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ON THE EFFECTIVENESS OF POROSITY ON TRANSIENT FLOW DUE TO A ROTATING DISK WITH HEAT TRANSFER AND DISSIPATION

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ABSTRACT. The transient flow with heat transfer through a porous medium of an incompressible viscous fluid due an infinite rotating disk is studied considering Darcy's model. The nonlinear partial differential equations that govern the motion of the fluid and the energy equation including the dissipation are solved numerically using finite differences. The effect of the porosity of the medium on both the velocity and temperature fields is investigated.

Keywords: Flow due to a rotating disk, porous medium, heat transfer, finite differences, unsteady state.

INTRODUCTION

The flow due to an infinite rotating disk was studied by von KARMAN in 1921 [1] who deduced a similarity transformation that reduced the governing partial differential equations to ordinary differential ones. Then, COCHRAN [2] obtained asymptotic solutions for the problem formulated by von Karman. BENTON [3] improved Cochran's solution and extended the problem to the transient state. The heat transfer from a rotating disk maintained at a constant temperature was studied in the steady state by MILLSAPS and POHLHAUSEN [4] for a variety of Prandtl numbers. SPARROW and GREGG [5] investigated the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids for all values of Prandtl number. The effect of an external axial uniform magnetic field on the flow due to a rotating disk was studied by many authors [6-8]. The effect of uniform suction or injection through the holes of a rotating porous disk on the steady hydrodynamic or hydromagnetic flow induced by the disk was studied [9-14]. The flow due a rotating disk through a porous medium was studied in the steady state [15].

In this paper, the transient laminar flow through a porous medium of a viscous incomepressible fluid due to the uniform rotation of a disk of infinite extent is studied with heat transfer and dissipation. The flow through the porous medium deals with the analysis in which the differential equations governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [16-18]. The temperature of the disk is impulsiely changed and then maintained at a constant value. The nonlinear partial differential equations that govern the motion of the fluid and the energy equation including the dissipation are solved numerically using finite differences with suitable coordinate transformations to remove a discontinuity between the initial and boundary conditions. The effect of the porosity of the medium on both the velocity and temperature distributions is presented.

BASIC EQUATIONS

It is assumed that the disk lies in the plane z=0 and the space z>0 is filled with a viscous incompressible fluid in a porous medium where the Darcy model is assumed [16-18]. The motion is due to the rotation of an infinite disk about an axis perpendicular to its plane with constant angular speed ω where otherwise the fluid is at rest under pressure p_∞. The equations of motion in the transient state are given by

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r})+\frac{\partial p}{\partial r}=\mu(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 u}{\partial z^2})-\frac{\mu}{K}u \tag{2}$$

$$\rho(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r})=\mu(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2})-\frac{\mu}{K}v \tag{3}$$

$$\rho(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z})+\frac{\partial p}{\partial z}=\mu(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2})-\frac{\mu}{K}w \tag{4}$$

where u, v, w are the velocity components in the directions of increasing r,φ, z respectively, P is denoting the pressure, μ is the coefficient of viscosity, ρ is the density of the fluid, and K is the Darcy permeability [16-18]. We introduce von Karman transformations [1],

$$u=r\omega F, v=r\omega G, w=\sqrt{\omega\nu}H, z=\sqrt{\nu/\omega}\zeta, p-p_\infty=-\rho\nu\omega P$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of ζ and t, and ν is the kinematic viscosity of the fluid, $\nu=\mu/\rho$. Using these definitions, Eqs. (1)-(4) take the form

$$\frac{\partial H}{\partial \zeta}+2F=0 \tag{5}$$

$$\frac{\partial F}{\partial t}-\frac{\partial^2 F}{\partial \zeta^2}+H\frac{\partial F}{\partial \zeta}+F^2-G^2+MF=0 \tag{6}$$

$$\frac{\partial G}{\partial t}-\frac{\partial^2 G}{\partial \zeta^2}+H\frac{\partial G}{\partial \zeta}+2FG+MG=0 \tag{7}$$

$$\frac{\partial H}{\partial t}-\frac{\partial^2 H}{\partial \zeta^2}+H\frac{\partial H}{\partial \zeta}-\frac{dP}{d\zeta}+MH=0 \tag{8}$$

