Kragujevac J. Sci. 40 (2018) 23-32. UDC 539.1:537.561

The effects of the perturbated ionization potential and the Magnetic Component on the relativistic transition rate

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(Received April 2, 2018; Accepted May 23, 2018)

ABSTRACT. In this paper we theoretically analyze the influence of the perturbated ionization potential and the component of the magnetic field on the transition rate, in a relativistic, linearly polarized laser field. The obtained results indicate that both effect play important roles during the ionization process, influence the rate, and must be considered. It is shown that a laser field influences the ionization potential of an electron strongly and causes that the rate curve shows significantly different behavior compared to the case with considered non-perturbated ionization potential.

Keywords: Gaussian shaped laser pulse, effective ionization potential, Lorentz ionization.

INTRODUCTION

Photoionization under intense laser irradiation is a fundamentally important process in interaction of atoms and molecules with strong field (MEUREN *et al.*, 2016; SONG *et al.*, 2016). The current attosecond streaking technologies allow one to study the photoionization process, both experimentally and theoretically (MARTINI *et al.*, 2017; YU *et al.*, 2017; SERKEZ *et al.*, 2018).

The theory of photoionization processes originates with the work by KELDYSH (1965), who showed for the first time that the tunneling and multiphoton ionization of atoms are two limiting cases of nonlinear photoionization, whose character depends strongly on the value of the adiabaticity parameter γ. This parameter, introduced by Keldysh, is defined as the ratio between the "tunneling time", τ_k, and laser oscillation period T_0 (KELDYSH, 1965), $\gamma = \frac{\tau_k}{T_0}$, or, expressed in frequency, the ration of the field frequency, ω and the tunneling frequency, ω_t, $\gamma = \frac{\omega}{\omega_t}$. The tunneling frequency is estimated by $\omega_t = eF/\sqrt{2mI_p}$, where e is electron charge, F is the laser field strength, m is the electron mass and I_p is the unperturbed ionization potential. In atomic units (MCWEENY, 1973) ($e = m = \hbar = 4\pi\varepsilon_0 = 1$), adiabaticity parameter γ has the following form $\gamma = \omega\sqrt{\frac{I_p}{2U_p}}$, where U_p is ponderomotive potential. The range $\gamma \ll 1$, corresponds to the tunneling ionization limit, while in the multiphoton regime $\gamma \gg 1$. Several theories resembling Keldysh's original proposal have been extended through the PPT theory (PERELOMOV *et al.*, 1966), the ADK theory (AMMOSOV *et al.*, 1986), and the KFR theory (KELDYSH, 1965; FAISAL, 1973; REISS, 1980; REISS, 1992]. According to strong-field theories an atom subjected to a specific strong laser field is entirely determined by its ionization potential energy.

With increasing laser field intensity also increases the ionization potential of an atom. It has been shown (PETROVIĆ and MILADINOVIĆ, 2015) that the influence of the ponderomotive potential and the Stark effect shifts up the ionization threshold of an atom. Ponderomotive potential is the mean energy stored in the quiver motion of a free electron in an external alternating electric field. This energy is defined as $U_p = \frac{F^2}{4\omega^2}\frac{1-\epsilon^2}{1+\epsilon^2}$, where ϵ is ellipticity (MILADINOVIĆ and PETROVIĆ, 2016), for non-relativistic domain. For a linearly polarized laser field ellipticity is $\epsilon = 0$, and U_p becomes $U_p = \frac{F^2}{4\omega^2}$. The influence of the ponderomotive potential on the field free ionization potential becomes larger and more significant with increasing laser field intensity (DELONE and KRAINOV, 1998). For relativistic intensities the ponderomotive potential may be written in the following form (GHEBREGZIABHER, 2008):

$$U_p^{rel} = \sqrt{c^4 + 2c^2 U_p} - c^2 = \sqrt{c^4 + 2c^2 \frac{F^2}{4\omega^2}} - c^2 \qquad (1)$$

where $\mathrm{c} = 137.02$ is the speed of light in atomic units.

