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HEAT TRANSFER IN A STAGNATION POINT FLOW OF A SECOND GRADE FLUID OVER A STRETCHING SURFACE WITH HEAT GENERATION/ABSORPTION

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ABSTRACT. The heat transfer in a steady planar stagnation point flow of an incompressible non-Newtonian second grade fluid impinging on a permeable stretching surface with heat generation or absorption is examined. The governing nonlinear momentum and energy equations are solved numerically using finite differences. The influence of the characteristics of the non-Newtonian fluid, the surface stretching velocity, the heat generation/absorption coefficient, and Prandtl number on both the flow and heat transfer is reported.

Keywords: Stagnation point flow, stretching sheet, non-Newtonian fluid, second grade fluid, heat transfer, heat generation.

INTRODUCTION

The flow of a viscous incompressible fluid near a planar stagnation point is a classical problem in fluid mechanics which was first handled by HIEMENZ (1911) who demonstrated that the governing Navier-Stokes partial differential equations can be transformed to an ordinary differential equation of third order using similarity transformation. Due to the nonlinear terms in the resulting differential equation, an analytical solution is hard to obtain and consequently the ordinary nonlinear equation is solved numerically.

Hiemenz problem of stagnation point flow was extended in numerous ways to add new physical effects and parameters. The axisymmetric stagnation point flow was examined by HOMANN (1936) which has importance practical applications in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows and also in the design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery etc. As in the planar case, the governing equations in the axisymmetric problem are transformed to an ordinary differential equation of third order using a similarity transformation. SCHLICHTING and BUSSMAN (1943) obtained numerical solution of the stagnation point flow in the presence of uniform suction. More detailed solutions were later given by PRESTON (1946) while an approximate solution to the problem of uniform suction is obtained by ARIEL (1994). In hydromagnetics, the problem of Hiemenz flow was studied by NA (1979) where the solution of the third-order boundary value problem was given using the method of finite difference. An approximate solution of the same problem has been provided by ARIEL (1994). The combined effect of an externally applied uniform magnetic field and the uniform suction across the surface on the planar or axisymmetric stagnation point flow was given, respectively, by Attia (ATTIA, 2003a, 2003b).

The study of heat transfer in boundary layer flows has important applications in many areas such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, thermal recovery of oil, etc. (MASSOUDI et al., 1990). MASSOUDI et al. (1990) used a perturbation technique to solve the stagnation point flow and heat transfer of a non-Newtonian fluid of second grade, however, the analysis was valid only for small values of the parameter that determines the behavior of the non-Newtonian fluid. Later MASSOUDI et al. (1992) extended the problem to nonisothermal surface. GARG (1994) improved the solution obtained by MASSOUDI et al. (1992) and solving numerically for the flow characteristics for any value of the non-Newtonian parameter using a pseudo-similarity solution.

Flow of an incompressible viscous fluid over stretching surface has important applications in polymer industry. For instance, a number of technical processes concerning polymers involves the cooling of continuous strips (or filaments) extruded from a die by drawing them through a stagnant fluid with controlled cooling system and in the process of drawing these strips are sometimes stretched. The quality of the final product depends on the rate of heat transfer at the stretching surface. CRANE (1970) obtained a similarity solution in closed form for steady planar incompressible boundary layer flow caused by the stretching of a sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. Temperature distribution in the flow over a stretching surface subject to uniform heat flux was examined by DUTTA et al. (1985). Steady flow of a non-Newtonian viscoelastic fluid (RAJAGOPAL et al., 1984; MAHAPATRA et al., 2004), micropolar fluid (NAZAR et al., 2004), or second grade fluid (MASSOUDI et al., 2004) past a stretching sheet was investigated.

In the present paper, the heat transfer in a steady planar stagnation point flow of an incompressible non-Newtonian second grade fluid impinging on a permeable stretching surface is studied with heat generation or absorption. The wall and stream temperatures are assumed to be constants. A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations. The numerical solution gives the flow and heat characteristics for the whole range of the non-Newtonian fluid parameter, the stretching velocity, the heat generation/absorption coefficient and the Prandtl number.

