internal heat generation is assumed to be a linear function of temperature. The semi-analytical solutions of the equations governing the fluid flow are obtained for velocity and temperature profiles using a rapidly convergent Adomian decomposition method (ADM). Attention is focused on the influence of internal heat generation on the velocity and temperature profiles and the effects of other various physical parameters such as porous medium permeability parameter, viscous heating parameter, pressure gradient and magnetic field intensity on the fluid flow are also presented and discussed.

Keywords: Internal heat generation, hydromagnetic, porous plates, Adomian Decomposition Method (ADM).

INTRODUCTION

The studies relating to flow of fluids between two parallel plates are on the increase due to its numerous applications in many industrial and engineering processes like in accelerators, aerodynamic heating, polymer technology, purification of crude oil and magnetohydrodynamics (MHD) generators. Lot of research works concerning the fluid flow between two parallel plates has been obtained under different physical effects (HAZEEM, 2006, 2008; JHA and AJIBADE, 2009; MAKINDE and ANWAR BEG, 2010; HASSAN and FENUGA, 2011; ADESANYA and MAKINDE, 2012; HASSAN and GBADEYAN, 2015). The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic and endothermic chemical reactions (HASSAN and GBADEYAN, 2015). Possible heat generation effects may alter the temperature distribution as well as the particle deposition rate of the fluid flow. For example, SEDDEK (2005) studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydromagnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Also, PATIL and KULKARNI (2008) studied the effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation.

In addition to the above mentioned researches in SEDDEK (2005), HAZEEM (2006, 2008), PATIL and KULKARNI (2008), JHA and AJIBADE (2009), MAKINDE and ANWAR BEG (2010), HASSAN and FENUGA (2011), ADESANYA and MAKINDE (2012) and HASSAN and GBADEYAN (2015), there have been increased interests on the study of electrically conducting fluid flow under the influence of magnetic intensity and other thermophysical properties. For instance, GBADEYAN *et al*. (2005) investigated the effect of radiation on temperature and velocity profiles of an electrodynamic froth flow process in vertical channels. Also, HERWIG and WICKEN (1986) studied the effect of variable properties on laminar boundary layer flow of fluid and showed that more accurate prediction for fluid flow can be obtained in the process of heat transfer and other physical properties on the temperature.

In spite of all these studies, there exists a need to examine the effect of internal heat generation on fluid flow within channels because of its importance as described in SEDDEK (2005), PATIL and KULKARNI (2008), JHA and AJIBADE (2009) and HASSAN and GBADEYAN (2013). However, from an application point of view in engineering and industries, the effect of a heat source cannot be totally neglected as HASSAN and GBADEYAN (2015) recently showed, that is, fluid involving exothermic/endothermic reactions in which reactant consumption is neglected, heat is being produced in accordance with respect to different chemical kinetics. Hence, the main objective of the present investigation is to study the combined effects of internal heat generation and magnetic influence on steady Poiseuille fluid flow between two parallel porous plates. It is assumed that the outcome of the study will be very useful in many geological and petro–chemical engineering systems.

The analytical solutions governing the flow field are obtained by using the rapidly convergent ADM. The method has been shown to be reliable, efficient and successfully applied to several linear and nonlinear mathematical models (WAZWAZ and EL-SAYED, 2001; ADESANYA and GBADEYAN, 2011; HASSAN and GBADEYAN, 2013).

MODEL FORMULATION

We consider the steady flow of an incompressible and electrically conducting fluid flowing through a channel consisting of two parallel porous plates located at $y = -a$ and $y = a$. Both plates are fixed with isothermal wall temperature T_0 under the influence of a transverse magnetic field strength B_0 in the presence of internal heat generation as shown in fig 1. Also, the fluid is assumed chemically active, in such a way that, chemical reactions occur in the middle of the channel and heat flows symmetrically from centreline of the channel. The heat source term is assumed to be a linear function of temperature (HASSAN and GBADEYAN, 2013; JHA and AJIBADE, 2009).

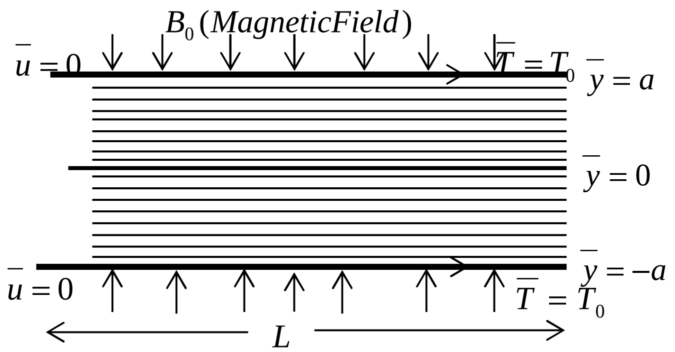

Figure 1. Geometry of the problem.

