Kragujevac J. Sci. 35 (2013) 11-14. UDC 532.546

STABILITY OF FLOW THROUGH A POROUS MEDIUM OF A VISCOELASTIC FLUID ABOVE A STRETCHING PLATE

Hazem Ali Attia[1] and Mostafa A. M. Abdeen[2]

[1]*Department of Engineering Mathematics and Physics,*
Faculty of Engineering, El-Fayoum University, El-Fayoum 63514, Egypt
e-mail: ah1113@yahoo.com
[2]*Department of Engineering Mathematics and Physics,*
Faculty of Engineering, Cairo University, Giza 12211, Egypt

(Received September 30, 2012)

ABSTRACT. The effect of the porosity of the medium on the stability characteristics of a viscoelastic fluid flow over a stretching plate is investigated. A three-dimensional linear stability analysis is performed by means of the Method of Weighted Residuals for disturbances of the Taylor-Gortler type. It is found that the porosity of the medium exerts a stabilizing influence on the flow.

INTRODUCTION

The flow above a stretching sheet has various interesting engineering applications such as in the polymer processing unit of a chemical engineering plant, and for the metal working process in metallurgy. CRANE [1] studied the steady two-dimensional boundary-layer flow caused by stretching the sheet which moves in its own plane with a velocity that varies linearly with the distance from a fixed point on the sheet. Later, many authors have extended this problem to non-Newtonian fluids without or with heat and mass transfer [2-7]. Extensions to the case of electrically conducting Newtonian and non-Newtonian fluids without or with heat transfer were examined by several researchers [8-13]. BHATTACHARYYA and GUPTA [14] and TAKHAR *et al*. [15] have studied the viscous flow of a Newtonian fluid over a stretching sheet with respect to three-dimensional disturbances of the Taylor-Gortler type. Later, DANDAPAT *et al*. handled the stability of a viscoelastic non-Newtonian fluid flow caused by stretching of a sheet in the hydrodynamic case [16] and in the magnetohydrodynamic case [17].

In the present study, the effect of the porosity of the medium on the stability characteristics of the viscoelastic compressible fluid flow due to a stretching sheet was discussed. The flow through the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy’s law which accounts for the drag exerted by the porous medium [18-20].

MATHEMATICAL FORMULATION

The flow of an incompressible viscoelastic fluid is due to the stretching of an impermeable flat plate in the *x*-*z* plane which is stretched along the *x*-axis. The viscoelastic fluid occupies the half space defined by $y>0$. The basic velocity field $(u_o(x,y), v_o(x,y), 0)$ developed due to the stretching of the plate will satisfy the boundary-layer equations

$$\frac{\partial u_o}{\partial x} + \frac{\partial v_o}{\partial y} = 0 \tag{1}$$

$$u_o \frac{\partial u_o}{\partial x} + v_o \frac{\partial u_o}{\partial y} = \nu \frac{\partial^2 u_o}{\partial y^2} - \chi_o^* \left(\frac{\partial}{\partial x}\left(u_o \frac{\partial^2 u_o}{\partial y^2}\right) + \frac{\partial u_o}{\partial y}\frac{\partial^2 v_o}{\partial y^2} + v_o \frac{\partial^3 u_o}{\partial y^3} \right) - \frac{\nu}{K} u_o \tag{2}$$

with the boundary conditions

$$y = 0 : u_o = cx, v_o = 0, x \geq 0, \tag{3}$$

$$y \to \infty : u_o \to 0, x \geq 0, \tag{4}$$

where c is a constant and ν, χ_o^*, ρ, and K denote the kinematic viscosity, viscoelastic parameter, density of the fluid, and Darcy permeability [20], respectively. Following DANDAPAT *et al*. [17] an exact similarity solution of the above system can be obtained as

$$u_o = cxe^{-Q\eta} = cxf'(\eta), \tag{5}$$

$$v_o = -(c\nu)^{1/2}(1 - e^{-Q\eta})/Q = -(c\nu)^{1/2} f(\eta), \tag{6}$$

where $Q = \left(\dfrac{1+N}{1-\gamma} \right)^{1/2}$, $N = \nu/(cK)$, $\gamma = \chi_o^* c/\nu$, and the similarity variable is $\eta = (c/\nu)^{1/2} y$.