$M=\nu/K\omega$ is the porosity parameter. The initial and boundary conditions for the velocity problem are given by

$$t=0, F=0, G=0, H=0, \tag{9a}$$

$$\zeta=0, F=0, G=1, H=0, \tag{9b}$$

$$\zeta \to \infty, F \to 0, G \to 0, P \to 0, \tag{9c}$$

The above system of Eqs. (5)-(7) with the prescribed initial and boundary conditions given by Eq. (9) are solved for the three unknown components of the flow velocity. Equation (8) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid, heat transfer takes place. The energy equation including the dissipation takes the form [4-5];

$$\rho c_p(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}) = k\frac{\partial^2 T}{\partial z^2} + \mu((\frac{\partial u}{\partial z})^2 + (\frac{\partial v}{\partial z})^2) + \frac{k}{r}\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial T}{\partial r}) \tag{10}$$

where T is the temperature of the fluid, c_p and k are, respectively the specific heat at constant pressure and the thermal conductivity of the fluid. The initial and boundary conditions for the energy problem are that the temperature is changed impulsively from rest and, by continuity considerations, it equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞ where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable $\theta=(T-T_\infty)/(T_w-T_\infty)$ and imposing von Karman transformations, Eq. (10) takes the form;

$$\frac{\partial \theta}{\partial t} - \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \zeta^2} + H\frac{\partial \theta}{\partial \zeta} = Ec((\frac{\partial F}{\partial \zeta})^2 + (\frac{\partial G}{\partial \zeta})^2) \tag{11}$$

where Pr is the Prandtl number, $Pr=c_p\mu/k$ and $Ec = \omega^2 r^2 / c_p(T_w - T_\infty)$ is the Eckert number. The initial and boundary conditions in terms of θ are expressed as

$$\theta(0,\zeta) = 0, \theta(t,0) = 1, \theta(t,\infty) = 0 \tag{12}$$

The heat transfer from the disk surface to the fluid is computed by application of Fourier's law

$$Q = -k(\frac{\partial T}{\partial z})_w .$$

Introducing the transformed variables, the expression for q becomes

$$Q = -k(T_w - T_\infty)\sqrt{\frac{\omega}{\nu}}\frac{\partial \theta(t,0)}{\partial \zeta} .$$

By rephrasing the heat transfer results in terms of a Nusselt number defined as, $N_u = Q\sqrt{\omega/\nu}/k(T_w - T_\infty)$ the last equation becomes

$$N_u = -\frac{\partial \theta(t,0)}{\partial \zeta} .$$

Numerical solution for the governing nonlinear Eqs. (5)-(7) with the conditions given by Eq. (9), using the method of finite-differences, leads to a numerical oscillation problem resulting from the discontinuity between the initial and boundary conditions (9a) and (9b). The same type of discontinuity happens between the initial and boundary conditions for the energy problem (see Eq. (12)). A solution for this numerical problem is obtained using proper coordinate transformations, as suggested by Ames [19] for similar problems. Expressing Eqs. (5)-(7) and (11) in terms of the modified coordinate $\eta=\zeta/2\sqrt{t}$ we get

$$\frac{\partial H}{\partial \eta} + 4\sqrt{t}F = 0 \tag{13}$$

$$\frac{\partial F}{\partial t}-\frac{\eta}{2t}\frac{\partial F}{\partial \eta}-\frac{1}{4t}\frac{\partial^2 F}{\partial \eta^2}+\frac{1}{2\sqrt{t}}H\frac{\partial F}{\partial \eta}+F^2-G^2+MF=0 \tag{14}$$

$$\frac{\partial G}{\partial t}-\frac{\eta}{2t}\frac{\partial G}{\partial \eta}-\frac{1}{4t}\frac{\partial^2 G}{\partial \eta^2}+\frac{1}{2\sqrt{t}}H\frac{dG}{d\zeta}+2FG+MG=0 \tag{15}$$