Atom's energy levels are altered in laser field and this effect is known as the Stark effect. This displacement of the energy level is determined by expression:
$E_{st} = \alpha F^2/4$ (VOLKOVA *et al.*, 2011), where α is the static polarizability of the atom (http://ctcp.massey.ac.nz/Tablepol2014.pdf). The Stark effect has the same form, E_{st}, as in the non-relativistic domain. Bearing all this in mind, the perturbated ionization potential can be expressed as: $I_{p,eff} = I_p + U_p + E_{st} = I_p + F^2/4\omega^2 + \alpha F^2/4$ (VOLKOVA *et al.*, 2011).

As the field intensity increases indefinitely, an electron in a laser field would exhibit relativistic behavior (KRAINOV, 1998; POPOV, 2004). It was found that the ADK theory (GHEBREGZIABHER, 2008) fits the experimental data very well. Also, in (MILOSEVIC *et al.*, 2002; REISS, 2008) in shown that, for these laser field intensities, the electric and magnetic fields become equally important in describing photoionization process.

In this paper, we observed and discussed the influence of the magnetic component on relativistic tunneling transition rate in linearly polarized laser field and how the relativistic ponderomotive and the Stark shift effect the rate.

INFLUENCE OF THE MAGNETIC FIELD COMPONENT ON THE TRANSITION RATE

Accordingly (LANDSMAN *et al.*, 2014) with increasing the laser intensity, it should be expected that the height of the tunneling barrier decreases and the shape of the barrier changes qualitatively. In intense laser fields an electron reaches relativistic velocities already within one laser period, the magnetic component of the Lorentz force becomes of the same order of magnitude of the electric one, and the electron's motion becomes highly nonlinear as a function of the laser's electromagnetic field. The magnetic component of the linearly polarized laser field induces a drift of the electron in the laser propagation direction. Such strong lasers can no longer be treated as pure electric fields and the laser magnetic field component must be considered, too (ZHAKENOVICH *et al.*, 2015). Described processes become significant for the electron dynamics during the ionization process. True relativistic effects do not set in before about 10^{17} W/cm^2 (REISS, 2000; REISS, 2001; KRAINOV and SOFRONOV, 2008).

It was shown in (MILADINOVIĆ and PETROVIĆ, 2015) that relativistic ionization rate along the field strength decreases exponentially with the electron kinetic energy, but more quickly than in the non-relativistic case. Criteria that characterize the onset of magnetic field effects as well as the onset for relativistic treatment of the ionization process have been formulated (REISS, 2014; JOACHAIN *et al.*, 2012). The onset of the influence of the magnetic field effects, becomes noticeable already at significantly smaller intensities and higher frequencies than those required to achieve this condition. In order to the widespread deployment of Ti:sapphire laser systems, the majority of experiments in strong field science are performed at wavelengths around 800 nm, where the onset of magnetic field effects can occur at about 10^{15} W/cm^2 (VOLKOVA *et al.*, 2011).

Under the frame of tunneling theory, the magnetic effects set a lower limit on Keldysh adiabaticity parameter γ (REISS, 2010). According to Reiss (REISS, 2018) the $\gamma \to 0$ limit is an extreme relativistic limit. In that case, the relativistic Keldysh parameter must be introduced (PETROVIĆ, 2015):

$$\gamma_{rel} = \frac{\omega c}{F}\sqrt{1 - \left(\left(c^2 - \frac{Z^2}{2}\right)/c^2\right)^2} \tag{2}$$

where Z is the ion charge.

To include relativistic treatment, we replaced the ground bound state I_p with the shifted energy for the relativistic domain $I_p^{rel} = c^2 - \sqrt{c^4 - Z^2c^2}$ (YAKABOYLU *et al.*, 2013).

Now, we can express effective ionization potential for the relativistic laser field intensity $I_{p,eff}^{rel}$ as (GHEBREGZIABHER, 2008):

$$I_{p,eff}^{rel} = I_p^{rel} + U_p^{rel} + E_{st} = c^2 - \sqrt{c^4 - Z^2c^2} + \sqrt{c^4 + 2c^2\frac{F^2}{4\omega^2}} - c^2 + \frac{\alpha F^2}{4} \tag{3}$$

