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UNSTEADY MHD FLOW AND HEAT TRANSFER BETWEEN PARALLEL POROUS PLATES WITH EXPONENTIAL DECAYING PRESSURE GRADIENT

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ABSTRACT. The unsteady magnetohydrodynamic flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non-conducting porous plates is studied with heat transfer. An external uniform magnetic field and a uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to an exponential decaying pressure gradient. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are included in the energy equation. The effect of the magnetic field and the uniform suction and injection on both the velocity and temperature distributions is examined.

INTRODUCTION

The magnetohydrodynamic flow between two parallel plates, known as Hartmann flow, is a classical problem that has many applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. HARTMANN and LAZARUS [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates. Then, a lot of research work concerning the Hartmann flow has been obtained under different physical effects [2-10].

In the present study, the unsteady magnetohydrodynamic flow and heat transfer of an incompressible, viscous, electrically conducting fluid between two infinite non-conducting horizontal porous plates are studied. The fluid is acted upon by an exponential decaying pressure gradient, a uniform suction and injection and a uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4, 5]. The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The equations of motion are solved analytically using the Laplace transform method while the energy equation is solved numerically taking the Joule and the viscous dissipations into consideration. The effect of the magnetic field, the Hall current, the ion slip, and the suction and injection on both the velocity and temperature distributions is studied.

DESCRIPTION OF THE PROBLEM

The two non-conducting plates are located at the $y=\pm h$ planes and extend from $x=-\infty$ to ∞ and $z=-\infty$ to ∞. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2>T_1$. The fluid flows between the two plates under the influence of an exponential decaying pressure gradient dP/dx in the x-direction, and a uniform suction from above and injection from below which are applied at t=0. The whole system is subjected to a uniform magnetic field B_o in the positive y-direction. This is the total magnetic field acting on the fluid since the induced magnetic field is neglected. From the geometry of the problem, it is evident that $\partial/\partial x=\partial/\partial z=0$ for all quantities apart from the pressure gradient dP/dx, which is assumed constant. The velocity vector of the fluid is

$$v(y,t) = u(y,t)i + v_o j$$

with the initial and boundary conditions u=0 at $t\leq 0$, and u=0 at $y=\pm h$ for $t>0$. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T=T_1$ at $t\leq 0$, $T=T_2$ at $y=+h$, and $T=T_1$ at $y=-h$ for $t>0$. The fluid flow is governed by the momentum equation

$$\rho\frac{\partial u}{\partial t} + \rho v_o\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \sigma B_o^2 u \tag{1}$$

where ρ , μ and σ are, respectively, the density, the coefficient of viscosity and the electrical conductivity of the fluid. To find the temperature distribution inside the fluid we use the energy equation [11]

$$\rho c\frac{\partial T}{\partial t} + \rho c v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_o^2 u^2, \tag{2}$$

where c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second and third terms on the right-hand side represent the viscous and Joule dissipations, respectively.

The problem is simplified by writing the equations in the non-dimensional form. The characteristic length is taken to be h, and the characteristic time is $\rho h^2/\mu^2$ while the characteristic velocity is $\mu/\rho h$. We define the following non-dimensional quantities

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{z} = \frac{z}{h}, \hat{u} = \frac{\rho h u}{\mu},, \hat{P} = \frac{P\rho h^2}{\mu^2}, t = \frac{t\mu}{\rho h^2},$$

$S = \rho v_o h / \mu,$ is the suction parameter,
$\Pr = \mu c / k$ is the Prandtl number,
$Ec = \mu^2 / \rho^2 c h^2 (T_2 - T_1)$ is the Eckert number,
$Ha^2 = \sigma B_o^2 h^2 / \mu$ where Ha is the Hartmann number,

In terms of the above non-dimensional variables and parameters, the basic Eqs. (1)-(2) are written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - Ha^2 u, \tag{3}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + EcHa^2 u^2, \tag{4}$$

