To Academic Council of the Doctoral School of Mathematics

EVALUATION REPORT OF THE DOCTORAL DISSERTATION

At the session of Academic Council of the Doctoral School of Mathematics, held on 18/04/2022, at the session of Academic Council of the Faculty of Science in Kragujevac, held on 27/04/2022 (No. 230/X-1), as well as at the session of the Council for Natural and Mathematical Sciences, University of Kragujevac, held on 18/05/2022 (No. IV-01-352/9), I was appointed as a member of the commission for evaluation and defense of the doctoral dissertation thesis written by the candidate **Bogdan Pirković** and titled:

"Contribution to the theory of random environment integer-valued autoregressive processes".

The candidate submitted the manuscript of his doctoral dissertation to the Academic Council of the Doctoral School of Mathematics and the Academic Council of the Faculty of Science in Kragujevac for evaluation. After a detailed review of the manuscript, I assessed the quality of the dissertation and pointed out to the candidate the corrections need to be made. The candidate adopted all the proposals and incorporated them into the final version of the dissertation, which allows me to submit the following

REPORT

1. Description of the doctoral dissertation

Doctoral dissertation "Contribution to the theory of random environment integer-valued autoregressive processes" belongs to the field of Mathematical statistics. Results of the previous research in the field of integer-valued autoregressive (INAR) time series created the starting point for conducted research. INAR processes were introduced, independently of each other, by McKenzie (1985) and Al-Osh and Alzaid (1987). Freeland (2010) was the first who introduced the INAR model that can take both positive and negative values, while Nastić et al. (2016) introduced the notion of random environment, and alongside with this, the random environment INAR model. Proposed dissertation discusses the development of INAR models that can take values over the entire set of integers, development of random environment INAR models and their possible combining.

Then text of the dissertation has been written in English. It contains 117 pages and consists of four chapters, a conclusion, a list of 50 references and three appendices. An integral part of the dissertation are abstracts in English and Serbian language, a word from the author and his biography. The manuscript itself contains 28 images and 21 tables.

Chapter 1 - INAR models-from the very beginnings to the present day. The first chapter is the introductory one and gives the theoretical basis for the research. It is designed to introduce the basic concepts and results that are further used in the dissertation. The chapter is divided into four sections. Section 1 provides some important definitions and the motivation for emergence of INAR models. Section 2 provides a historical overview of the INAR models development from its inception to the

present day. Section 3 deals with important distributions widely used in the following text. The final section lists the theorems, proved by other authors, which will be used for proving the theorems and propositions given in the following chapters of this dissertation.

Original results of the doctoral dissertation are presented in Chapter 2, Chapter 3 and Chapter 4, as well as in Appendix A, Appendix B and Appendix C.

Chapter 2 - Extracting and predicting latent components of the skewed TINAR(1) time series. This chapter is heavily relied on results given in Freeland (2010). The first two sections of this chapter describe in detail the symmetric and skewed TINAR(1) time series. These time series form the strong foundation for original results presented in the following sections. Statistics for extracting and predicting latent components of the skewed TINAR(1) time series are proposed in Section 3. An application of those statistics to the simulated data sequences is given in Section 4. Section 5 shows the application to the properly selected real-life data.

Chapter 3 - Random environment integer-valued autoregressive model with discrete Laplace marginal distributions. In this chapter, the candidate approached the construction of a new non-stationary INAR model in random environment that can take values from the entire set of integers. The construction of the model is given in Section 1. Section 2 describes some features of the model. Section 3 estimates unknown parameters of the model. The simulation study is presented in Section 4. In Section 5, the quality of the model is additionally examined on real data sequences.

Chapter 4 - Random environment estimation (RENES) method for generalized random environment INAR models of higher order. This chapter contains a unique adaptation of the K-means clustering technique. The modified technique, called RENES method, proved to be very suitable for estimating the environment states of realizations that correspond to generalized random environment INAR time series of higher order. Section 1 presents the construction of the RENES method. The next two sections deal with application of the newly introduced clustering technique to simulated data sets. The efficiency of the RENES method is confirmed on real-life data within the last section. In addition, many results concerning the application of the RENES method to simulated and real-life data are presented in Appendix A, Appendix B and Appendix C. In this way, the candidate has successfully increased the readability of the entire dissertation manuscript.

