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MHD MIXED CONVECTION VISCO-ELASTIC SLIP FLOW THROUGH A POROUS MEDIUM IN A VERTICAL POROUS CHANNEL WITH THERMAL RADIATION

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ABSTRACT. Investigation of an oscillatory fully developed MHD convective flow of a viscoelastic, incompressible and electrically conducting fluid through a porous medium bounded by two infinite vertical parallel porous plates is carried out. The two porous plates with slip-flow condition and the no-slip condition are subjected respectively to a constant injection and suction velocity. The pressure gradient in the channel oscillates periodically with time. A magnetic field of uniform strength is applied in the direction perpendicular to the plates. The induced magnetic field is neglected due to the assumption of small magnetic Reynolds number. The temperature difference of the two plates is also assumed high enough to induce heat transfer due to radiation. Adopting complex variable notations, a closed form analytical solution of the problem is obtained. The analytical results are evaluated numerically and then presented graphically to discuss in detail the effects of different parameters entering into the problem. The dotted curves shown in figures correspond to the particular case viscous fluids.

Keywords: injection/suction; magnetohydrodynamic (MHD); convective; slip-flow; radiation.

INTRODUCTION

Magnetohydrodynamic (MHD) or magneto-fluid-dynamics (MFD) is the field of fluid mechanics which deals with the dynamics of an electrically conducting fluid under the influence of magnetic field. MHD or MFD has gained considerable importance because of its wide ranging applications in physics and engineering. In geophysics and astrophysics it is applied to study the flow and heat transfer through porous medium when the fluid is electrically conducting. In engineering it finds its application in electromagnetic lubrication, boundary cooling, biophysical systems etc. Magnetohydrodynamic (MHD) has attracted the attention of large number of scholars due to its diverse applications such as MHD generators, ion propulsion, MHD bearings, MHD pumps, MHD boundary layer control of reentry of space-vehicle and its surface heating etc. Unsteady mixed convection MHD flows through porous medium also remained of considerable interest to scholars because of their occurrence in nature and varied applications in many branches of science and technology such as in the fields of chemical engineering for filtration and purification processes, in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs etc. ATTIA and KOTB [3] studied the MHD flow between two parallel porous plates. SANYAL and ADHIKARI [18] analyzed the effects of radiation on MHD vertical channel flow. HAKIEM [11] studied an oscillatory MHD flow on free convection and radiation through a porous medium with constant suction velocity. ALAGOA *et al.* [1] investigated radiative and free convection effects of a MHD flow through a porous medium between infinite parallel plates with time dependent suction. MAKINDE and MHONE [15] studied combined effects of transverse magnetic field and heat transfer due to radiation on MHD oscillatory flow in channel filled with porous medium. GHOLIZADEH [8] investigated the MHD oscillatory flow past a vertical porous plate through porous medium in th presence of thermal and mass diffusion with constant heat source. ALDOSS *et al*. [2] studied Magnetohydrodynamic mixed convection from a vertical plate embedded in a porous medium. CHOUDHORY and DAS [6] studied heat transfer to MHD oscillatory visco-elastic flow in a channel with impermeable walls filled with porous medium.

In this modern era of industrialization there are many practical applications, where the particles adjacent to a solid surface no longer acquire the velocity of the surface. The particle has a finite tangential velocity; it 'slips' along the surface. The flow regime is called slip-flow regime and this effect cannot be neglected. In order to achieve maximum efficiency of mechanical devices various techniques are employed. Lubrication of mechanical devices is one of the processes when a thin film of lubricant is attached to the surfaces slipping over one another or when the surfaces are coated with special coatings to minimize the friction between them. Many viscous and visco-elastic fluids like oils and greases are used to lubricate the solid surface, thus, fluid particles adjacent to surface no longer move with the velocity of the surface but have a finite tangential velocity and, hence the fluid slips along the surface. MARQUES *et al*. [16] have considered the effect of fluid slippage at the plate for Couette flow. In view of the practical applications of the slip-flow regime it remained of paramount interest for several scholars e.g. SHARMA [19]; SHARMA and CHAUDHARY [20]; JAIN and GUPTA [11]; JOTHIMANI and ANJALIDEVI [12] KHALED and VAFAI [13] obtained exact solutions of oscillatory Stokes and Couette flows under slip flow condition. MEHMOOD and ALI [17] extended the problem of oscillatory MHD flow in a channel filled with porous medium studied by MAKINDE and MHONE [15] to slip-flow regime. Further, by applying the perturbation technique KUMAR *et al*. [14] investigated the same problem of slip-flow regime for the unsteady MHD periodic flow of viscous fluid through a planer channel. HAYAT *et al*. [10] studied heat transfer and slip flow of a second grade fluid past a stretching sheet through a porous space. SINGH [21] analyzed hydromagnetic forced convection oscillatory slip flow through porous medium in a vertical channel with thermal radiation.