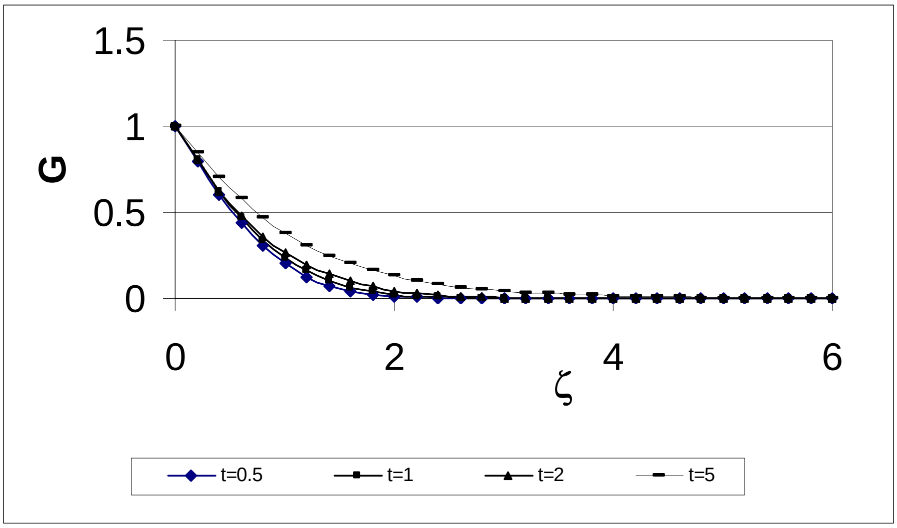
$$\frac{\partial \theta}{\partial t}-\frac{\eta}{2t}\frac{\partial \theta}{\partial \eta}-\frac{1}{4t\Pr}\frac{\partial^2 \theta}{\partial \eta^2}+\frac{1}{2\sqrt{t}}H\frac{\partial \theta}{\partial \eta}=\frac{Ec}{4t}((\frac{\partial F}{\partial \eta})^2+(\frac{\partial G}{\partial \eta})^2) \tag{16}$$

Equations (13)-(16) represent coupled system of non-linear partial differential equations which can be solved numerically under the initial and boundary conditions (9) and (12) using the method of finite differences. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. The Crank-Nicolson implicit method is then used at two successive time levels [19]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for the next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tri-diagonal system is solved using the generalized Thomas-algorithm [19]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the η-direction. The diffusion terms are replaced by the average of the central differences at two successive time-levels. The computational domain is divided into meshes each of dimension Δt and $\Delta\eta$ in time and space respectively. The modified Eqs. (13)-(16) are integrated from t=0 to t=1. Then, the solution obtained at t=1 is used as the initial condition for integrating Eqs. (5)-(7) and (11) from t=1 towards the steady state.