Conslusion. In conclusion, results of the entire dissertation are summarized. In addition, possible directions for further research are presented.

2. Significance and contribution of the doctoral dissertation regarding the current situation in a certain scientific field

Doctoral dissertation "Contribution to the theory of random environment integer-valued autoregressive processes" belongs to the field of Mathematics, and the corresponding subfield is Mathematical statistics. The first random environment INAR model has been introduced in 2016, and numerous generalizations of this model have emerged later. The given results from the literature created a good starting point for making new research and obtaining original scientific results.

In this dissertation, candidate Bogdan Pirković keeps forcing two research directions. The first one refers to the construction of the random environment integer-valued autoregressive time series that can take both, positive and negative values. The realization of this goal would create new possibilities

in integer-valued data modeling. In addition, since the environment state estimation of each individual realization is one of the crucial steps in real-life modeling by usage of models in random environment, the second direction refers to adapting existing clustering procedures in order to make environment state estimates as accurate as possible.

The dissertation presents original results obtained as a product of the candidate's scientific research. First of all, new statistics for extracting and predicting latent components of the skewed TINAR(1) time series have been introduced. Further, a new non-stationary random environment INAR model with discrete Laplace marginal distributions has been constructed. This model enabled more efficient modeling of the data that take both positive and negative values. Furthermore, the dissertation offers the new method for estimating environment states of the data corresponding to the generalized random environment INAR models of higher order. This method, called RENES, has proven to be more efficient than the standard K-means technique mainly used for environment state estimation so far.

All hypotheses rely on well known results from the field of INAR time series. First of all, candidate assumed that is possible to construct new random environment INAR models that can take values over entire set of integers by combining stationary INAR models with positive and negative values and nonnegative random environment INAR models. The second hypothesis assumed that is possible to improve performances of the K-means clustering method, used to estimate environment states of random environment INAR models of higher order, by including estimates of all parameters of the model (distribution parameter, thinning parameter and model order) in the clustering procedure.

The dissertation contains methods widely used for new models introduction. First of all, new models are constructed in the way to satisfy the set criteria regarding the stationarity and allowable model values. Further, new techniques for unknown parameter estimation are derived. New techniques represent an adaptation of familiar techniques to the specific problem of parameter estimation in newly introduced random environment INAR models. The efficiency of estimates such obtained is tested on simulated data. Finally, the model quality is examined on adequately selected real-life data, whereby the level of the model quality is determined using the root mean square (RMS) error value.

3. Evaluation of the originality of results presented in the doctoral dissertation

Based on the insight into the existing research and scientific contributions in the field of Mathematical statistics, I confirm that the doctoral dissertation of the candidate Bogdan Pirković is an original scientific work. The results presented in doctoral dissertation have not been the subject of previous research.

4. Review of achieved scientific results from the candidate's doctoral dissertation

Candidate Bogdan Pirković has two scientific papers in international journals from the SCI list (category M22), as well as one statement from the international confererence (category M34). **All scientific attaivements that the candidate has achieved so far are based on results presented in his doctoral dissertation.** In this way, the candidate successfully fulfilled all the conditions for defending the dissertation provided by the University of Kragujevac Rulebook on application, preparation and defense of the doctoral dissertation and Doctoral School of Mathematics Rulebook on

realization of joint doctoral academic studies. In addition, the candidate has one more scientific paper that relies on results from the dissertation. This paper is in the process of being published.