In the present paper it is proposed to analyze an oscillatory MHD convection flow of a visco-elastic fluid through the porous medium bounded by two infinite vertical porous plates. The two plates respectively with slip-flow condition and the no-slip condition are subjected to constant injection and the same constant suction velocities. A magnetic field of uniform strength is also applied perpendicular to the plates of the channel. The magnetic Reynolds number is assumed small enough so that the induced magnetic field is neglected. The fluid is acted upon by an oscillatory pressure gradient in the vertically upward direction.

FORMULATION OF THE PROBLEM

An unsteady MHD convective flow of a viscous, viscoelastic, incompressible and electrically conducting fluid through porous medium filled in a vertical porous channel is consider. The two porous plates of the vertical channel are distance 'd' apart. A Cartesian coordinate system is introduced such that the X^*-axis lies vertically upwards in the direction of the buoyancy force along the centerline of the channel and Y^*-axis is perpendicular to the parallel plates. The schematic configuration of the physical problem is shown in Figure 1.

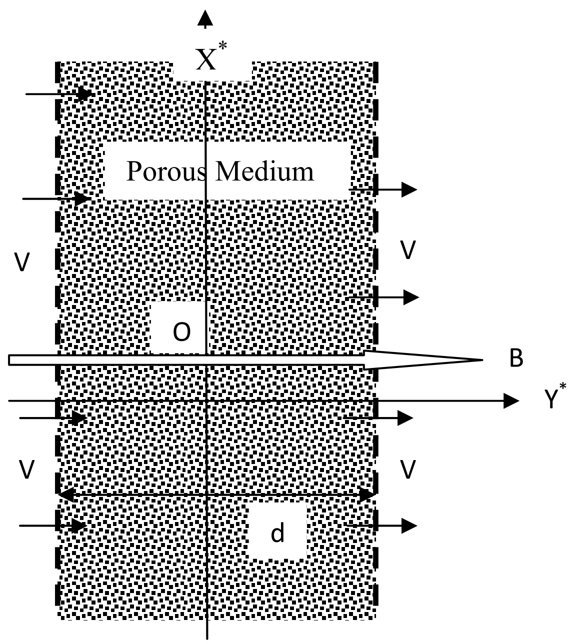

Figure 1. Schematic diagram of the physical problem.

In order to derive the basic equations for the problem under consideration, following assumptions are made:

(i) The two infinite vertical parallel plates of the channel are permeable.

(ii) There is a slip-flow condition at one of the plates while the no-slip condition at the other plate.

(iii) The fluid is injected with constant velocity V through the porous plate with slip-flow condition and simultaneously removed with the same suction velocity through the other plate with no-slip condition.

(iv) All fluid properties are assumed to be constant except that of the influence of density variation with temperature is considered only in the body force term.

(v) The flow considered to be fully developed laminar and oscillatory.
(vi) The pressure gradient in the channel oscillates periodically with time.
(vii) A magnetic field of uniform strength B is applied perpendicular to the plates of the channel.
(viii) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(ix) Hall effect, electrical and polarization effects are also neglected.
(x) The temperature of the plate with no-slip condition is non-uniform and oscillates periodically with time.
(xi) The temperature difference of the two plates is also assumed to be high enough to induce heat transfer due to radiation.
(xii) The fluid is assumed to be optically thin with relatively low density.

Under the above assumptions with usual Boussinesq approximation and following ATTIA [4] and ATTIA and ABDEEN [5] the oscillatory MHD convective flow through the porous medium is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial y^{*2}} + \vartheta_2 \frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} - \frac{\sigma B^2}{\rho} u^* - \frac{\vartheta}{K^*} u^* + g\beta(T^* - T_1), \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q^*}{\partial y^*}, \tag{3}$$

Following COGLEY *et al.* [7] it is assumed that the fluid is optically thin with relatively low density and the radiative heat flux is given by

$$\frac{\partial q^*}{\partial y^*} = 4\alpha^2 (T^* - T_1), \tag{4}$$

where α is the mean radiation absorption coefficient.

