

EVALUATION REPORT OF THE DOCTORAL DISSERTATION

Pursuant to the Decision by the Council for Natural and Mathematical Sciences of the University of Kragujevac no. IV-01-352/9 from 18 May 2022, and based on the Recommendation by the Teaching and Scientific Council of the Faculty of Science in Kragujevac no. 230/X-1 from 27 April 2022 and the Recommendation by the Council of the PhD School of Mathematics from 18 April 2022, I have been appointed member of the committee tasked with writing a report on the evaluation of the scientific merit of the dissertation with topic **“Contribution to the Theory of Random Environment Integer-Valued Autoregressive Processes”** and as a member of the thesis defence committee.

The PhD candidate Bogdan Pirković has submitted the manuscript of his doctoral dissertation to the Council of the PhD School of Mathematics and the Scientific Council of the Faculty of Science in Kragujevac for evaluation. After a detailed review of the manuscript, I assessed the quality of the dissertation and pointed out to the candidate the corrections need to be made. The candidate adopted all the proposals and incorporated them into the final version of the dissertation, I hereby submit the following

REPORT

1. Description of the doctoral dissertation

The doctoral dissertation „Contribution to the theory of random environment integer-valued autoregressive processes” belongs to the field of mathematical statistics. It is based on the research of the integer-valued autoregressive models (INAR). Some new INAR models are introduced, also the improvement of the already known models has been discussed. The practical aspect of the obtained results has been discussed through the application on real data.

The dissertation has been written in English. It contains 117 pages and consists of four chapters, a conclusion, a list of 50 references and three appendices. An integral part of the dissertation are abstracts in English and Serbian language, a word from the author and his biography. The dissertation itself contains 28 images and 21 tables.

Chapter 1 - INAR models-from the very beginnings to the present day. The first chapter is the introductory one and gives the theoretical basis for the research. It is designed to introduce the basic concepts and results that are further used in the dissertation. The chapter is divided into four sections. Section 1 provides some important definitions and the motivation for emergence of INAR models. Section 2 provides a historical overview of the INAR models development from its inception to the present day. Section 3 deals with important distributions widely used in the following text. The fourth section lists the theorems, proved by other authors, which will be used for proving the theorems and propositions given in the following chapters of this dissertation.

Original results of the doctoral dissertation are presented in Chapter 2, Chapter 3 and Chapter 4, as well as in Appendix A, Appendix B and Appendix C.

Chapter 2 - Extracting and predicting latent components of the skewed TINAR(1) time series. The first two sections give detail discussion about INAR models for time series with symmetric and asymmetric Skellam marginal distribution, where the time series can have positive and negative integer values. Statistics for extracting and predicting latent components of the skewed TINAR(1) time series are proposed in Section 3. An application of those statistics to the simulated data sequences is given in Section 4. Section 5 contains the application to real-life data.

Chapter 3 - Random Environment Integer-Valued Autoregressive model with discrete Laplace marginal distribution. In this chapter, the candidate approached the construction of a new non-stationary INAR model in random environment that can take values from the entire set of integer numbers. The construction of the model is given in Section 1. Section 2 describes some features of the model. Section 3 estimates unknown parameters of the model. The simulation study is presented in Section 4. In Section 5, the quality of the model is additionally examined on real data sequences.

Chapter 4 - Random environment estimation (RENES) method for generalized random environment INAR models of higher order. This chapter contains a unique adaptation of the K-means clustering technique. The modified technique, called RENES method, proved to be very suitable for estimating the environment states of realizations that correspond to generalized random environment INAR time series of higher order. Section 1 presents the construction of the RENES method. The next two sections deal with application of the newly introduced clustering technique to simulated data sets. The efficiency of the RENES method is confirmed on real-life data within the last section. In addition, many results concerning the application of the RENES method to simulated and real-life data are presented in Appendix A, Appendix B and Appendix C. In this way, the candidate has successfully increased the readability of the entire dissertation manuscript.

Conclusion. In conclusion, results of the entire dissertation are summarized. In addition, possible directions for further research are presented.

2. Significance and contribution of the doctoral dissertation regarding the current situation in a certain scientific field

The doctoral dissertation "Contribution to the theory of random environment integer-valued autoregressive processes" belongs to the field of Mathematics, and the corresponding subfield is Mathematical statistics. The research subject are random environment integer-valued autoregressive models. This type of models can be found in the literature in the past decade. Some of the main results from this topic have been mentioned, and they represent the foundation for the dissertation research. New random environment INAR models have been introduced. Also, much of the attention and the novelties have been given that concern the estimation of the random states, as one of the most important component of these models.

The dissertation presents original results obtained as a product of the candidate's scientific research. First of all, new statistics for extracting and predicting latent components of the skewed TINAR(1) time series have been introduced. Further, a new non-stationary random environment INAR model with discrete Laplace marginal distributions has been constructed. This model enabled more efficient modeling of the data that take both positive and negative values. Furthermore, the dissertation offers the new method for estimating environment states

of the data corresponding to the generalized random environment INAR models of higher order. This method, called RENES, has proven to be more efficient than the standard K-means technique mainly used for environment state estimation so far.