Neglecting the consumption of the reactant, the momentum and energy equations governing the fluid flow in non–dimensional form may be written as:

$$-\frac{d\overline{P}}{d\overline{x}}+\mu\frac{d^2\overline{u}}{d\overline{y}^2}-\sigma B_0{}^2\overline{u}-\frac{\mu}{k}\overline{u}=0 \tag{1}$$

$$K\frac{d^2\overline{T}}{d\overline{y}^2}+\mu\left(\frac{d\overline{u}}{d\overline{y}}\right)^2+\sigma B_0{}^2\overline{u}^2+\frac{\mu}{k}\overline{u}^2+Q_0(\overline{T}-T_0)=0 \tag{2}$$

The flow is symmetric about the horizontal y–axis. Hence, the corresponding boundary conditions along the channel centreline is given as

$$\frac{d\overline{u}}{d\overline{y}}=\frac{d\overline{T}}{d\overline{y}}=0 \text{ on } \overline{y}=0 \tag{3a}$$

and at the plate surfaces to be

$$\overline{u}=\overline{T}=0 \text{ on } \overline{y}=\pm a. \tag{3b}$$

It implies that the fluid velocity (u) and wall temperature (T_0) are both zero at the isothermal parallel plates that are stationary at $\overline{y}=\pm a$.

Here $\overline{u}$ is the dimensional fluid velocity, $\overline{T}$ is the dimensional fluid temperature, σ is the electrical conductivity, $\overline{P}$ is the dimensional pressure, μ represents the fluid viscosity and Q_0 is the dimensional heat generation coefficient. Also, a is the channel half width, $\overline{x}$ and $\overline{y}$ are the dimensional spacial coordinates, K is thermal conductivity coefficient and k represents the Darcy permeability coefficient.

Introducing the following dimensionless parameters and variables:

$$y=\frac{\overline{y}}{a},\ x=\frac{\overline{x}}{a},\ u=\frac{\overline{u}}{U},\ T=\frac{E(\overline{T}-T_0)}{RT_0{}^2},\ P=\frac{a\overline{P}}{\mu U},\ G=-\frac{dP}{dx},$$

$$H^2=\frac{\sigma B_0{}^2a^2}{\mu},\ \alpha=\frac{a^2}{k},\ Br=\frac{E\mu U^2}{KRT_0{}^2} \text{ and } \beta=\frac{Q_0a^2}{K} \tag{4}$$

we obtain the dimensionless governing equations for the momentum and energy with appropriate boundary conditions as:

$$\frac{d^2u}{dy^2}-(H^2+\alpha)u+G=0 \tag{5}$$

$$\frac{d^2T}{dy^2}+Br\left[\left(\frac{du}{dy}\right)^2+(H^2+\alpha)u^2\right]+\beta T=0 \tag{6}$$

together with the boundary conditions

$$\frac{du}{dy}=\frac{dT}{dy}=0 \text{ on } y=0 \text{ and } u=T=0 \text{ on } y=\pm 1 \tag{7}$$

where G is the dimensionless pressure gradient taken to be unity, U is the characteristic velocity, E represents activation energy parameter, R is the universal gas constant, α is the porous medium permeability parameter, H is the Hartmann number, Br is the viscous heating Brinkman number and β is the heat source parameter.

METHOD OF SOLUTION

The fluid velocity equation (5) is a linear second order non-homogeneous differential equation that can be solved by splitting into complimentary function and particular integral together with the boundary conditions (7) give a general exact solution:

$$\mathrm{u(y)} = -\frac{\mathrm{e}^{-\mathrm{y}\sqrt{\mathrm{H}^2+\alpha}}\left(\mathrm{e}^{\sqrt{\mathrm{H}^2+\alpha}} - \mathrm{e}^{\mathrm{y}\sqrt{\mathrm{H}^2+\alpha}} - \mathrm{e}^{2\sqrt{\mathrm{H}^2+\alpha}+\mathrm{y}\sqrt{\mathrm{H}^2+\alpha}} + \mathrm{e}^{\sqrt{\mathrm{H}^2+\alpha}+2\mathrm{y}\sqrt{\mathrm{H}^2+\alpha}}\right)G}{(1+\mathrm{e}^{2\sqrt{\mathrm{H}^2+\alpha}})(\mathrm{H}^2+\alpha)} \tag{8}$$