The boundary conditions of the problem are

$$y^* = \frac{d}{2}, \quad u^* = 0, \ v^* = V, \ T^* = T_1 + (T_2 - T_1)\cos\omega^* t^*, \tag{5}$$

$$y^* = -\frac{d}{2}, \quad u^* = L_1 \frac{\partial u^*}{\partial y^*}, \quad v^* = V, \quad T^* = T_1, \tag{6}$$

where V is the injection/suction velocity, ω^* is the frequency of oscillations, $L_1 = \left(\frac{2-r_1}{r_1}\right)L$, L being mean free path and r_1 is the Maxwell's reflexion coefficient. For the oscillatory internal flow in the channel the periodic pressure gradient variations are assumed to be of the form

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = A\cos\omega^* t^* , \tag{7}$$

where A is a constant and superscript '*' in all the above equations denote dimensional quantities.

Because of the assumption of constant injection and suction velocity V at the left ($y^* = -\frac{d}{2}$) and the right ($y^* = \frac{d}{2}$) plates respectively, continuity equation (1) integrates to

$$v^* = V. \tag{8}$$

Substituting equation (8) and introducing the following non-dimensional quantities

$$x = \frac{x^*}{d}\,,\; y = \frac{y^*}{d}\,,\; u = \frac{u^*}{V}\,,\; T = \frac{(T^* - T_1)}{(T_0 - T_1)}\,,\; t = \omega^* t^*\;\; \omega = \frac{\omega^* d}{V}\,,\; p = \frac{p^*}{\rho V^2}\,, \tag{9}$$

into equations (2) and (3), we get

$$\lambda\left(\omega\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y}\right) = -\lambda\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \gamma\omega\frac{\partial^3 u}{\partial y^2\,\partial t} - M^2 u - \frac{1}{K}u + GrT\,, \tag{10}$$

$$\lambda Pr\left(\omega\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y}\right) = \frac{\partial^2 T}{\partial y^2} - N^2 T, \tag{11}$$

where Grashof number $Gr = \frac{g\beta d^3 T_0}{\vartheta_1 V}$,

Viscoelastic parameter $\gamma = \frac{\alpha_2\lambda}{d^2}$,

Permeability of the porous medium $K = \frac{K^*}{d^2}$,

Prandtl number $Pr = \frac{\rho\vartheta_1 c_p}{k}$,

Radiation parameter $N = 2\alpha\frac{d}{\sqrt{k}}$,

slip-flow parameter $\delta = \frac{L_1}{d}$,

Hartmann number $M = Bd\sqrt{\frac{\sigma}{\rho\vartheta_1}}$,

Injection/suction parameter $\lambda = \frac{Vd}{\vartheta_1}$.

The boundary conditions in dimensionless form reduce to

$$y = \tfrac{1}{2};\quad u = 0,\quad T = \cos t, \tag{12}$$

$$y = -\tfrac{1}{2};\quad u = \delta\frac{\partial u}{\partial y},\quad T = 0\,, \tag{13}$$

where $\delta = \frac{L_1}{d}$ is the slip-flow parameter.

SOLUTION OF THE PROBLEM

We assume the solution in complex variable notations for this problem of unsteady periodic flow in the porous channel when the fluid is also acted upon by a periodic drop in pressure, as

$$u(y,t) = u_0(y)e^{it}, \quad T(y,t) = \theta_0(y)e^{it}, \quad -\frac{\partial p}{\partial x} = Ae^{it}, \tag{14}$$

The real part of the solution will have physical significance.
The boundary conditions (12) and (13) can also be written in complex notations as

$$y = \frac{1}{2}; \quad u = 0, \quad T = e^{it}, \tag{15}$$

$$y = -\frac{1}{2}; \quad u = \delta\frac{\partial u}{\partial y}, \quad T = 0. \tag{16}$$

Substituting expressions (14) into equations (10) and (11), we obtain following equations

$$(1 + i\omega\gamma)u_0'' - \lambda u_0' - (M^2 + K^{-1} + i\omega\lambda)u_0 = -\lambda A - Gr\theta_0, \tag{17}$$

$$\theta_0'' - \lambda Pr\theta_0' - (N^2 + i\omega\lambda Pr)\theta_0 = 0, \tag{18}$$

where primes in these ordinary differential equations denote differentiation with respect to y. The boundary conditions (15) and (16) reduce to

$$y = \frac{1}{2}; \quad u_0 = 0, \quad \theta_0 = 1, \tag{19}$$

$$y = -\frac{1}{2}; \quad u_0 = \delta\frac{du_0}{dy}, \quad \theta_0 = 0. \tag{20}$$