For the relativistic intensities, the ADK expression, W_{rel}, with the correction for non-zero initial momentum of the photoelectron has the form (DELONE *et al.*, 1993):

$$W_{rel} = W_{nonrel}\mathrm{Exp}\left[-\frac{2E_e{\gamma_{rel}}^3}{3\omega} - \frac{E_e^2\gamma_{rel}}{c^2\omega}\right] \tag{4}$$

where E_e is the relativistic kinetic energy of ejected photoelectrons, $E_e = \sqrt{p^2c^2 + c^4} - c^2$ (KRAINOV, 1998) and W_{nonrel} is the non-relativistic total tunneling ionization rate $W_{nonrel} = \left(\frac{4Z^3}{Fn^{*4}}\right)^{n^*}\mathrm{Exp}\left[-\frac{2Z^3}{3Fn^{*3}} - \frac{p^2{\gamma}^3}{3\omega}\right]$ (AMMOSOV *et al.*, 1986), where n^* is the effective principal quantum number, $n^* = \frac{Z}{\sqrt{2I_p}}$, and p denotes the longitudinal component of the initial momentum (BAUER, 2006).

For purpose of incorporating the magnetic component of the laser field in the relativistic transition rate, we shall extend Eq. (4). One of the semi-analytical analyses of tunneling process that examine how the magnetic component of the laser field influences the relativistic transition rate is formulated as the Lorentz ionization (ZHAKENOVICH *et al.*, 2015):

$$W_L = \left(1 - v^2\right)^{1/2} S\, W_{rel} \tag{5}$$

where, W_{rel} is already defined (Eq.4) relativistic transition rate of the atom under the influence of an electric field only (DELONE *et al.*, 1993), v is the electron velocity and S is the stabilization factor (ZHAKENOVICH *et al.*, 2015). To express electron velocity v, we focused on the momentum of ejected photoelectrons in the form (KRAINOV, 1998):

$$p = -\frac{2I_{p,eff}^{rel}}{3c} = -\frac{2}{3c}\left(I_p^{rel} + U_p^{rel} + E_{st}\right) = \frac{2}{3}\left(\sqrt{c^2 - Z^2} - \sqrt{c^2 + \frac{F^2}{2\omega^2} - \frac{\alpha F^2}{4c}}\right) \quad (6)$$

By definition, relativistic momentum p is classical momentum multiplied by the relativistic factor γ_L, $p = m\, v\, \gamma_L = (m = 1) = v\, \gamma_L$, where $\gamma_L = 1/\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ is the relativistic factor (also known as Lorentz factor) (LORENTZ *et al.*, 1952).

For the sake of performing necessary calculations for Lorentz transition rate, we needed an expression for the electron velocity v. We obtained it by combining the expression for the momentum of the ejected photoelectron, Eq. 6, and the electron moment definition:

$$v^2 = \frac{4c^2\left(\sqrt{c^2-Z^2} - \sqrt{c^2+\frac{F^2}{2\omega^2}-\frac{\alpha F^2}{4c}}\right)^2}{\left(9c^2+4\left(\sqrt{c^2-Z^2} - \sqrt{c^2+\frac{F^2}{2\omega^2}-\frac{\alpha F^2}{4c}}\right)^2\right)}. \quad (7)$$

Finally, substituting Eq. 4 and Eq. 7 into Eq. 5 the following expression is obtained:

$$W_L{}^{Corr} = \left(1 - \frac{4\left(I_{p,eff}^{rel}\right)^2}{9c^2+\frac{4}{c^2}\left(I_{p,eff}^{rel}\right)^2}\right)^{1/2} \times S \times \left(\frac{4Z^3 e}{Fn^{*4}}\right)^{n^*} \times \mathrm{Exp}[-\frac{2Z^3}{3Fn^{*3}} - \frac{4\omega^3 p^2}{F^{3/2}c^2} +$$

$$+\frac{2E_e\gamma_{rel}{}^3}{c^2\omega} - \frac{E_e{}^2\gamma_{rel}}{c^2\omega}] \quad (8)$$

i.e. in developed form:

$$W_L{}^{Corr} = \left(1 - \frac{4c^2\left(\sqrt{c^2-Z^2} - \sqrt{c^2+\frac{F^2}{2\omega^2}-\frac{\alpha F^2}{4c}}\right)^2}{9c^2+4\left(\sqrt{c^2-Z^2} - \sqrt{c^2+\frac{F^2}{2\omega^2}-\frac{\alpha F^2}{4c}}\right)^2}\right)^{1/2} \times S \times \left(\frac{4Z^3 e}{Fn^{*4}}\right)^{n^*} \times$$

$$\times \mathrm{Exp}\left[-\frac{2Z^3}{3F_G(\rho)n^{*3}} - \frac{4\omega^3 p^2}{F^{3/2}c^2} + \frac{2\left(\sqrt{p^2c^2+c^4}-c^2\right)\gamma_{rel}{}^3}{c^2\omega} - \frac{(\sqrt{p^2c^2+c^4}-c^2)^2\gamma_{rel}}{c^2\omega}\right] \quad (9)$$