Scientific papers published in scientific journals of international importance (M20)

[1] M. S. Đorđević, M. M. Ristić, **B. A. Pirković** (2021) Identifying latent components of the TINAR model, *Filomat*, **35**(13), 4469-4482. (**ISSN**:0354-5180, **M22**, **IF**2020=0.844, 197/330, **Category**: Mathematics)
https://doi.org/10.2298/FIL2113469D

[2] **B. A. Pirković,** P. N. Laketa, A. S. Nastić (2021) On generalized random environment INAR models of higher order: estimation of random environment states, *Filomat*, **35**(13), 4545-4576. (**ISSN**:0354-5180, **M22**, **IF**2020=0.844, 197/330, **Category**: Mathematics)
https://doi.org/10.2298/FIL2113545P

Statements from conferences of international importance (M34)

[1] **B. A. Pirković,** P. N. Laketa, A. S. Nastić (2021) Random Environment Estimation Method for Generalized Random Environment INAR Models of Higher Order, *Third International Workshop on Nonlinear Analysis and its Applications,* Niš, Serbia, October 13-16, 2021 Book of Abstracts-page 40.

5. Assessment of the fulfillment of the scope and quality of the dissertation in relation to the applied topic

The scope of the research work, goals, hypotheses and methodological approach in the research realization, planned within the application of dissertation topic, have been fully realized.

6. Applicability of results in theory and practice

The most important theoretical (scientific) results, that represent the scientific contribution of this dissertation, are:

- formulation of statistics for extracting and predicting latent components of the skewed TINAR(1) time series, provided that one realization sequence of the mentioned time series is known;
- estimation of unknown parameters of the skewed TINAR(1) time series;
- construction of a new non-stationary INAR model in random environment that can take both positive and negative values;
- collection of theorems describing the properties of the newly introduced model;
- adaptation of the Yule-Walker method and Conditional least squares method in order to successfully estimate unknown model parameters;
- construction of a new clustering technique used to estimate environment states of realizations corresponding to the generalized random environment INAR models of higher order.

Applicability of results in theory. A generalization of results from the second and third chapter of the doctoral dissertation leads to solving the problem of modeling non-stationary integer-valued data

sequences with more complex correlation structure. In addition, the process of adapting the K-means technique, outlined in the fourth chapter, can be applied to other clustering techniques.

Applicability of results in practice. All results presented in the doctoral dissertation are successfully applied to the appropriate real-life data sequences. In this way, the applicability of results in practice is unequivocally confirmed. Beside the applications described in the dissertation text, given results can find practical application in medicine, telecommunications, gambling and many other aspects of life.

7. Ways of presenting the results to the scientific public

Scientific results of the doctoral dissertation were presented to the public through two scientific papers in prominent international journals (category M22). Some of the results are presented in the third paper, which is in the process of being published. Beside this, a portion of results from the doctoral dissertation was presented to the scientific public at international conference *Third International Workshop on Nonlinear Analysis and its Applications.*

CONCLUSION

Dissertation manuscript of the candidate Bogdan Pirković entitled **"Contribution to the theory of random environment integer-valued autoregressive processes"** is an original science achievement in the field of Mathematical statistics under the mentorship of **prof. dr Aleksandar Nastić.**

Original scientific results presented in the dissertation, such as: new statistics for extracting and predicting latent components of the skewed TINAR(1) time series, new non-stationary INAR model in random environment with discrete Laplace marginal distributions and improved method for estimating environment states of the data that correspond to the generalized random environment INAR models of higher order, brought new possibilities in modeling data sequences with integer values and data sequences of non-stationary nature.

Results presented in this doctoral thesis have a potential to contribute to the development of new research, primarily within the area of Mathematical Statistics, but also in the other areas of mathematics, such as Stochastic Analysis among others.

The quality of scientific results of the doctoral dissertation has been confirmed by publishing two papers in prominent international journals from the SCI list (category M22) and by the statement from the conference of international importance (category M34).

Accordingly, I am of the opinion that all scientific, professional and administrative conditions for accepting the proposed doctoral dissertation as an original scientific work have been fulfilled. **In that sense, my position on the proposed doctoral dissertation is: THE DISSERTATION IS ACCEPTED.** In this regard, I suggest to Academic Council of the Doctoral School of Mathematics to approve the public defense of the doctoral dissertation to Bogdan Pirković under the stated title.

In Niš, July 01, 2022

Marija Milošević

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