To discuss the stability of the above solutions (5) and (6) we follow the procedure of DANDAPAT *et al*. [17] and then we obtain the following perturbed equations after eliminating all variables except *u* and *v*:

$$\begin{aligned} &u_1'' + fu_1' - (\overline{\alpha}^2 + 2f' + \overline{\beta} + N)u_1 + f''v_1 + \gamma[u_1'''f - (\overline{\beta} + 2f')u_1'' - (\overline{\alpha}^2 f - 3f'')u_1' \\ &+ \overline{\beta}\overline{\alpha}^2 u_1 - f''v_1'' - 2f'''v_1' + (\overline{\alpha}^2 f'' + f^{iv})v_1] = 0 \end{aligned} \tag{7}$$

and

$$\begin{aligned} &u_1^{iv} + fv_1''' + (f' - 2\overline{\alpha}^2 - \overline{\beta} - N)v_1'' + (f'' - \overline{\alpha}^2 f)v_1' + [f''' + \overline{\alpha}^2(\overline{\beta} + \overline{\alpha}^2 - f')]v_1 - 2f''u_1' \\ &- 2f'''u_1 + \gamma[4f''u_1'' + 4f'''u_1' + fv_1^{v} - (\overline{\beta} + f')v_1^{iv} - (4f'' + 2\overline{\alpha}^2 f)v_1''' + (2\overline{\beta}\overline{\alpha}^2 + 2\overline{\alpha}^2 f' - 4f''')v_1'' \\ &+ \{\overline{\alpha}^2 f'' - f^{iv} + \overline{\alpha}^2(\overline{\alpha}^2 f + 3f'')\}v_1' + \{f^{v} + \overline{\alpha}^2 f''' - \overline{\alpha}^2(\overline{\beta}\overline{\alpha}^2 + f''' + \overline{\alpha}^2 f')\}v_1] = 0 \end{aligned} \tag{8}$$

where $\overline{\alpha}^2 = \alpha^2 \nu / c$ and $\overline{\beta} = \beta / c$. It should be pointed out that in deriving Eqs. (7) and (8) we have assumed that all the perturbed quantities have periodicity in the direction normal to the basic flow with the usual exponential time-dependence. Here α and β denote the wave number and the phase speed, respectively, and u_1 and v_1 are the corresponding normal mode components for u and v [17]. The corresponding boundary conditions are

$$u_1 = v_1 = v_1' = 0 \text{ at } \eta = 0 \text{ and } \eta \to \infty . \tag{9}$$

Following Dandapat et al. [17] we have transformed the η variable into T according to

$$T = e^{-Q\eta}, L = -T\frac{d}{dt} \tag{10}$$

and then applying the Method of Weighted Residuals to the transformed equations by expanding u_1 and v_1 as

$$u_1 = \sum_{i=1}^{\infty} A_i u_i(T), v_1 = \sum_{i=1}^{\infty} B_i v_i(T) . \tag{11}$$

$u_i(T)$ and $v_i(T)$ are chosen by satisfying the transformed boundary conditions

$$u_1 = v_1 = Lv_1 = 0 \text{ at } T = 0 \text{ and } T = 1,$$

and

$$u_i = T^i(1-T) \text{ and } v_i = T^i(1-T)^2 . \tag{12}$$

The growth rate $\overline{\beta}$ is calculated as the eigenvalue of the matrix which is formed using (11) in the transformed equations and restricting to the orthogonality condition as demanded by the Method of Weighted Residuals [21]. In this calculation we have restricted ourselves to 10-term trial functions, although good results may be obtained with even a 2-term approximation.

DISCUSSION

The analysis in the preceding section helps in determining the combined effects of fluid viscoelasticity and the porosity on the stability of the boundary layer flow along a stretching plate. However, for an inelastic fluid ($\gamma = 0$) the problem reduces to the Newtonian flow problem considered by TAKHAR *et al*. [15], whereas the non-porous case examined by DANDAPAT *et al*. [16] is recovered for *N*=0. This shows that the flow is stable for disturbances of the Taylor-Gortler type. The porosity stabilizes the flow for both Newtonian ($\gamma = 0$) and non-Newtonian ($\gamma \neq 0$) fluids for the whole range of wave-number which can be explained as the disturbance kinetic energy gets exhausted in order to overcome the resistive porosity force.