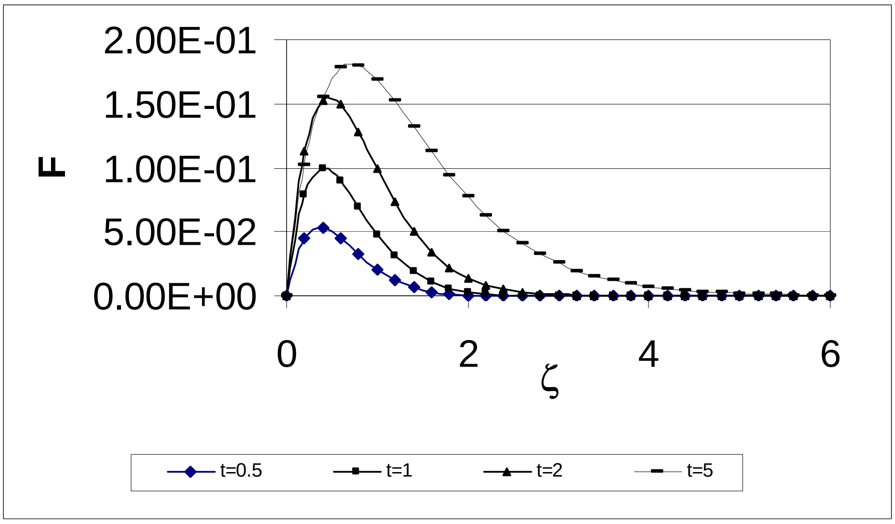
The resulting system of equations has to be solved in the infinite domain $0<\eta<\infty$. A finite domain in the η-direction can be used instead with η chosen large enough such that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain as well as the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for η_∞=10 and step size $\Delta\eta$=0.04 which are found adequate for the ranges of the parameters studied here. Larger finite distance or smaller step size do not show any significant change in the results. Convergence of the scheme is assumed when all of the variables F, G, H, θ, $\partial F/\partial\eta$, $\partial G/\partial\eta$, and $\partial\theta/\partial\eta$ for the last two approximations differs from unity by less than 10^{-6} for all values of η in $0<\eta<10$ and all t.

RESULTS AND DISCUSSION

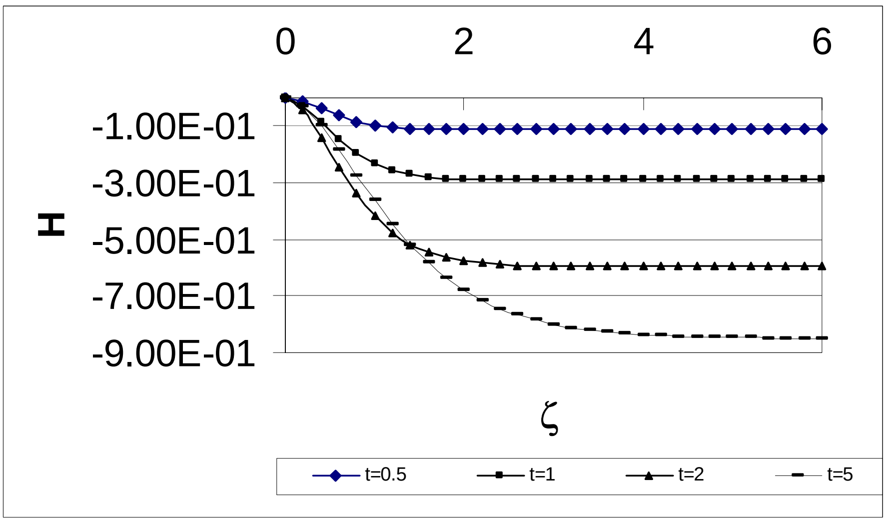
Figures 1 and 2 show the evolution of the azimuthal, radial, and vertical velocity profiles towards the steady state, respectively, for M=0 and M=1. The figures show that the vertical velocity component reaches the steady state slower than the radial velocity component and much slower than the azimuthal velocity component. This is due to the fact that the centrifugal effect is the source of the radial motion which is the source of the vertical motion. Comparison between Figs. 1 and 2 indicates the resistive effect of the porosity of the medium on the flow and its influence on reducing the time required for the velocity profiles to approach their steady state profiles. It is also clear from Fig. 1 that the velocity components F and H do not reach their steady state profiles monotonically with time. As time develops, both F and H decrease near the disk and increase far from it, which accounts for the crossing of each of the *F* and *H* profiles with time which is more pronounced for *H* than for *F*. Figure 2 indicates the marked effect of the porosity on shifting the crossover occurs in both *F* and *H* profiles far from the disk.