FORMULATION OF THE PROBLEM

Consider the planar stagnation point flow of an incompressible non-Newtonian second grade fluid near a stagnation point at a surface located y=0 plane where the flow is confined to the region y>0, as shown in Fig. 1. The surface is stretched in the x-axis direction using two equal and opposite forces applied along the x-axis such that keeping the origin fixed and the x-component of the velocity varies linearly along it, $u_w(x) = bx$ where b(>0) is an arbitrary constant. The potential flow that comes from the y-axis and falls on the surface divides into two streams on the wall and leaves in both directions. The viscous flow must adhere to the wall, whereas the potential flow slides along it. The components for the potential flow of velocity at any point (x,y) for the viscous flow are (u,v) whereas (U,V) are the velocity components for the potential flow. The velocity distribution in the frictionless flow in the neighborhood of the stagnation point is given by

$U(x)=ax$, $V(y)=-ay$

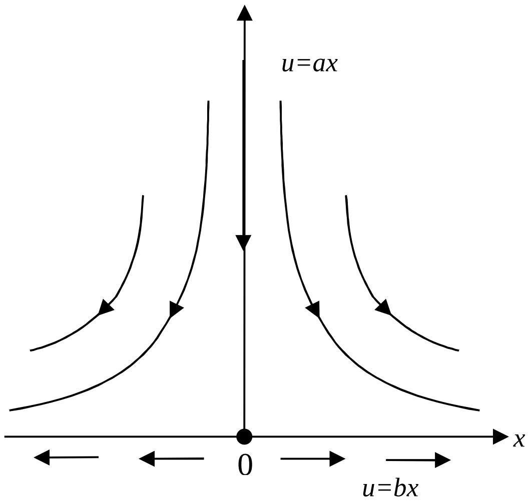

Fig. 1. Physical model and coordinate system.

where the constant a(>0) is proportional to the free stream velocity far away from the surface. A second grade fluid is defined such that the Cauchy stress tensor is related to the fluid motion in the following manner (ATTIA, 2000)

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \quad (1)$$

where p denotes the hydrostatic pressure, I is the identity tensor, μ is the viscosity of the fluid, α_1 and α_2 are scalar constants named as normal stress moduli, and A_1 and A_2 are the first two-Rivlin-Ericksen tensors. For $\alpha_1 = \alpha_2 = 0$, Eq. (1) describes a common Newtonian fluid. Then, A_1 represents the usual deformation tensor. All the stress components have to be introduced into the equations of motion. Then, for the planar steady-state flow, the continuity and momentum equations, using the usual boundary layer approximations (WHITE, 1991) and by introducing the stress components, reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = U\frac{dU}{dx} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) + \alpha_1\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right)\right) = 0, \quad (3)$$

where ρ is the density of the fluid, and $U(x)$ is the potential flow velocity over the body surface. The appropriate boundary conditions of the flow problem are

$$u(x,0) = bx, v(x,0) = 0, \quad (4a)$$

$$y \to \infty : u(x,y) \to U(x) = ax, v(x,y) \to 0, \quad (4b)$$

Introducing the following transformation

$$\eta = \sqrt{\frac{b}{\nu}}\,y, u = bxf'(\eta), v = -\sqrt{b\nu}\,f(\eta), \quad (5)$$

Eqs. (2), (3) and (4) transform to the single equation

$$K(ff^{iv} - 2f'f''' + f''^2) - f''' - ff'' + f'^2 - C^2 = 0 \quad (6)$$

$$f(0) = 0, f'(0) = 1, f'(\infty) = C, \quad (7)$$

where $K = \alpha_1 a/\mu$ is the dimensionless normal stress modulus, $C = b/a$ is the stretching parameter, and primes denote differentiation with respect to η.