From Eq. 9 we concluded that the tunneling rate, $W_L{}^{Corr}$ among other, strongly depends on the field intensity, F the initial momentum, p and effective ionization potential $I_{p,eff}^{rel}$. The minimal change of those parameters strongly affects changes in the tunneling signal.

A common standard for an ideal laser is one with a single Gaussian spatial mode. Because of that, we assumed the Gaussian shaped laser pulse which is the simplest and often the most desirable type of laser beam provided by a laser source which allows the highest concentration of light in the following form (BAUER and MULSER, 1999):

$$F_G(\rho) = F \times \mathrm{Exp}\left[-\frac{t_0^2}{4\sigma^2}\right] \tag{10}$$

where $t_0 = N\pi/\omega$ (N is number of laser cycles, we supposed $N = 1$) and $\sigma = {t_0}^2/(4\ln(20))$ (BAUER and MULSER, 1999). By implementing this dependence into the equation for Lorentz transition rate (Eq. 9) we obtained:

$$W_L{}^{Corr} = \left(1 - \frac{4c^2\left(\sqrt{c^2-Z^2} - \sqrt{c^2+\frac{F_G(\rho)^2}{2\omega^2}} - \frac{\alpha F_G(\rho)^2}{4c}\right)^2}{9c^2+4\left(\sqrt{c^2-Z^2} - \sqrt{c^2+\frac{F_G(\rho)^2}{2\omega^2}} - \frac{\alpha F_G(\rho)^2}{4c}\right)^2}\right)^{1/2} \times S \times \left(\frac{4Z^3 e}{Fn^{*4}}\right)^{n^*} \times$$

$$\times\, \mathrm{Exp}\left[-\frac{2Z^3}{3F_G(\rho)n^{*3}} - \frac{4\omega^3 p^2}{F_G(\rho)^{3/2}c^2} + \frac{2\left(\sqrt{p^2c^2+c^4}-c^2\right){\gamma_{rel}}^3}{c^2\omega} - \frac{(\sqrt{p^2c^2+c^4}-c^2)^2\gamma_{rel}}{c^2\omega}\right] \tag{11}$$

Eq. 11 presents the corrected formula for a Lorentz transition rate. It describes the exponential dependence of the amplitude of the Gaussian shaped laser pulse $F_G(\rho)$, the effective quantum number n^*, as well as a relativistic Keldysh parameter, γ_{rel}. Additional terms, which can be seen in Eq.11, compared to the standard ADK formula (DELONE *et al.*, 1993), are directly related to the contribution of magnetic field component and relativistically corrected ionization potential.

DISCUSSION

In this paper, we performed the analysis of the Lorentz relativistic transition rate of ejected photoelectrons and the influence of the relativistic ponderomotive and the Stark shift on it, in a linearly polarized laser field. We assumed the Gaussian beam profile with included fully corrected ionization potential. The laser field intensity varied within the range $I = 10^{16} - 10^{19}\,\mathrm{Wcm}^{-2}$. According to (LANDSMAN *et al.*, 2014) in this range it can expect the influence of the relativistic effect in photoionization process. We considered the noble, single ionized, $Z = 1$, atoms. In the regime of very low Keldysh parameter $\gamma \ll 1$ and the wavelength of the incident light $\lambda = 800$ nm ($\omega = 0.05696$ a.u.), tunneling is a highly successful concept used to understand the ionization process.