(a)

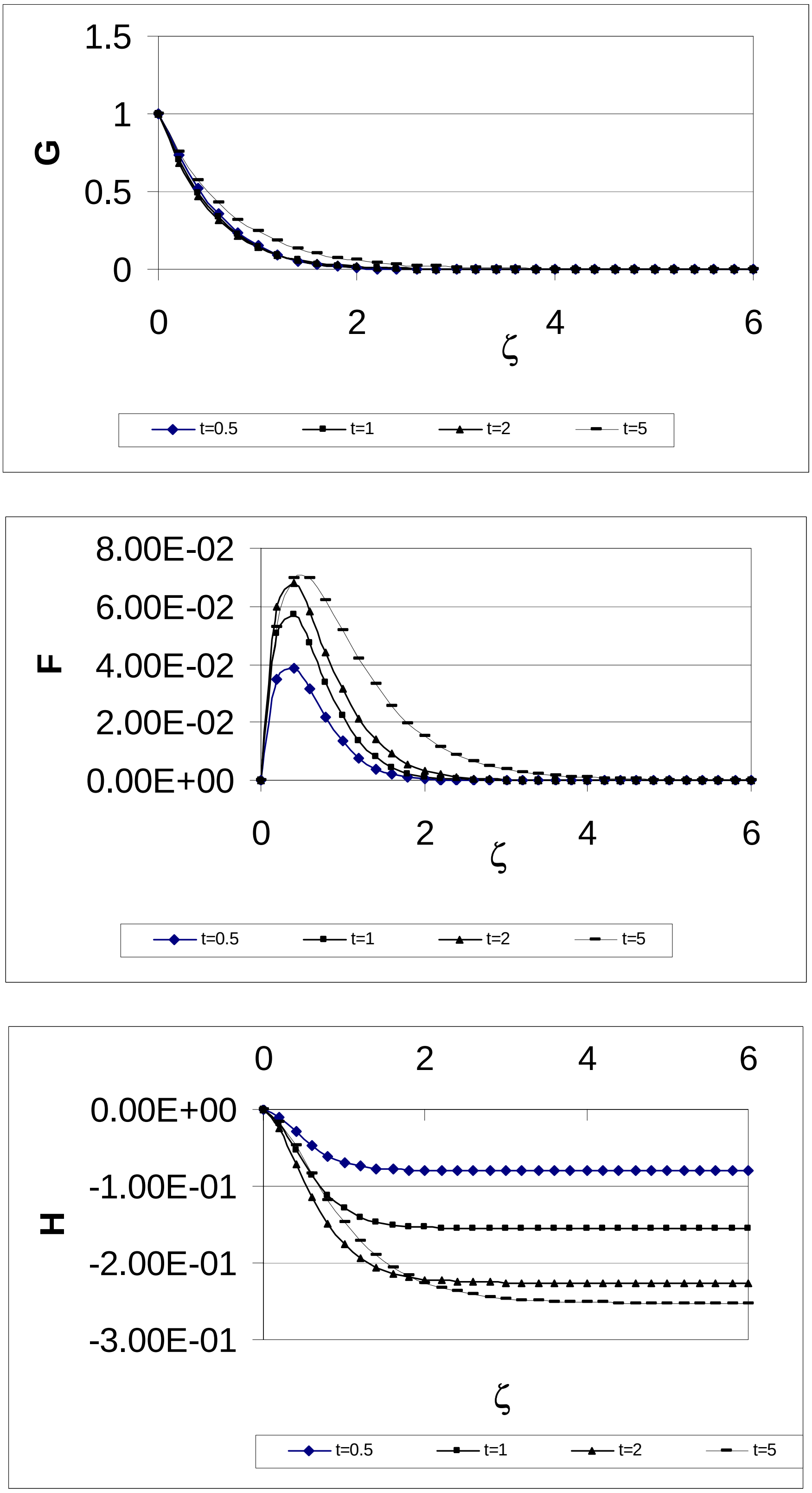


(b)



(c)

Fig. 1. - Time variation of the velocity profiles for M=0.