Using the boundary layer approximations and neglecting the dissipation due to its small effect especially in the presence of heat generation (MASSOUDI et al., 1990), the equation of energy for temperature T is given by MASSOUDI et al. (1990),

$$\rho c_p (u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = k\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) \quad (8)$$

where c_p is the specific heat capacity at constant pressure of the fluid, k is the thermal conductivity of the fluid, T_∞ the constant temperature of the fluid far away from the sheet, Q is the volumetric rate of heat generation/absorption, and T is the temperature profile. A similarity solution exists if the wall and stream temperatures, T_w and T_∞ are constants–a realistic approximation in typical stagnation point heat transfer problems (WHITE, 1991).
The thermal boundary conditions are

$$y = 0 : T = T_w, \quad (9a)$$

$$y \to \infty : T \to T_\infty, \quad (9b)$$

By introducing the non-dimensional variable

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},$$

and using Eq. (5), Eqs. (8) and (9) reduce to,

$$\theta'' + \Pr f\theta' + \Pr B\theta = 0 \quad (10)$$

$$\theta(0) = 1, \theta(\infty) = 0, \quad (11)$$

where $\Pr = \mu c_p / k$ is the Prandtl number and $B = Q / b\rho c_p$ is the dimensionless heat generation/absorption coefficient.

The flow Eqs. (6) and (7) are solved numerically using finite difference approximations. A quasi-linearization technique is first applied to replace the non-linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence. The quasi-linearized form of Eq. (6) is,

$$K(f_n f_{n+1}^{iv} + f_n^{iv} f_{n+1} - f_n f_n^{iv} - 2(f_n' f_{n+1}''' + f_n''' f_{n+1}' - f_n''' f_n') + 2 f_n'' f_{n+1}'' - f_n''^2) - f_{n+1}''' -$$
$$f_n f_{n+1}'' - f_n'' f_{n+1} + f_n'' f_n + 2 f_n' f_{n+1}' - f_n'^2 - C^2 = 0$$

where the subscript n or n+1 represents the n^{th} or $(n+1)^{th}$ approximation to the solution. Then, Crank-Nicolson method is used to replace the different terms by their second order central difference approximations. An iterative scheme is used to solve the quasi-linearized system of difference equations. The solution for the Newtonian case is chosen as an initial guess and the iterations are continued till convergence within prescribed accuracy. Finally, the resulting block tri-diagonal system was solved using generalized Thomas' algorithm.

The energy Eq. (10) is a linear second order ordinary differential equation with variable coefficient, $f(\eta)$, which is known from the solution of the flow Eqs. (6) and (7) and the Prandtl number Pr is assumed constant. Equation (10) is solved numerically under the boundary condition (11) using central differences for the derivatives and Thomas' algorithm for the solution of the set of discritized equations. The resulting system of equations has to be solved in the infinite domain $0<\eta<\infty$. A finite domain in the η-direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid-independence studies show that the computational domain $0<\eta<\eta_\infty$ can be divided into intervals each is of uniform step size which equals 0.02. This reduces the number of points between $0<\eta<\eta_\infty$ without sacrificing accuracy. The value η_∞=10 was found to be adequate for all the ranges of parameters studied here. Convergence is assumed when the ratio of every one of f, f', f'', or f''' for the last two approximations differed from unity by less than 10^{-5} at all values of η in $0<\eta<\eta_{\infty}$.

RESULTS AND DISCUSSION

Figures 2 and 3 show the velocity profiles of f and f', respectively, for various values of C and K. The figures indicate that increasing the parameter C increases both f and f' and that the effect of K on both f and f' depends upon C. For $C<1$, increasing K increases f and f' while for $C>1$, increasing K decreases both. The figures tell also that the effect of C on f and f' is more apparent for smaller values of K. Also, increasing C decreases the velocity boundary layer thickness. Figure 4 presents the profile of temperature θ for various values of C and K and for $Pr=0.7$ and $B=0.1$. It is clear that increasing C decreases θ and its effect on θ is more clear for smaller values of K. The figure indicates that the thermal boundary layer thickness decreases when C increases. The effect of K on θ depends on C. For $C<1$, increasing K decreases θ, but for $C>1$, increasing K increases θ. The same effect for the parameter K holds on the thickness of the thermal boundary layer.