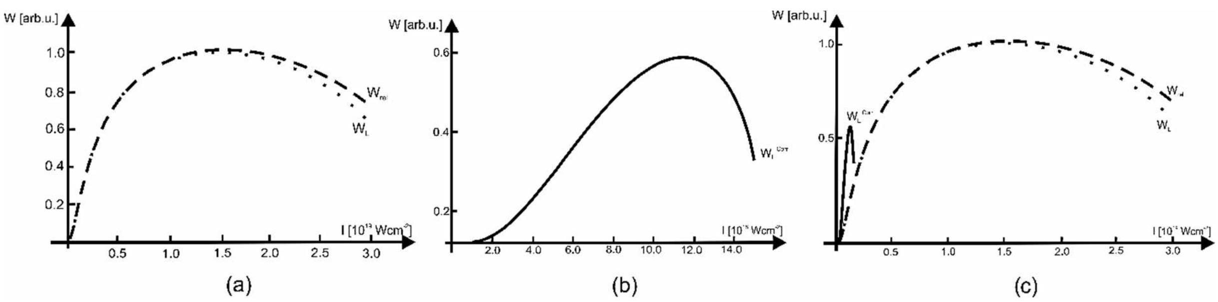

Figure 1. Comparative review of the transition rate of: a) $\boldsymbol{W_{rel}}$ and $\boldsymbol{W_L}$ (the dotted curve represents calculations by the formulas of (DELIBAŠIĆ *et al.*, 2018)), b) $\boldsymbol{W_L{}^{Corr}}$ and c) $\boldsymbol{W_L{}^{Corr}}$, $\boldsymbol{W_L}$ and $\boldsymbol{W_{rel}}$. The laser intensity is within the range: $\boldsymbol{I = 0.5 \times 10^{19} - 3 \times 10^{19}\ \mathrm{Wcm}^{-2}}$.

In Fig 1, we gave comparative review of the theoretical curves, obtained based on relativistic ADK formula, W_{rel} (Eq. 4), uncorrected Lorentz transition rate, W_L (DELIBAŠIĆ *et al.*, 2018) and our corrected formula for the Lorentz transition rate, ${W_L}^{Corr}$ (Eq. 11). The analysis was performed for the argon atom, (Ar).

From Fig. 1 is obvious that curve which representing corrected Lorentz transition rate, ${W_L}^{\mathrm{Corr}}$, differ strongly from the curve without any correction, W_L, and relativistic ADK curve, W_{rel}. One can observe (Fig. 1(a)) that curves W_L and W_{rel} have almost the same "flow". Both curves on some definite intensity range increase exponentially (in this range all curves have the same behavior), reach the maximum and then approach to intensity axis but with different asymptotic slopes. This rate's behavior is in accordance with (DICHIARA *et al.*, 2008). The differences between W_L and W_{rel} appears to be larger for laser field intensities approximately $I \sim 1.2 \times 10^{18}\ \mathrm{Wcm}^{-2}$. For corrected Lorentz transition rate, ${W_L}^{\mathrm{Corr}}$ (Fig. 1(b)), the maximum is shifted to the left, i.e. to the lower field intensity. In addition, shift of the transition rate to the lower value is also obvious (Fig. 1(c)). This shift could be due to the effect of the influence of the incorporated Stark shift. This is in accordance with prediction result in (DELONE and KRAINOV, 1998). Fewer theoretical and experimental results are available for Ar and the other noble gases and our theoretically obtained results can be compared with them. In [30] it is clearly shown that besides the laser pulse intensity and shape, the magnetic component is an important parameter with respect how an ionization process occurs. The laser field intensity of the maximum is comparable to the (DICHIARA *et al.*, 2008) and the shape of the curve (slope) is the same as in (MAJETY and SCRINZI, 2015).

To obtain a more complete analysis, we gave comparative review of the relativistic transition rates with included correction of the ionization potential, for the helium (He), neon (Ne) and argon (Ar) atoms, respectively. We plotted the Lorentz ionization rate ${W_L}^{Corr}$ (Eq. 11) as function of the field intensity, I, (2D graph) and the field intensity, I, and the stabilization factor S (3D graph). Fig. 2 illustrates obtained results.