The differential equation (6) can be written in the integral form as:

$$T(y) = a_0 - \int_0^y\int_0^y \left(Br\left[\left(\frac{du}{dy}\right)^2 + (H^2+\alpha)u^2\right] + \beta T\right) dydy \tag{9}$$

where $a_0 = T(0)$ is to be determined by using the thermal boundary condition at $y = 1$.
We assume an infinite series solution of equation (9) in the form of:

$$T(y) = \sum_{n=0}^{\infty} T_n(y) \tag{10}$$

Inserting (10) in (9) leads to the following recursive relation with the zeroth component as:

$$T_0(y) = a_0 - Br\int_0^y\int_0^y \left[\left(\frac{du}{dy}\right)^2 + (H^2+\alpha)u^2\right] dydy \tag{11}$$

$$T_{n+1}(y) = -\beta\int_0^y\int_0^y (T_n)\, dydy\,, \qquad n \geq 0 \tag{12}$$

Equations (11) – (12) are then coded on Mathematica software using (8) to obtain the partial sum

$$T(y) = \sum_{n=0}^{m} T_n(y) \tag{13}$$

as the approximate solutions and the graphical results are presented and discussed in the next section.

ENTROPY GENERATION RATE

The total entropy change observed in a closed system is the sum of the entropy change which can be attributed to reversible heat transfer and the entropy change attributable to irreversibility. The entropy generation is due to heat transfer and the combined effects of fluid friction and Joules dissipation. Following [6] and [15], the general equation for the entropy generation per unit volume in the presence of a magnetic field and porous medium is given by:

$$S^m = \frac{k}{T_0^2}\left(\frac{d\bar{T}}{d\bar{y}}\right)^2 + \frac{\mu}{T_0}\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 + \frac{\sigma B_0^2 \bar{u}^2}{T_0} + \frac{\mu \bar{u}^2}{KT_0} \tag{14}$$

The first term in (14) is the irreversibility due to heat transfer; the second term is the entropy generation due to viscous dissipation and the last two are the local entropy generation due to the effects of magnetic field and porosity respectively. The entropy generation number in dimensionless form using the existing dimensionless variables and parameter in (4) is expressed as follows:

$$N_s = \frac{S^m a^2 E^2}{kR^2 T_0^2} = \left(\frac{dT}{dy}\right)^2 + \frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2 + (H^2 + \alpha)u^2\right] \tag{15}$$

The first term, $\left(\frac{dT}{dy}\right)^2$ is assigned N_1 which is the irreversibility due to heat transfer and the second term, $\frac{Br}{\Omega}\left[\left(\frac{du}{dy}\right)^2 + (H^2 + \alpha)u^2\right]$ referred to as N_2 is the entropy generation due to the combined effects of viscous dissipation, magnetic field and porosity of the flow regime where $\Omega = \frac{RT_0}{E}$ is the wall temperature parameter.

We defined the irreversibility distribution ratio as

$$\phi = \frac{N_2}{N_1} \tag{16}$$

The interpretation of relation (16) shows that heat transfer dominates when $0 \le \phi < 1$ and fluid friction dominates when $\phi > 1$. This is used to determine the contribution of heat transfer in many engineering designs. An alternative to irreversibility parameter, the Bejan number (Be) is defined as

$$Be = \frac{N_1}{N_s} = \frac{1}{1+\phi} \quad where \quad 0 \le Be \le 1. \tag{17}$$

DISCUSSION OF RESULTS

In this section, we discuss the effects of the internal heat generation together with other important flow parameters on the hydromagnetic fluid flow between two parallel porous plates with an isothermal wall temperature. These are presented in tables and graphs using code on Mathematica software. The computation of rapid convergence of ADM which has been applied to a wide class of problems in the sciences is shown in Table 1. The results clearly converge at the fourth iteration. The ADM has shown reliable results in supplying analytical approximations that converge rapidly (HASSAN and GBADEYAN, 2013) – (WAZWAZ and EL-SAYED, 2001).