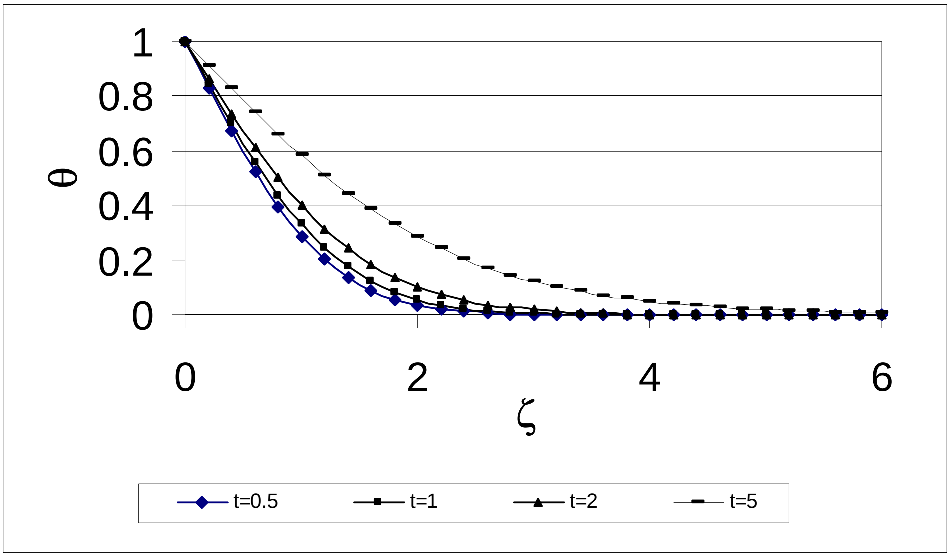
(a)

(b)

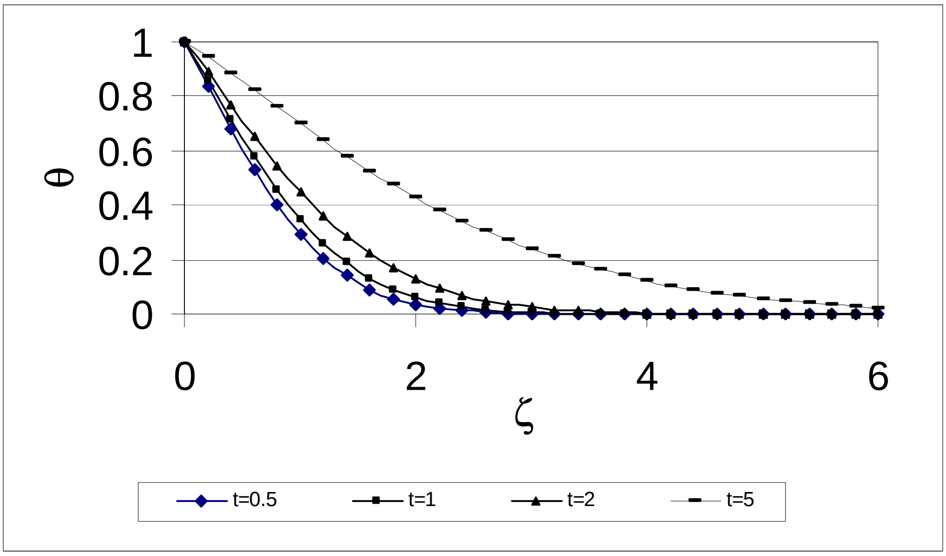
(c)

Fig. 2. - Time variation of the velocity profiles for M=1.

Figure 3 shows the evolution of the profile of the temperature θ with time for the cases M=0 and M=1, respectively and for Pr=0.7. It is shown in the figure that θ reaches the steady state monotonically. Also the figure indicates the influence of increasing the porosity parameter on increasing θ as a result of the effect of the porosity in preventing the fluid at near-ambient temperature from reaching the surface of the disk.



(a)



(b)

Fig. 3. - Time variation of the profiles θ for: (a) M=0; (b) M=1.