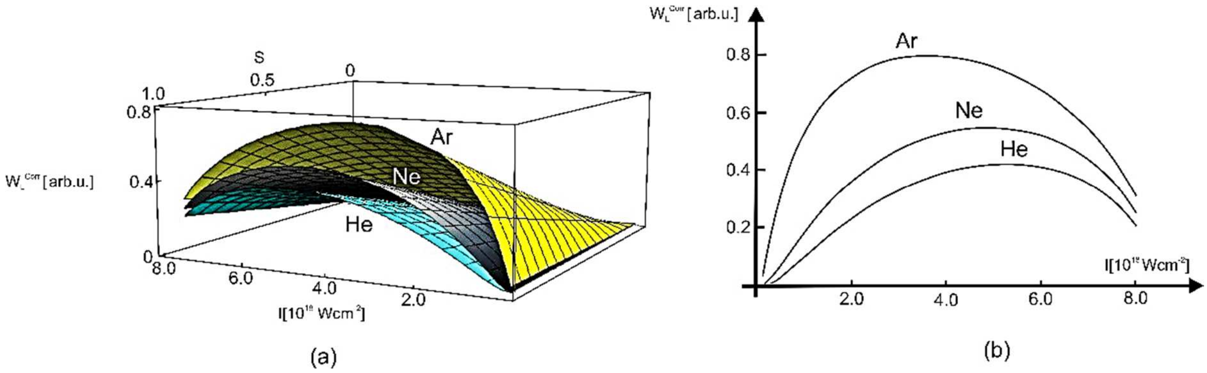

Figure 2. a) 3D graph for $\boldsymbol{W_L}^{\boldsymbol{Corr}}$ as a function of the field intensity, $\boldsymbol{I}$ and the stabilization factor $\boldsymbol{S}$, for noble, single ionized, $\boldsymbol{Z=1}$, $\boldsymbol{Ar}$, $\boldsymbol{Ne}$ and $\boldsymbol{He}$ atoms. The stabilization factor is within the range $\mathbf{0 \le \boldsymbol{S} \le 1}$, b) 2D graph, $\boldsymbol{W_L}^{\boldsymbol{Corr}}$ as a function of the field intensity, $\boldsymbol{I}$. For both graphs intensity varies within the range $\boldsymbol{I} = \mathbf{1 \times 10^{18} - 8 \times 10^{18}\ Wcm^{-2}}$.

Fig. 2(a) (3D) shows how the corrected relativistic Lorentz transition rate, ${W_L}^{\mathrm{Corr}}$, depends on field intensity, I when the stabilization factor S varied in the range $0 \le \mathrm{S} \le 1$, for different noble atoms. It can be seen that ${W_L}^{\mathrm{Corr}}$ decreases going from Ar to He atom. This order is completely expected since the static polarizability has different values for corresponding noble atoms: $\alpha \sim 11$ for Ar atom, $\alpha \sim 2.6$ for Ne atom and $\alpha \sim 1.3$ for He atom (http://ctcp.massey.ac.nz/Tablepol2014.pdf). Because of that it appears natural that the Ar atom ionization curves are more strongly influenced than the He atom ionization curves. Also, there is a shift of the plot's maxima to the lower field intensity, going from Ar to He atom.

Fig. 2(b) (2D) shows how the corrected relativistic Lorentz transition rate, ${W_L}^{\mathrm{Corr}}$, depends on field intensity, I, for fixed value of the stabilization factor, $S = 1$, for different noble atoms. All plotted curves exhibit more or less similar behavior. They do not have prominent peaks, but have maxima, which is for the Ar atom shifted to the lower field intensity. The corresponding maximal value for the Ar atom appears at $I \sim 3{,}6 \times 10^{18}$ Wcm^{-2}, for the Ne atom maximum is achieved at $I \sim 4{,}8 \times 10^{18}$ Wcm^{-2} and in the case of He atom $I \sim 5{,}3 \times 10^{18}$ Wcm^{-2}. After reaching maximal intensities curves decrease and approach the intensity axis.

CONCLUSION

In this paper, we reported our calculations of Lorentz transition rate in frame of the relativistic ADK theory with inclusion of the magnetic component of laser field as well as perturbated ionization potential. The influence of the perturbated ionization potential is caused by the relativistically corrected the ponderomotive and the Stark shift. We performed calculations with supposed the Gaussian spatial laser profile. We found that incorporation of these effects significant influence the transition rate of ejected photoelectrons.

Acknowledgments

This work was supported by the Serbian Ministry of Education, Science and Technological Development for financial support through Projects 171020.

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