The solution of equation (17) for the velocity field under the boundary conditions (19) and (20) is obtained as

$$u(y,t) = \left[\begin{array}{c} \frac{\lambda A}{i}\left\{1 + \frac{\left((1-\delta m)e^{-\frac{m}{2}} - e^{\frac{m}{2}}\right)e^{ny} - \left((1-\delta n)e^{-\frac{n}{2}} - e^{\frac{n}{2}}\right)e^{my}}{(1-\delta n)e^{\frac{m-n}{2}} - (1-\delta m)e^{-\frac{m-n}{2}}}\right\} + \\ \frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l} \left(\frac{e^{\frac{r-s}{2}}}{C_1} - \frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(\frac{(1-\delta n)e^{my-\frac{n}{2}} - (1-\delta m)e^{ny-\frac{m}{2}}}{(1-\delta n)e^{\frac{m-n}{2}} - (1-\delta m)e^{-\frac{m-n}{2}}}\right) \\ + \left(\frac{1-\delta s}{C_2} - \frac{1-\delta r}{C_1}\right)\left(\frac{e^{my+\frac{n}{2}} - e^{ny+\frac{m}{2}}}{(1-\delta n)e^{\frac{m-n}{2}} - (1-\delta m)e^{-\frac{m-n}{2}}}\right)e^{-\frac{\lambda Pr}{2}} \end{array}\right\} \\ -\frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{ry-\frac{s}{2}}}{C_1} - \frac{e^{sy-\frac{r}{2}}}{C_2}\right) \end{array}\right]e^{it}, \tag{21}$$

where $C_1 = (1 + i\omega\gamma)r^2 - \lambda r - l$, $C_2 = (1 + i\omega\gamma)s^2 - \lambda s - l$, $l = M^2 + K^{-1} + i\omega\lambda$,

$$m = \frac{\lambda + \sqrt{\lambda^2 + 4l(1+i\omega\gamma)}}{2(1+i\omega\gamma)}, \qquad n = \frac{\lambda - \sqrt{\lambda^2 + 4l(1+i\omega\gamma)}}{2(1+i\omega\gamma)},$$

$$r = \frac{\lambda Pr + \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}, \qquad s = \frac{\lambda Pr - \sqrt{\lambda^2 Pr^2 + 4(N^2 + i\omega\lambda Pr)}}{2}.$$

Similarly, the solution of equation (18) for the temperature field under the boundary conditions (19) and (20) is obtained as

$$T(y,t) = \left(\frac{e^{ry-\frac{s}{2}} - e^{sy-\frac{r}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}\right) e^{it}. \tag{22}$$

From the velocity field obtained in equation (21) we can get the skin-friction τ at the left plate (y = -0.5) in terms of its amplitude $|F|$ and phase angle φ as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=-\frac{1}{2}} = |F| \cos(t + \varphi) \text{, with} \tag{23}$$

$$F = F_r + i\, F_i = \left[\begin{array}{c} \frac{\lambda A}{l}\left(\frac{n\left((1-\delta m)e^{-\frac{m}{2}} - e^{\frac{m}{2}}\right)e^{-\frac{n}{2}} - m\left((1-\delta n)e^{-\frac{n}{2}} - e^{\frac{n}{2}}\right)e^{-\frac{m}{2}}}{(1-\delta n)e^{\frac{m-n}{2}} - (1-\delta m)e^{-\frac{m-n}{2}}}\right) + \\ \frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l} \left(\frac{e^{\frac{r-s}{2}}}{C_1} \quad \frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(\frac{m(1-\delta n) - n(1-\delta m)}{(1-\delta n)e^{\frac{m-n}{2}} - (1-\delta m)e^{-\frac{m-n}{2}}}\right) e^{-\frac{\lambda}{2(1+i\omega\gamma)}} \\ + \left(\frac{1-\delta s}{C_2} - \frac{1-\delta r}{C_1}\right)\left(\frac{m e^{-\frac{m-n}{2}} - n e^{\frac{m-n}{2}}}{(1-\delta n)e^{\frac{m-n}{2}} - (1-\delta m)e^{-\frac{m-n}{2}}}\right) e^{-\frac{\lambda Pr}{2}} \end{array}\right\} \\ - \frac{Gr}{2\sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1} - \frac{s}{C_2}\right) e^{-\frac{\lambda Pr}{2}} \end{array}\right], \tag{24}$$