Figures 2 and 3 show the effects of the porosity and magnetic field intensity on the velocity profile respectively using (8). As observed, the fluid velocity reduces with increasing values of the porous term (α) and a magnetic field intensity (H). This is due to the retarding effects of porosity in nature and magnetic force present in the flow channel.

The temperature profile for variations in the internal heat generation parameter (β) is displayed in figure 4. From the plot, it is observed that an increase in the internal heat absorption increases the fluid temperature within the parallel porous plates. It is important to note that the internal heat generation parameter (β) which shows an increase in fluid temperature is due to reduction in the thermal conductivity of the fluid. Also, the heat dissipation will continue to rise within the channel as the heat generated due to chemical reactions increases the fluid temperature.

Table 1. Computation showing rapid convergence of ADM

$[\mathrm{Br} = 0.1, H = 1, G = 1, \alpha = 0.1, \beta = 0.1, y = 1]$

n	T_n	$\sum_{n=0}^{k} T_n$
0	0. 530553	0.530553
1	0. 495475	1.026028
2	− 0. 0262473	0.999781
3	0. 000220139	1.000000
4	-7.35235×10^{-7}	1.000000
5	1.31403×10^{-9}	1.000000
6	-1.46068×10^{-12}	1.000000

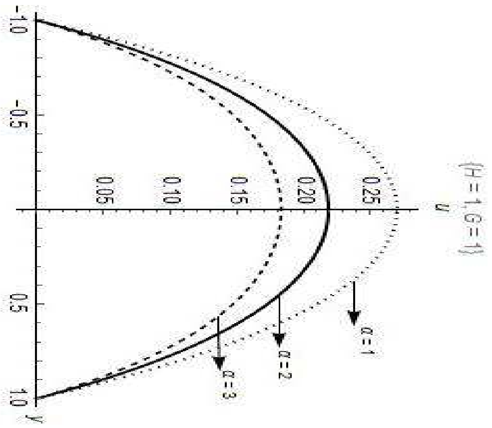

Figure 2: Effect of α on the velocity profile.

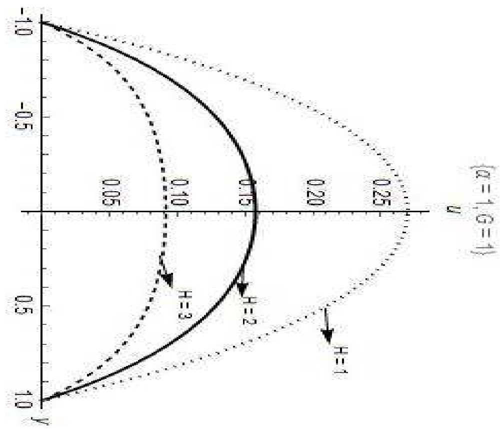

Figure 3: Effect of the H on the velocity profile.

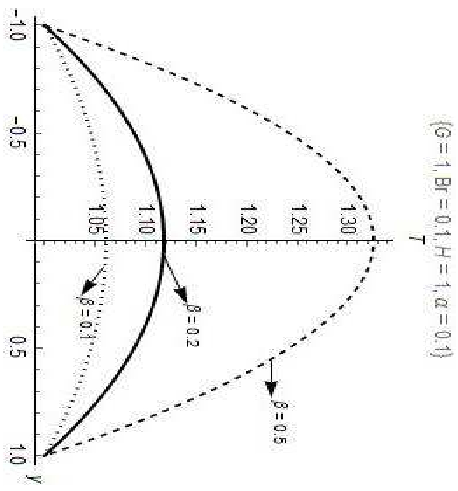

Figure 4. Effect of β on the temperature profile.

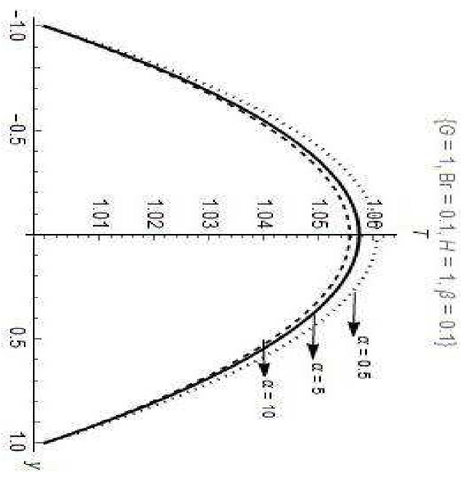

Figure 5. Effect of α on the temperature profile.