The initial and boundary conditions for the velocity become

$$u = 0, t \le 0, u = 0, y = \pm 1, t > 0 \tag{5}$$

and the initial and boundary conditions for the temperature are given by

$$t \le 0 : T = 0, t > 0 : T = 1, y = +1, T = 0, y = -1. \tag{6}$$

Analytical solution of the equations of motion

Equation (3) is the equation of motion which, if solved, give the velocity field as functions of space and time. Equation (3) is a linear, inhomogeneous partial differential equation, which can be solved analytically using the Laplace transform (LT) method, under the initial and boundary conditions given by Eq. (5). Taking the LT of Eq. (3) gives

$$\frac{d^2 U(y,s)}{dy^2} - S\frac{dU(y,s)}{dy} - K(s)U(y,s) = -F(s) \tag{7}$$

where $U(y,s)$=L($u(y,t)$), $-F(s)$ is the LT of the pressure gradient and $K(s) = Ha^2 + s$. The solution of Eq. (7) with y as an independent variable is given as

$$U(y,s) = \frac{F(s)}{K}\left(1 + \exp(Sy/2)\left[\frac{\sinh(S/2)\sinh(qy)}{\sinh(q)} - \frac{\cosh(S/2)\cosh(qy)}{\cosh(q)}\right]\right)$$

where $q^2 = S^2/4 + K$. Using the complex inversion formula and the residue theorem [12], the inverse transform of $U(y,s)$ is determined as

$$u(y,t) = C\sum_{n=1}^{\infty}\Big(\frac{PI_1}{PN_1 + \alpha}\left(\exp(PN_1 xt) - \exp(-\alpha t)\right) + \frac{PI_2}{PN_2 + \alpha}\left(\exp(PN_2 xt) - \exp(-\alpha t)\right)$$

$$+\frac{PI_3}{PN_3+\alpha}\left(\exp(PN_3xt)-\exp(-\alpha t)\right)+\frac{PI_4}{PN_4+\alpha}\left(\exp(PN_4xt)-\exp(-\alpha t)\right)\ ) \qquad (8)$$

where

$$-\frac{dP}{dx}=C\exp(-\alpha t),$$

$$PN_1=PN_2=NN_1/2,$$

$$PN_3=PN_4=NN_2/2,$$

$$PI_1=\frac{NN_3}{Ha^2+PN_1},$$

$$PI_2=\frac{NN_3}{Ha^2+PN_2},$$

$$PI_3=\frac{NN_4}{Ha^2+PN_3},$$

$$PI_4=\frac{NN_4}{Ha^2+PN_4},$$

$$NN_1=-\pi^2(n-1)^2-S^2/4,$$

$$NN_2=-\pi^2(n-0.5)^2-S^2/4,$$

$$NN_3=2\pi(-1)^n(n-1)\exp(Sy/2)\sinh(S/2)\sin(\pi(n-1)y),$$

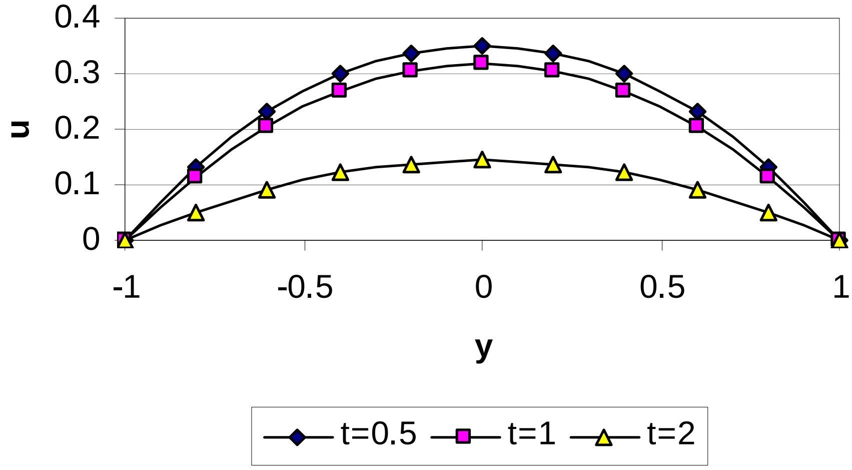
$$NN_4=2\pi(-1)^{n+1}(n-0.5)\exp(Sy/2)\cosh(S/2)\cos(\pi(n-0.5)y),$$