The amplitude is $|F| = \sqrt{F_r^2 + F_i^2}$ and the phase angle $\varphi - \tan^{-1}\frac{F_i}{F_r}$. (25)

Similarity, we can get the rate of heat transfer q, in terms of its amplitude $|H|$ and the phase angle ψ from equation (22) for the temperature field as

$$q = |H| \cos(t + \psi) , \tag{26}$$

$$\text{with} \quad H = Hr + i\, Hi = \frac{(r-s)e^{-\frac{\lambda Pr}{2}}}{2\sinh\left(\frac{r-s}{2}\right)}, \tag{27}$$

where the amplitude $|H|$ and the phase angle β of the rate of heat transfer are given as

$$|H| = \sqrt{Hr^2 + Hi^2}, \qquad \psi = \tan^{-1}\frac{Hi}{Hr} . \tag{28}$$

DISCUSSION OF THE RESULTS

The problem of an oscillatory mixed convection MHD flow in a porous vertical channel filled with porous medium is solved under the no-slip and slip-flow conditions in the presence of a transverse uniform magnetic field. The visco-elastic, incompressible and electrically conducting fluid is injected with constant velocity through the porous plate with slip-flow condition and is simultaneously removed with the same suction velocity through the other porous plate with no-slip condition. The pressure gradient in the channel also oscillates periodically with time. The velocity field, the amplitude and the phase angle of the skin-friction are calculated numerically and then shown graphically for different values of visco-elastic parameter γ , slip-flow parameter δ, injection/suction parameter λ, Grashof number Gr, Hartmann number M, the permeability of the porous medium K, the Prandtl number Pr, radiation parameter N, the pressure gradient A, and the frequency of oscillations ω.

The effects of different parameters on the velocity profiles are displayed in figure 2. The dotted curve (….) corresponds to the case of viscous fluid ($\gamma = 0$). Comparison of dotted curve and curve I reveals that the velocity in the case of visco-elastic fluid is less than the velocity of viscous fluid. The Figure depicts that the velocity increases tremendously with the increase of injection/suction parameter λ (curves III & I). The velocity goes on increasing manifolds as λ increases from 0.5 to 1. The injection/suction parameter λ enhances the flow velocity maximum. The influence of the Grashof number Gr on the velocity profiles is illustrated by curves IV & I. It is evident that the velocity increases with increasing Grashof number. Physically it means that the buoyancy force enhances the flow velocity. From curves I & V of this figure it can be observed easily that the velocity decreases rapidly with the increase of Hartmann number M. Physically, it means that with the increasing M, the strength of the magnetic field, the Lorentz force increases which drags the flow backward. The velocity variations due to the increase of permeability of the porous medium K are displayed by the curves I & VI. The velocity goes on increasing as the permeability increases. Physically, this means that with the increasing permeability of the porous medium the resistance posed by the porous matrix goes on decreasing which consequently leads to the gain in the velocity. This is in agreement with the fact that the velocity is more in the ordinary medium than in porous medium. Fig. 2 also shows that the velocity decreases with the increase of Prandtl number Pr (curves I & VII). This is because increasing Prandtl number means (Prandtl number being the ratio of the viscous to the thermal diffusion) the dominance of the viscous diffusion over the thermal diffusion. Thus, the fluid flow is resisted because of this predominance property of the viscous fluid that leads to the decrease in velocity.