References:

[1] CRANE, L. J. (1970): Flow past a stretching plate. *ZAMP* **21**: 645-647.

[2] CHIAM, T. C. (1982): Micropolar fluid flow over a stretching sheet. *ZAMM* **62**: 565-568.

[3] RAJAGOPAL, K. R., NA, T. Y. and GUPTA, A. S. (1984): Flow of a viscoelastic fluid over a stretching sheet. *Rheol. Acta* **24**: 213-215.

[4] ANDERSSON, H. I. and DANDAPAT, B.S. (1991): Flow of a power-law fluid over a stretching sheet. *Stability. Appl. Anal. Continuous Media* **1**: 339-347.

[5] DANDAPAT, B. S. and GUPTA, A. S. (1989): Flow and heat transfer in a viscoelastic fluid over a stretching sheet. *Int. J. Non-Linear Mech*. 24: 215-219.

[6] ANDERSSON, H. I. and HANSEN, O. R. (1993): Chemical reactions in the viscoelastic boundary layer along a stretching sheet. In: *Some applied problems in fluid mechanics* (MAZUMDAR, H. P., ed.), pp. 37-44. Calcutta, Ind. Stat. Institute.

[7] ANDERSSON, H. I., HANSEN, O. R. and HOLMEDAL, B. (1994): Diffusion of a chemically reactive species from a stretching sheet. *Int. J. Heat Mass Trans*. **37**: 659-664.

[8] PAVLOV, K. B. (1974): Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. *Magnitnaya Gidrodinamika* 4: 146-147.

[9] CHAKRABARTI, A. and GUPTA, A. S. (1979): Hydromagnetic flow and heat transfer over a stretching sheet. *Q. Appl. Math*. **37**: 73-78.

[10] MANDAL, G., GUPTA, A. S., and POP, I. (1987): Magnetohydrodynamic flow of an incompressible viscous fluid caused by the axisymmetric stretching o a plane sheet. *Magnitnaya Gidrodinamika* **1**: 10-14.

[11] ANDERSSON, H. I. (1992): MHD flow of a viscoelastic fluid past a stretching surface. *Acta Mech*. **95**: 227-230.

[12] ANDERSSON, H. I. (1995): An exact solution of the Navier-Stokes equations for magnetohydrodynamic flow. *Acta Mech*. **113**: 241-244.

[13] ANDERSSON, H. I., BECH, K. H., and DANDAPAT, B. S. (1992): Magnetohydrodynamic flow of a power-law fluid over a stretching sheet. *Int. J. Non-Linear Mech*. **27**: 929-936.

[14] BHATTACHARYYA, S. N., and GUPTA, A. S. (1985): On the linear stability of viscous flow over a stretching sheet. *Q. Appl. Math*. **43**: 359-367.

[15] TAKHAR, H. S., ALI, M. A., and GUPTA, A. S. (1989): Stability of magnetohydrodynamic flow over a stretching sheet. In: *Liquid metal hydrodynamics* (Lielpeteris, J., Moreau, R., eds.), pp. 465-471. Dordrecht: Kluwer.

[16] DANDAPAT, B. S., HOLMEDAL, L. E., and ANDERSSON, H. I. (1994): Stability of flow of a viscoelastic fluid over a stretching sheet. *Arch. Mech*. **46**: 829-838.

[17] DANDAPAT, B. S., HOLMEDAL, L. E., and ANDERSSON, H. I. (1998): On the stability of MHD flow of a viscoelastic fluid over a stretching sheet. *Acta Mechanica*, **130**: 143-146.

[18] JOSEPH, D. D., NIELD, D. A., and PAPANICOLAOU, G. (1982): Nonlinear equation governing flow in a saturated porous medium. *Water Resources Research*, **18**(4): 1049-1052.

[19] INGHAM, D. B. and POP, I. (2002): *Transport phenomena in porous media*. Pergamon, Oxford.

[20] KHALED, A. R. A., and VAFAI, K. (2003): The role of porous media in modeling flow and heat transfer in biological tissues. *Int. J. Heat Mass Transf*. **46**: 4989-5003.

[21] FINLAYSON, B. A. (1972): The method of weighted residuals and variational principles: With Applications in Fluid Mechanics. New York: Academic Press.