Figure 5 presents the temperature profile for various value of the porous medium term (α), it is clearly observed that the fluid temperature reduces as the porous medium term increases; this is due to the reduction in the fluid flow and the time taken for fluid flow within the porous medium thereby reduces the temperature.

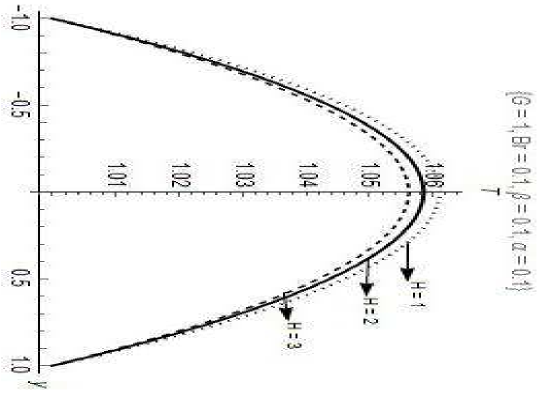

Figure 6. Effect of *H* on the temperature profile.

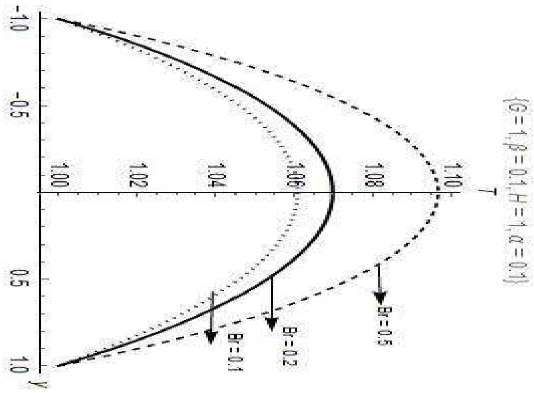

Figure 7. Effect of *Br* on the temperature profile.

The effect of the magnetic field intensity (H) on the temperature distribution within the channel is displayed in figure 6. It is noticed that the maximum fluid temperature is obtained at the minimum values of the magnetic field intensity parameter (*H*). Meanwhile, the effect of the viscous heating parameter (*Br*) on the temperature profile of the flow regime is presented in figure 7. It is observed that the fluid temperature rises as the viscous heating parameter (*Br*) increases; this is caused by the conversion of kinetic energy in the moving fluid to internal energy, hence, the temperature increases.

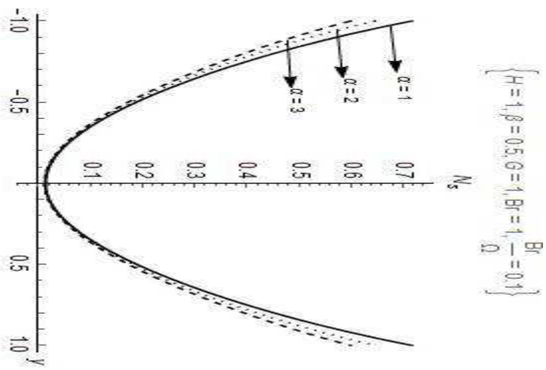

Figure 8. Entropy generation rate with variations in α.

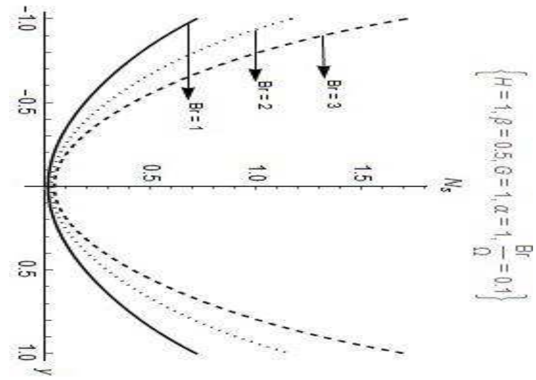

Figure 9. Entropy generation rate with variations in *Br*.

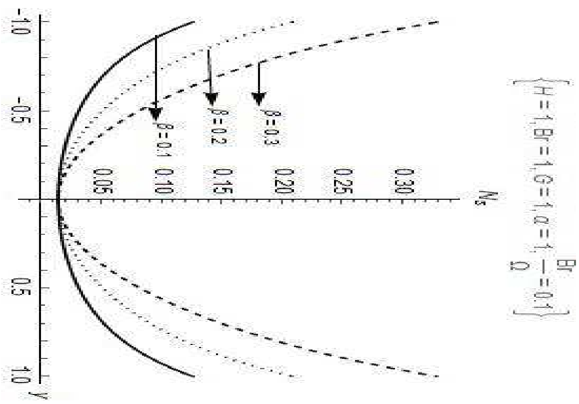

Figure 10. Entropy generation rate with variations in β.