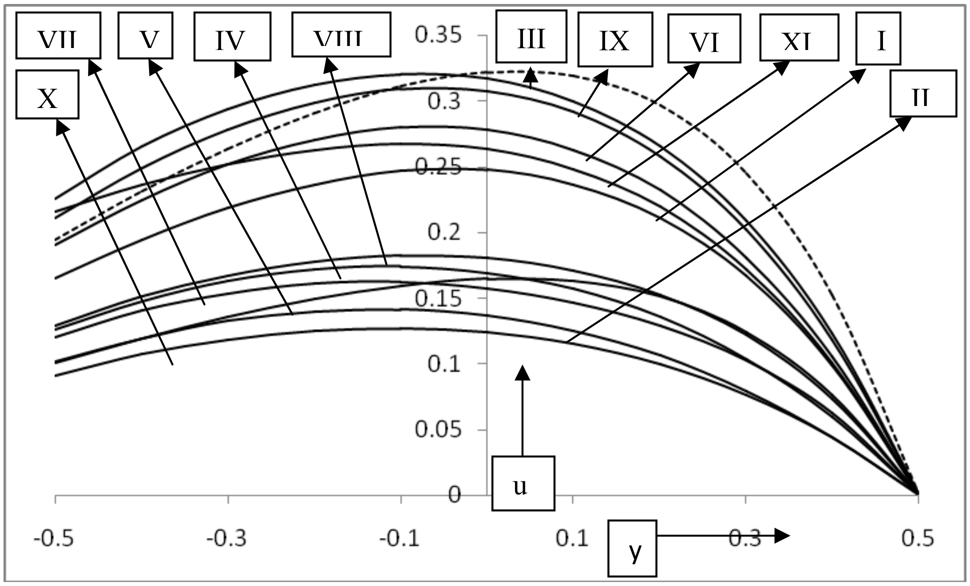

Fig. 2. Effects of different parameters on velocity profiles for t=0.

γ	λ	Gr	M	K	Pr	N	A	ω	δ	
0	0.5	5	2	0.2	0.7	1	5	5	0.5	….
0.2	0.5	5	2	0.2	0.7	1	5	5	0.5	I
0.5	0.5	5	2	0.2	0.7	1	5	5	0.5	II
0.2	1.0	5	2	0.2	0.7	1	5	5	0.5	III
0.2	0.5	1	2	0.2	0.7	1	5	5	0.5	IV
0.2	0.5	5	4	0.2	0.7	1	5	5	0.5	V
0.2	0.5	5	2	1.0	0.7	1	5	5	0.5	VI
0.2	0.5	5	2	0.2	7.0	1	5	5	0.5	VII
0.2	0.5	5	2	0.2	0.7	5	5	5	0.5	VIII
0.2	0.5	5	2	0.2	0.7	1	7	5	0.5	IX
0.2	0.5	5	2	0.2	0.7	1	5	10	0.5	X
0.2	0.5	5	2	0.2	0.7	1	5	5	1.0	XI

The thermal radiations effect on the velocity profiles are shown by curves I & VIII. The velocity decreases with the increase of radiation parameter N. The effects of pressure gradient on the velocity profiles are shown by curves I & IX. It is obvious from the figure that the velocity increases with increasing favorable pressure gradient A. It is because of the fact that more is the drop in pressure gradient faster is the flow. As is evident from curves I & X, the velocity decreases with increasing frequency of oscillations ω. The figure clearly shows that the velocity increases with the increase of slip-flow parameter δ (curves I & XI). The effect of slip-flow parameter is more pronounced in the left half of the channel in which the plate is subjected to the slip-flow condition.

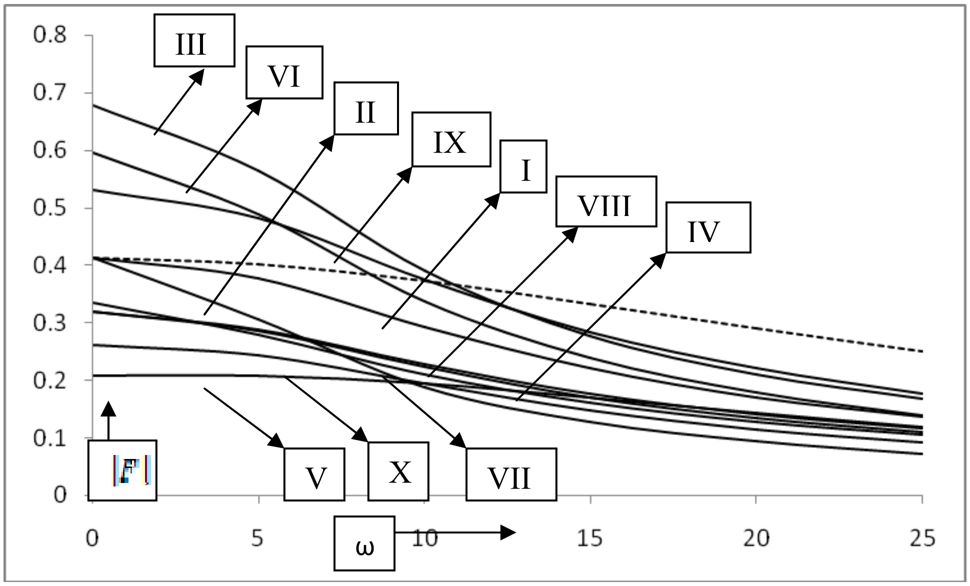

Fig. 3. Amplitude of |F| the skin-friction.