Numerical Solution of the Energy Equation

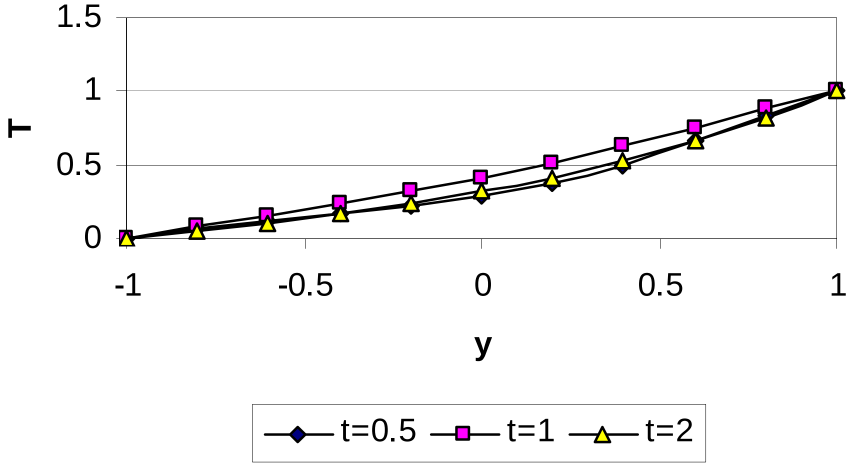
The exact solution of the equation of motion, given by Eq. (8), determines the velocity field for different values of the parameters *Ha* and *S*. The values of the velocity components, when substituted in the right-hand side of the inhomogeneous energy equation (4), make it too difficult to solve analytically. he energy equation is to be solved numerically with the initial and boundary conditions given by Eq. (6) using finite differences [13]. The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the *y*-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The viscous and Joule dissipation terms are evaluated using the velocity components and their derivatives in the *y*-direction which are obtained from the exact solution. Finally, the block tri-diagonal system is solved using Thomas' algorithm. Unlike the velocity *u*, the temperature distribution depends on *C*. All calculations have been carried out for C=1, α=1, Pr=1 and Ec=0.2.

RESULTS AND DISCUSSION

Figure 1 presents the velocity and temperature distributions as functions of y for different values of the time starting from t=0 to the steady state. Figures 1a and 1b are evaluated for Ha=1 and S=1. The velocity curves are asymmetric about the y=0 plane because of the suction as shown in Fig. 1a. It is observed that the velocity component u decreases monotonically with time, although the temperature T increases monotonically with t.

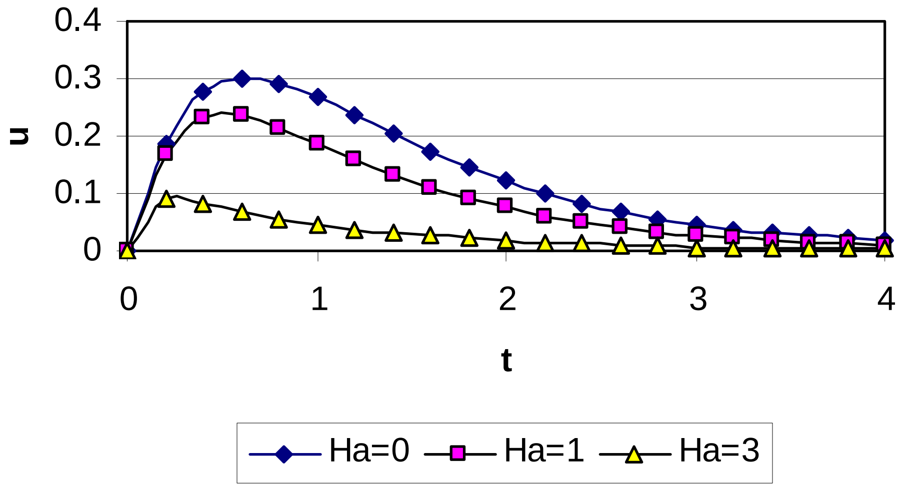


(a)