Figure 5 indicates that the temperature profiles for various values of C and Pr and for $K=1$ and $B=0.1$. Figure 5 brings out clearly the effect of the Prandtl number on the thermal boundary layer thickness. As shown in Fig. 5, increasing Pr decreases the thermal boundary layer thickness for all C. Increasing C decreases θ and its effect is more apparent for smaller Pr. Figure 6 shows the temperature profiles for various values of C and B and for $K=1$ and $Pr=0.7$. Increasing B increases the temperature θ and the boundary layer thickness. The effect of B on θ is more apparent for smaller C.

Figure 7 indicates the variation of dimensionless heat transfer rate at the wall $-\theta'(0)$ for different values of B and Pr and for $C=0.5$ and $K=0$. It is seen that increasing the parameter B decreases $-\theta'(0)$ for all Pr. However, increasing Pr increases $-\theta'(0)$ for all B. The influence of B on $-\theta'(0)$ becomes more pronounced for higher values of Pr.

Tables 1 and 2 present the variation of the dimensionless shear stress at the wall $f''(0)$ and the dimensionless heat transfer rate at the wall $-\theta'(0)$, respectively, for various values of C and K and for $Pr=0.7$ and $B=0.1$. Increasing C increases $f''(0)$ for all K. Increasing K decreases the magnitude of $f''(0)$ for all C. It is of interest to see the reversal of the sign of $f''(0)$ for $C<1$ for all K. Table 2 depicts that, increasing C increases $-\theta'(0)$. The effect of K on $-\theta'(0)$ depends on C. For $C<1$, increasing K increases $-\theta'(0)$ but for $C>1$ increasing K decreases $-\theta'(0)$.

Table 1. Variation of the wall shear stress $f''(0)$ with C and K.

K	C=0.1	C=0.2	C=0.5	C=1	C=1.1	C=1.2	C=1.5
0	-1.4541	-1.3772	-1.0009	0	0.2464	0.5066	1.3642
1	-0.6634	-0.6077	-0.4027	0	0.0855	0.1721	0.4374
2	-0.5353	-0.4848	-0.3139	0	0.0649	0.1305	0.3292
3	-0.4619	-0.4158	-0.4259	0	0.0545	0.1093	0.2750

Table 2. Variation of the rate of heat transfer at the wall $-\theta'(0)$ with C and K (Pr=0.7, B=0.1).

K	C=0.1	C=0.2	C=0.5	C=1	C=1.1	C=1.2	C=1.5
0	0.3913	0.4254	0.5089	0.6201	0.6399	0.6589	0.7136
1	0.4613	0.4844	0.5434	0.6201	0.6333	0.6459	0.6811
2	0.4915	0.5098	0.5573	0.6201	0.6310	0.6415	0.6708
3	0.5089	0.5246	0.5654	0.6201	0.6297	0.6389	0.6648

Table 3 presents the effect of *C* on $-\theta'(0)$ for various values of *Pr* and for *K*=1 and *B*=0.1. Increasing *C* increases $-\theta'(0)$ for all *Pr* and its effect becomes more clear for higher *Pr*. Increasing *Pr* increases $-\theta'(0)$ for all *C* and its effect is more apparent for higher *C*. Table 4 presents the effect of the parameters *C* and *B* on $-\theta'(0)$ for *K*=1 and *Pr*=0.7. Increasing *C* increases $-\theta'(0)$ for all *B*, however, increasing *B* decreases $-\theta'(0)$ for all *C* and its effect is more clear for smaller *C*.

Table 3. Variation of the rate of heat transfer at the wall $-\theta'(0)$ with *C* and *Pr* (*K*=1, *B*=0.1).