γ	λ	Gr	M	K	Pr	N	A	δ	
0	0.5	5	2	0.2	0.7	1	5	0.5	….
0.2	0.5	5	2	0.2	0.7	1	5	0.5	I
0.5	0.5	5	2	0.2	0.7	1	5	0.5	II
0.2	1.0	5	2	0.2	0.7	1	5	0.5	III
0.2	0.5	1	2	0.2	0.7	1	5	0.5	IV
0.2	0.5	5	4	0.2	0.7	1	5	0.5	V
0.2	0.5	5	2	1.0	0.7	1	5	0.5	VI
0.2	0.5	5	2	0.2	7.0	1	5	0.5	VII
0.2	0.5	5	2	0.2	0.7	5	5	0.5	VIII
0.2	0.5	5	2	0.2	0.7	1	7	0.5	IX
0.2	0.5	5	2	0.2	0.7	1	5	1.0	X

The amplitude $|F|$ of the skin-friction τ on the left plate ($y = -0.5$) with slip-flow condition is plotted in Fig. 3 against ω the frequency of oscillations. The dotted curve represent the skin-friction amplitude $|F|$ for the viscous fluid i.e. for $\gamma = 0$. Comparison of this dotted curve with the curves I and II shows that the amplitude goes on decreasing with increasing visco-elastic parameter γ. In order to study the effect of each of the parameter every curve is compared with curve I ($\gamma = 0.2$).From the respective comparison of curves III, IV, VI and IX with curve I it is gathered that the skin-friction amplitude $|F|$ increases with the increase of injection/suction parameter λ, Grashof number Gr, the permeability of the porous medium K and the pressure gradient A. It is true otherwise also because with the increase of all these parameters the flow velocity increases (Fig. 2) and this increased velocity gives rise to more friction. However, it is noticed from the respective comparison of curves V, VII, and VIII with curve I that the skin-friction amplitude $|F|$ decreases with the increase of Hartmann number M, Prandtl number Pr, and the thermal radiation parameter N. This reduction in skin-friction is because of the fact that the flow velocity decreases due to the increase of these parameters (Fig. 2), thus, for slower flows friction is less. The amplitude $|F|$ also decreases with the increase of the slip-flow parameter δ (curves I & IX). Slip flow is the only parameter because of which the velocity increases and at the same time the skin-friction reduces. Slightly careful observation of the figure reveals that the skin-friction amplitude decreases quite significantly for smaller values of the frequency ω (varying from 0 to 10) in comparison to the larger values (varying from 15 to 25). The skin-friction goes on reducing further with the increase of ω although the rate of reduction declines.

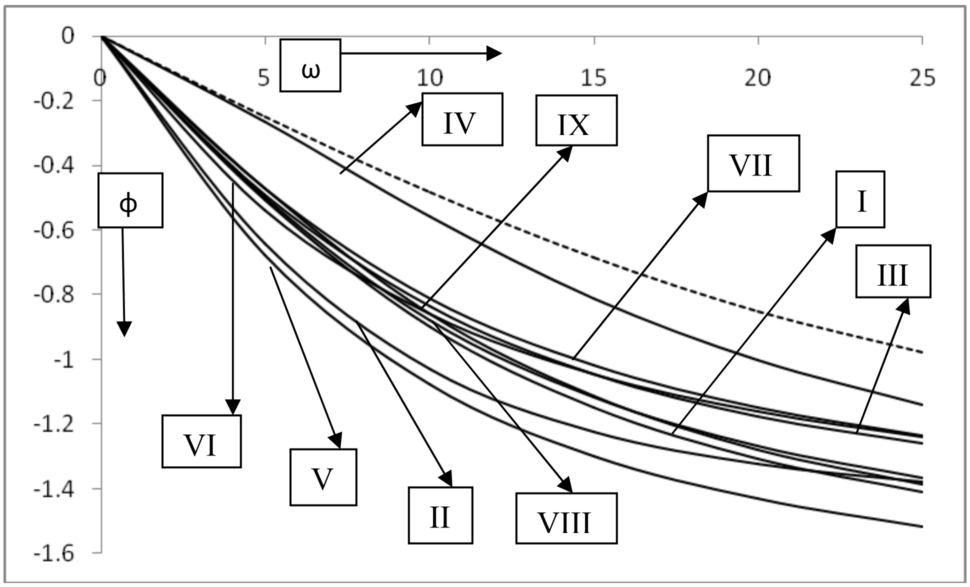

Fig.4. Phase angle ϕ of the skin-friction.