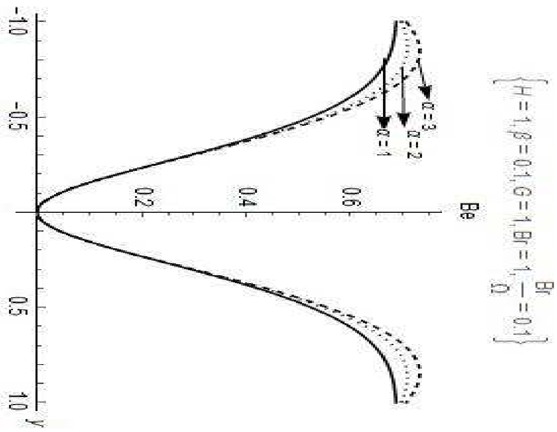

Figure 11. Bejan number with variations in α.

The entropy generation rates of the flow regime are displayed in figures 8–10. It is observed that the entropy generation rate is at maximum at both upper and lower fixed plate surfaces and minimum around the centreline. The entropy generation reduces with increasing values of the porous medium term (α) in figure 8. The entropy rates increase with increasing values of the viscous heating parameter (Br), and the internal heat generation parameter (β) in figures 9 and 10.

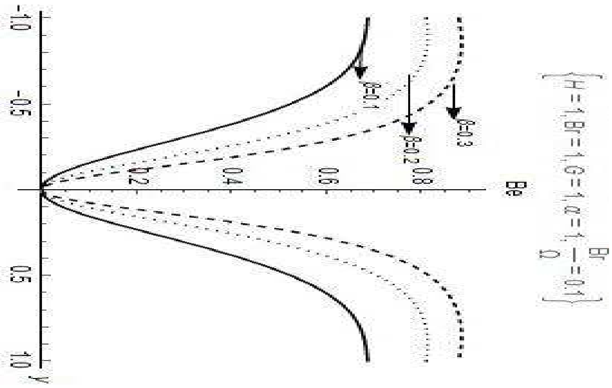

Figure 12. Bejan ratio with variations in β.

Figures 11 and 12 show the Bejan number versus the channel width. The heat transfer irreversibility dominates at both the fixed upper and lower plate surfaces while the fluid friction irreversibility dominates at the central line. The heat transfer irreversibility domination at both ends of the channel increases with increasing values of porous and internal heat generation parameters.

CONCLUSION

The effect of internal heat generation on a steady hydromagnetic fluid flow between two parallel porous plates with isothermal wall temperature is investigated. Analytical solutions of the equations governing the fluid flow are obtained using the rapidly convergent ADM. In view of the above results, the following are the major contributions in this study:

(i) the effects of the porosity (α) and the magnetic intensity (H) reduce the fluid velocity,

(ii) due to collision of the fluid particles especially with the effect of the porosity (α), heat is being generated which eventually leads to an increase in the fluid temperature,

(iii) an increase in the porous medium permeability parameter (α) reduces the temperature due to the time taken for fluid flow within the porous medium,

(iv) the entropy generation rate is maximum at both upper and lower fixed plate surfaces and minimum around the centreline and

(v) the heat transfer irreversibility dominates at both ends of the channel.

Finally, we showed that the internal heat generation, magnetic intensity field and porous medium have significant effects on the velocity profiles, temperature distributions and entropy generation analysis.

NOMENCLATURE

L	Channel characteristic length	U	Characteristic velocity
a	Channel half width	E	Activation energy
T_o	Wall temperature	R	Universal gas constant
B_o	Magnetic field strength	G	Pressure gradient
P	Pressure	H	Hartmann number
μ	Fluid viscosity	Be	Bejan number
u	Fluid velocity	β	Heat source parameter
T	Fluid Temperature	N_s	Entropy generation rate
k	Darcy permeability coefficient	Br	Brinkman number
Ω	Wall temperature parameter		
K	Thermal conductivity coefficient		
Q_o	Dimensional heat generation coefficient		
S^m	Entropy generation per unit volume		
ϕ	Irreversibility distribution ratio		
α	Porous medium permeability parameter		

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