Figure 4 presents the time variation of the Nusselt number *Nu* respectively, for various values of the porosity parameter *M* and for *Pr*=0.7. It is clear from Fig. 4 that increasing *M*, which decreases the axial flow towards the disk, decreases *Nu* since the absence of the fluid at near-ambient temperature close to the surface of the disk increases the heat transfer. For small values of time *t*, *Nu* increases with time up till a maximum value which does not depend greatly on *M* due to the very small variation in θ. As time develops, the variation in θ with *M* increases and then *Nu* decreases.

Tables 1 and 2 present the variation of the radial wall shear, azimuthal wall shear, the axial inflow at infinity and the Nusselt number at the surface of the disk for various values of the parameter *n*, *m* and, respectively, and for *Ec*=0.2 , *Pr*=0.72.

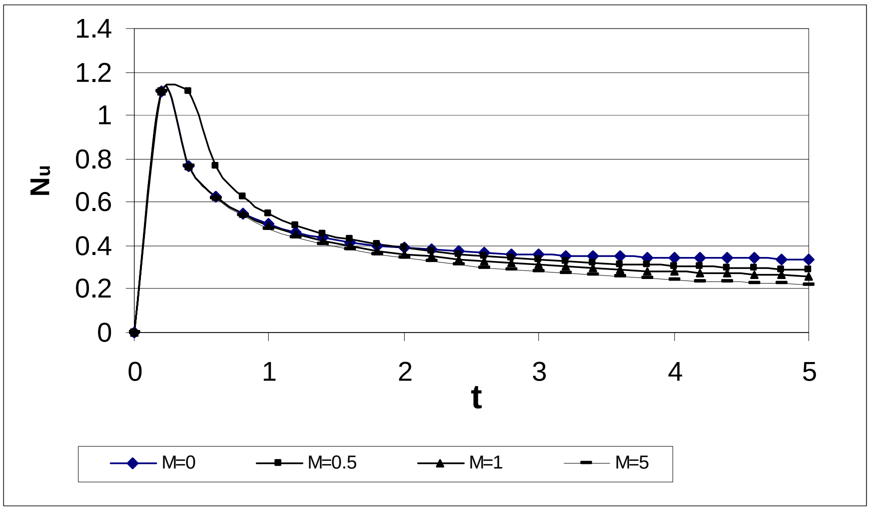


Fig. 4. - Effect of M on the time variation of Nu.

Table 1. - Variation of $F'(0)$ and $G'(0)$ for various values of M.

$F'(0)$			$-G'(0)$		
M=0	M=1	M=2	M=0	M=1	M=2
0.5102	0.3093	0.2306	0.6159	1.0691	1.4421

It is clear from Table 1 that increasing n increases the radial wall shear while decreases the azimuthal wall shear and its effect becomes more apparent for the non-porous case.

Table 2. - Variation of $H(\infty)$ and $\theta'(0)$ for various values of M and for Pr=0.72 and Ec=0.20.

$-H'(\infty)$			$-\theta'(0)$		
m=0	m=1	m=2	m=0	m=1	m=2
0.8845	0.2533	0.1086	0.2761	0.0715	0.0276

On the other hand, the influence of the parameter n on the axial inflow at infinity depends on the porosity parameter as illustrated in Table 2. Therefore, increasing n decreases the axial inflow towards the disk in the non-porous case, while slightly increases it in the porous case. Table 2 presents that increasing the parameter n decreases the heat transfer at the surface of the disk and, in turn, decreases the Nusselt number Nu. Increasing the porosity parameter m decreases the radial wall shear, the axial inflow at infinity, the heat transfer at the surface of the disk, but increases the azimuthal wall shear.

CONCLUSION

In this paper the transient flow through a porous medium due to a rotating disk was studied with heat transfer including dissipation. The presented results show the restraining effect of the porosity on the transient flow. On the other hand, increasing the porosity parameter increases the temperature of the fluid. It is observed that the radial and vertical components of the velocity do not reach their steady state profiles monotonically which results in crossing of the charts of these velocity components with time. The porosity of the medium has an important effect on pushing these crossing points far from the disk.

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