γ	λ	Gr	M	K	Pr	N	A	δ	
0	0.5	5	2	0.2	0.7	1	5	0.5	….
0.2	0.5	5	2	0.2	0.7	1	5	0.5	I
0.2	1.0	5	2	0.2	0.7	1	5	0.5	II
0.2	0.5	1	2	0.2	0.7	1	5	0.5	III
0.2	0.5	5	4	0.2	0.7	1	5	0.5	IV
0.2	0.5	5	2	1.0	0.7	1	5	0.5	V
0.2	0.5	5	2	0.2	7.0	1	5	0.5	VI
0.2	0.5	5	2	0.2	0.7	5	5	0.5	VII
0.2	0.5	5	2	0.2	0.7	1	7	0.5	VIII
0.2	0.5	5	2	0.2	0.7	1	5	1.0	IX

The effects of the variations of different flow parameters on the phase angle φ of the skin-friction τ are illustrated in Fig.4. It is obvious from this figure that there is always a phase lag because the values of φ plotted against ω are negative throughout. The dotted curve representing the case of viscous fluid (γ = 0) shows that the phase lag is minimum for the viscous fluids in comparison to the visco-elastic fluids. Comparing curves; dotted, I and II we find that the phase lag goes on increasing with increasing values of the visco-elastic parameter γ. In order to know the effect of each of the flow parameter every curve in the figure is compared with the basic curve I ($\gamma = 0.2$). By the comparison of curves II, III and V with curve I it is found that the lag in phase angle increases with the increase of injection/suction parameter λ, the Grashof number Gr and the permeability of the porous medium K. However, since all other curves IV, VI, VII , VIII and IX lie above curve I which imply that the phase lag decreases with the increase of all rest of the parameters like Hartmann number M, Prandtl number Pr, the radiation parameter N, the pressure gradient A and the slip-flow parameter δ. The decrease in the phase lag due to the Prandtl number and the slip-flow parameters is marginal.

CONCLUSIONS

The problem of oscillatory MHD convection flow through the porous medium bounded by two infinite vertical porous plates is analyzed. The two plates respectively with slip-flow condition and the no-slip condition are subjected to constant injection and the same constant suction velocities. A magnetic field of uniform strength is also applied perpendicular to the plates of the channel. The fluid is acted upon by an oscillatory pressure gradient in the vertically upward direction. From the analysis following conclusions are made:

(i) The velocity increases when the slip-flow parameter δ, injection/suction λ, Grashof number Gr, permeability of the porous medium K and the pressure gradient A increase.
(ii) The velocity decreases due to the increase of viscoelastic parameter γ, Hartmann number M, Prandtl number Pr, radiation parameter N and the frequency of oscillations ω.
(iii) The skin-friction amplitude $|F|$ increases with the increase of injection/suction parameter λ, Grashof number Gr, the permeability of the porous medium K and the pressure gradient A. Physically also it is expected because of the increase of these parameters velocity increases and this increased velocity give rise to more friction.
(iv) The skin-friction amplitude $|F|$ decreases with the increase of viscoelastic parameter γ, Hartmann number M, Prandtl number Pr, and the thermal radiation parameter N. This reduction in skin-friction is because of the fact that the flow velocity decreases due to the increase of these parameters, thus, for slower flows friction is less.
(v) The slip-flow parameter δ is the only parameter the increase of which increases the velocity and at the same time decreases the skin-friction amplitude. This is expected from physical point of view also.
(vi) There is always a phase lag and the lag in phase in minimum when the fluid considered is viscous.

NOMENCLATURE

A	a constant	L	mean free path
B	magnetic field applied	M	Hartmann number
c_p	specific heat at constant pressure	N	heat radiation parameter
g	gravitational force	p	pressure
Gr	Grashof number	Pr	Prandtl number
k	thermal conductivity	r_1	the Maxwell's reflexion coefficient.
K	permeability of the porous medium	t	time variable
		T	fluid temperature

T_0 constant temperature
u the axial velocity
V injection/suction velocity
x axial variable
y transverse variable

Greek symbols
α mean radiation absorption coefficient
β coefficient of volume expansion
γ visco-elastic parameter
δ non-dimensional slip parameter
λ injection/suction parameter
ω frequency of oscillations
ϑ_1 kinematic viscosity
ρ fluid density
τ_L skin-friction at the left wall
φ phase angle of the skin-friction
ψ phase angle of rate of heat transfer
θ_0 mean non-dimensional temperature
* superscript representing dimensional quantities

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