Pr	*C*=0.1	*C*=0.2	*C*=0.5	*C*=1	*C*=1.1	*C*=1.2	*C*=1.5
0.05	0.1176	0.1235	0.1418	0.1712	0.1767	0.1821	0.1973
0.1	0.1404	0.1526	0.1874	0.2349	0.2429	0.2507	0.2718
0.5	0.3669	0.3917	0.4509	0.5241	0.5365	0.5483	0.5810
1	0.5816	0.6029	0.6613	0.7412	0.7552	0.7686	0.8064

Table 4. Variation of the rate of heat transfer at the wall $-\theta'(0)$ with C and B (K=1, Pr=0.7).

B	*C*=0.1	*C*=0.2	*C*=0.5	*C*=1	*C*=1.1	*C*=1.2	*C*=1.5
-0.1	0.5949	0.6096	0.6518	0.7127	0.7237	0.7343	0.7644
0	0.5321	0.5501	0.5995	0.6676	0.6796	0.6911	0.7236
0.1	0.4613	0.4844	0.5434	0.6201	0.6333	0.6459	0.6811

CONCLUSIONS

The heat transfer in a steady planar stagnation point flow of an incompressible non-Newtonian second grade fluid impinging on a permeable stretching surface with heat generation/ absorption was studied. A numerical solution for the governing nonlinear momentum and energy equations was given which allows the computation of the flow and heat transfer characteristics for different values of the non-Newtonian parameter *K*, the stretching velocity *C*, the heat generation/absorption coefficient *B*, and the Prandtl number *Pr*. The results indicate that increasing the stretching velocity increases the velocity components but decreases the velocity boundary layer thickness. On the other hand, increasing the stretching velocity decreases the temperature as well as the thermal boundary layer thickness. The effect of the stretching velocity on the velocity and temperature is more pronounced for smaller values of the non-Newtonian parameter. The variation of velocity components and the temperature as well as the rate of heat transfer at the wall with the non-Newtonian parameter depends on the magnitude of the stretching velocity. The sign of the wall shear stress was shown to depend on the stretching velocity. The effect of the heat generation/absorption parameter *B* on the rate of heat transfer at the wall becomes more apparent for smaller *C*.

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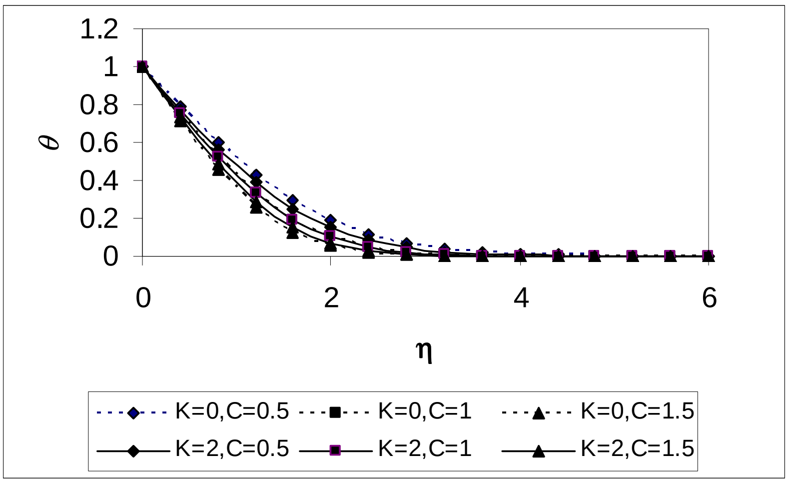

Fig. 2. Effect of the parameters *C* and *K* on the profile of θ (*Pr*=0.7, *B*=0.1).