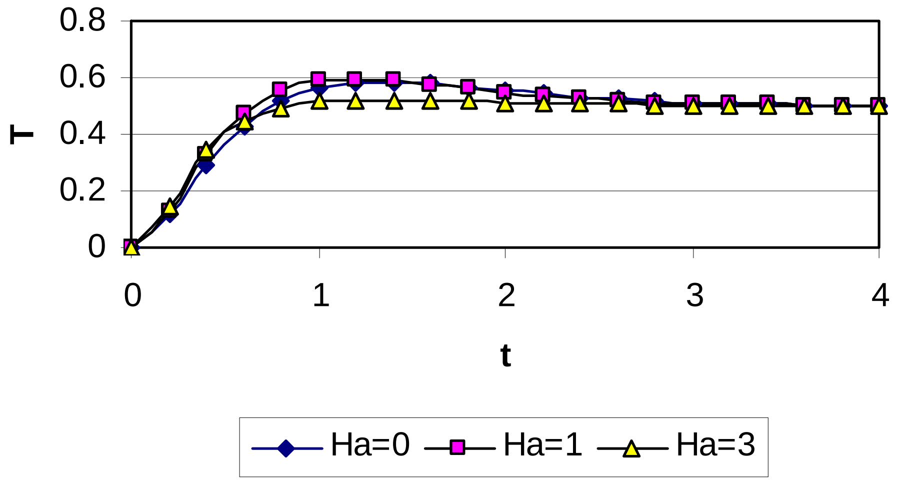


(b)

Fig. 1. - Time development of the profile of: (a) u; and (b) T (Ha=1 and S=1)



(a)



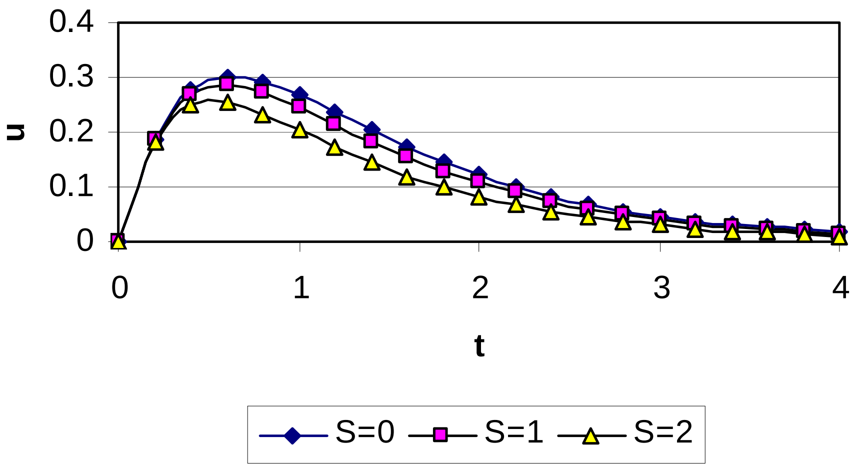
(b)

Fig. 2. - Effect of *Ha* on the time variation of: (a) *u* at *y*=0; (b) *T* at *y*=0. (*S*=0)