All hypotheses rely on well known results from the field of INAR time series. First of all, candidate assumed that is possible to construct new random environment INAR models that can take values over entire set of integers by combining stationary INAR models with positive and negative values and nonnegative random environment INAR models. The second hypothesis assumed that is possible to improve performances of the K-means clustering method, used to estimate environment states of random environment INAR models of higher order, by including estimates of all parameters of the model (distribution parameter, thinning parameter and model order) in the clustering procedure.

3. Evaluation of the originality of results presented in the doctoral dissertation

Based on the insight into the existing research and scientific contributions in the field of Mathematical statistics, I confirm that the doctoral dissertation of the candidate Bogdan Pirković is an original scientific work. The results presented in doctoral dissertation have not been the subject of previous research.

4. Review of achieved scientific results from the candidate's doctoral dissertation

Bogdan Pirković is engaged in research work in the field of Mathematical Statistics. He has published two scientific paper related to his doctoral dissertation field of research in a journals from the SCI list (category M22), as well as one statement from the international conference (category M34). **All scientific achievements that the candidate has accomplished so far are based on the results presented in his doctoral dissertation.** In this way, the candidate successfully fulfilled all the conditions for defending the dissertation provided by the University of Kragujevac Rulebook on application, preparation and defense of the doctoral dissertation and PhD School of Mathematics Rulebook on realization of joint doctoral academic studies.

Scientific papers published in scientific journals of international importance (M20)

- [1] M. S. Đorđević, M. M. Ristić, **B. A. Pirković** (2021) Identifying latent components of the TINAR model, *Filomat*, **35**(13), 4469-4482. (ISSN:0354-5180, **M22**, IF2020=0.844, 197/330, **Category: Mathematics**)
<https://doi.org/10.2298/FIL2113469D>
- [2] **B. A. Pirković**, P. N. Laketa, A. S. Nastić (2021) On generalized random environment INAR models of higher order: estimation of random environment states, *Filomat*, **35**(13), 4545-4576. (ISSN:0354-5180, **M22**, IF2020=0.844, 197/330, **Category: Mathematics**)
<https://doi.org/10.2298/FIL2113545P>

Statements from conferences of international importance (M34)

- [1] **B. A. Pirković**, P. N. Laketa, A. S. Nastić (2021) Random Environment Estimation Method for Generalized Random Environment INAR Models of Higher Order, *Third*

5. Assessment of the fulfillment of the scope and quality of the dissertation in relation to the applied topic

The scope of the research work, goals, hypotheses and methodological approach in the research realization, planned within the application of dissertation topic, have been fully realized.

6. Applicability of results in theory and practice

The most important theoretical (scientific) results, that represent the scientific contribution of this dissertation, are:

- formulation of statistics for extracting and predicting latent components of the skewed TINAR(1) time series, provided that one realization sequence of the mentioned time series is known;
- estimation of unknown parameters of the skewed TINAR(1) time series;
- construction of a new non-stationary INAR model in random environment that can take both positive and negative values;
- collection of theorems describing the properties of the newly introduced model;
- adaptation of the Yule-Walker method and Conditional least squares method in order to successfully estimate unknown model parameters;
- construction of a new clustering technique used to estimate environment states of realizations corresponding to the generalized random environment INAR models of higher order.

Applicability of results in theory. The dissertation contains a generalization of some INAR models which increase the usefulness of these models on the entire set of integer numbers. The results presented through the theorems can be used for some future researches for this field of science. In addition, the process of adapting the K-means technique, outlined in the fourth chapter, can be applied to other clustering techniques.

Applicability of results in practice. All results presented in the doctoral dissertation are successfully applied to the appropriate real-life data sequences. In this way, the applicability of results in practice is unequivocally confirmed. Beside the applications described in the dissertation text, given results can find practical application in medicine, telecommunications, gambling and many other aspects of life.

7. Ways of presenting the results to the scientific public

Scientific results of the doctoral dissertation were presented to the public through two scientific papers in prominent international journals (category M22). Beside this, a portion of results from the doctoral dissertation was presented to the scientific public at international conference *Third International Workshop on Nonlinear Analysis and its Applications*.

CONCLUSION

Dissertation manuscript of the candidate Bogdan Pirković entitled "**Contribution to the theory of random environment integer-valued autoregressive processes**" is an original scientific achievement in the field of Mathematical statistics under the supervision of **prof. dr Aleksandar Nastić**.

The quality of scientific results of the doctoral dissertation has been confirmed by publishing two papers in prominent international journals from the SCIE list (category M22) and by the statement from the conference of international importance (category M34).

Accordingly, I am of the opinion that all scientific, professional and administrative conditions for accepting the proposed doctoral dissertation as an original scientific work have been fulfilled. In that sense, my position on the proposed doctoral dissertation is: **THE DISSERTATION IS ACCEPTED**. In this regard, I suggest to Academic Council of the Doctoral School of Mathematics to approve the public defense of the doctoral dissertation to Bogdan Pirković under the stated title.

Therefore, it is my pleasure to recommend to the Council of the PhD School of Mathematics that they approve the topic entitled "**Contribution to the Theory of Random Environment Integer-Valued Autoregressive Processes**" as the doctoral dissertation topic and that they allow the candidate, **Bogdan Pirković**, to write the said dissertation under the supervision of **professor Aleksandar Nastić, PhD**.

In Kragujevac,
10 July 2022

COMMITTEE MEMBER

Predrag Popović, PhD, associate professor
Faculty of Civil Engineering and Architecture, University of Niš
Narrow scientific field: Mathematics