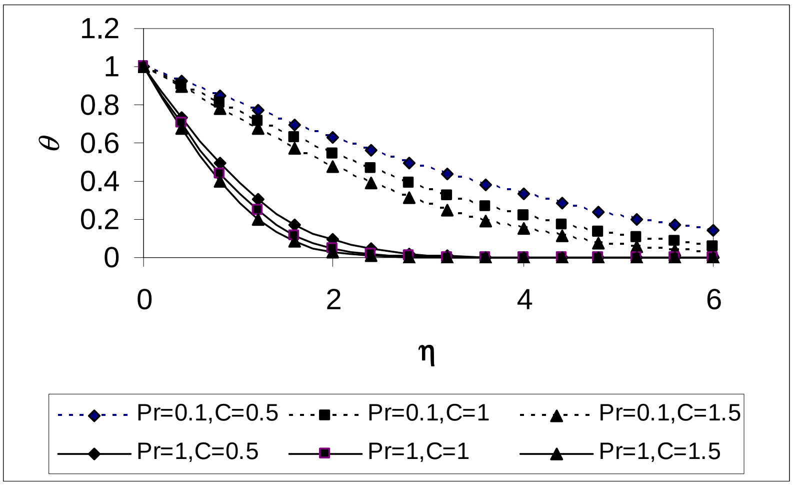

Fig. 3. Effect of the parameters *C* and *Pr* on the profile of θ (*K*=1, *B*=0.1).

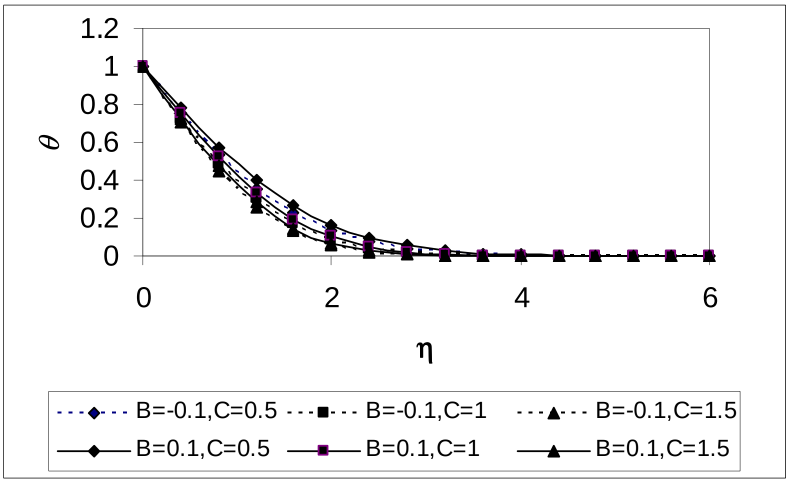

Fig. 4. Effect of the parameters *C* and *B* on the profile of θ (*K*=1, *Pr*=0.7)

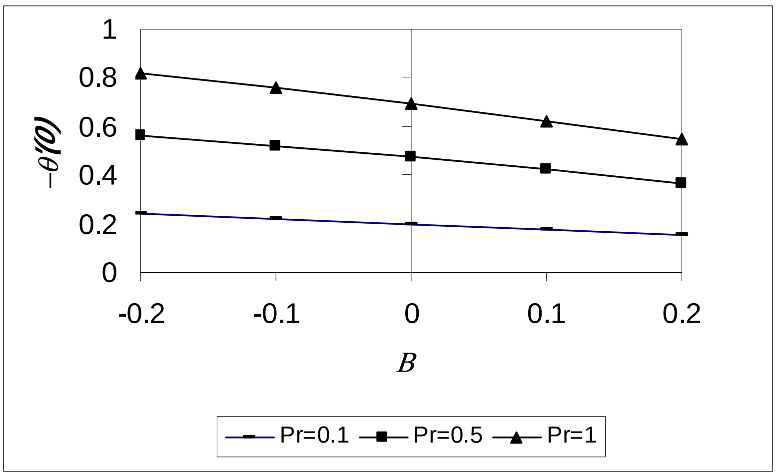

Fig. 5. Variation of the wall heat transfer rate $-\theta'(0)$ with *B* various values of Pr (*C*=0.5, K=0).