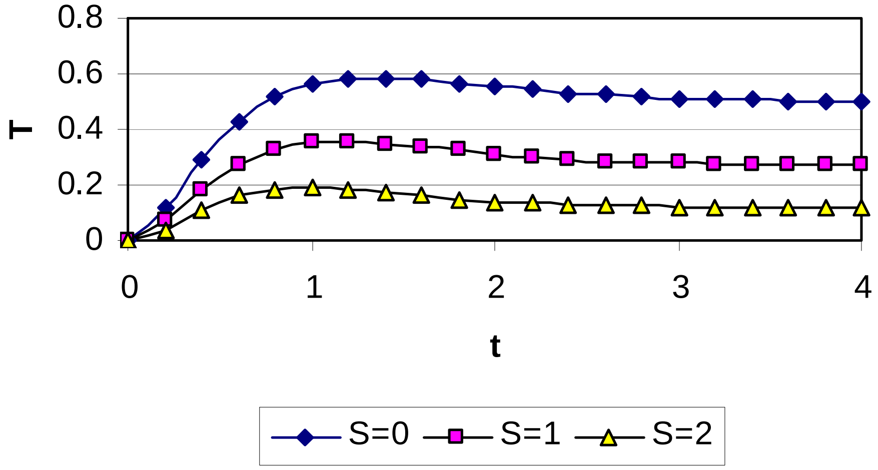
Figure 2 shows the effect of the Hartmann number *Ha* on the time development of the velocity *u* and temperature *T* at the centre of the channel (*y*=0). In this figure, *S*=0 (suction suppressed). It is clear from Fig. 3a that increasing the parameter *Ha* decreases *u* and its steady state time. This is due to increasing the magnetic damping force on *u*. Figure 2b and Table 1 indicate that increasing *Ha* increases *T* at small times but decreases it at large times. This can be attributed to the fact that, for small times, *u* is small and an increase in *Ha* increases the Joule dissipation which is proportional to *Ha* and therefore, the temperature increases. For large times, increasing *Ha* decreases *u* and, in turn, decreases the Joule and viscous dissipations and consequently decreases *T*. This accounts for crossing the curves of *T* with time for various values of *Ha*.

Table 1. - Time variation of the temperature at *y*=0 for various values *Ha* (*S*=0).

T	t=0.2	t=0.4	T=0.6	t=0.8	t=1	t=1.2	t=1.4	t=1.6	t=1.8	t=2
Ha=0	0.1156	0.2663	0.3599	0.4173	0.4519	0.4726	0.4847	0.4917	0.4957	0.4979
Ha=1	0.1159	0.2675	0.3617	0.4187	0.4529	0.4730	0.4848	0.4916	0.4955	0.4976
Ha=3	0.1168	0.2682	0.3608	0.4165	0.4500	0.4701	0.4822	0.4894	0.4937	0.4963



(a)



(b)

Fig. 3. - Effect of *S* on the time variation of: (a) *u* at *y*=0; (b) *T* at *y*=0. (*Ha*=0)

Figure 3 shows the effect of the suction parameter on the time development of the velocity *u* and temperature *T* at the centre of the channel (*y*=0). In this figure, *Ha*=0 (hydrodynamic case). In Fig. 3a, it is observed that increasing the suction decreases the velocity *u* at the center and its steady state time due to the convection of fluid from regions in the lower half to the center, which has higher fluid speed. In Fig. 3b, the temperature at the center is affected more by the convection term, which pumps the fluid from the cold lower half towards the centre.

CONCLUSION

The unsteady Hartmann flow of a conducting fluid under the influence of an applied uniform magnetic field and an exponential decaying pressure gradient has been studied in the presence of uniform suction and injection. The effect of the magnetic field and the suction and injection velocity on both the velocity and temperature distributions has been investigated. It is found that the magnetic field has a marked effect on the velocity distribution more than its effect on the temperature distribution. On the other hand, the suction and injection velocity has a more apparent effect on the temperature distribution than on the velocity distribution. It is of interest to see that the effect of the magnetic field on the temperature at the center of the channel depends on time. For small time, increasing the magnetic field increases the temperature, however, for large time, increasing the magnetic field decreases the temperature.

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